Past state of probed quantum systems: ”spooky action” in the past ? Olomouc, December, 2014.

Theory:

”*Past quantum states*”, Phys. Rev. Lett. 111 (2013)

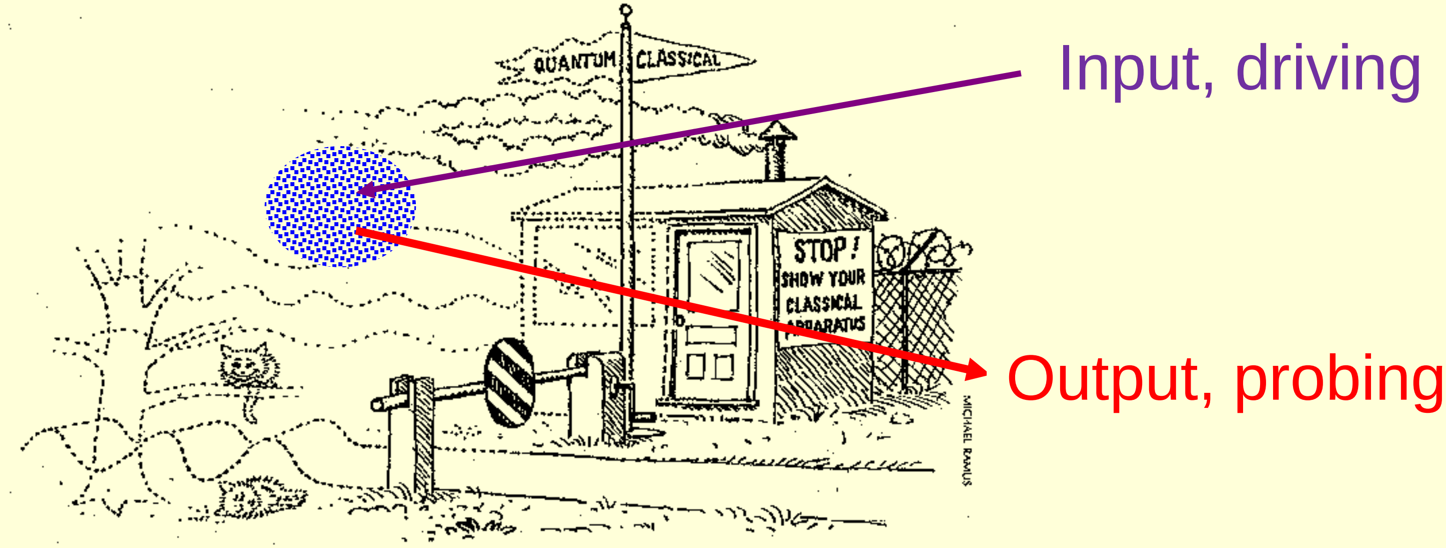
Søren Gammelmark, Brian Julsgaard, Klaus Mølmer

Examples:

Experiments from Bonn and ENS

european social fund in the czech republic
EUROPEAN UNION
MINISTRY OF EDUCATION, YOUTH AND SPORTS
OP Education for Competitiveness
INVESTMENTS IN EDUCATION DEVELOPMENT

Evolution of quantum systems



Measurements on a quantum system imply

- wave function collapse - back action - state reduction

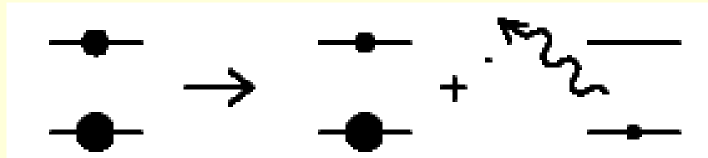
This *conditional* time evolution is

non-unitary, non-linear, non-local,

unpredictable, counter-intuitive,

… indispensable to describe repeated/continuous measurements

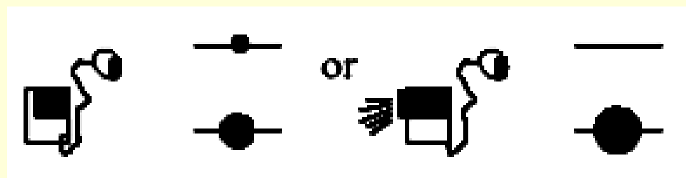
Open quantum systems



Example:

A laser driven atom emits fluorescence photons.

Master equation → damped Rabi oscillations, steady state

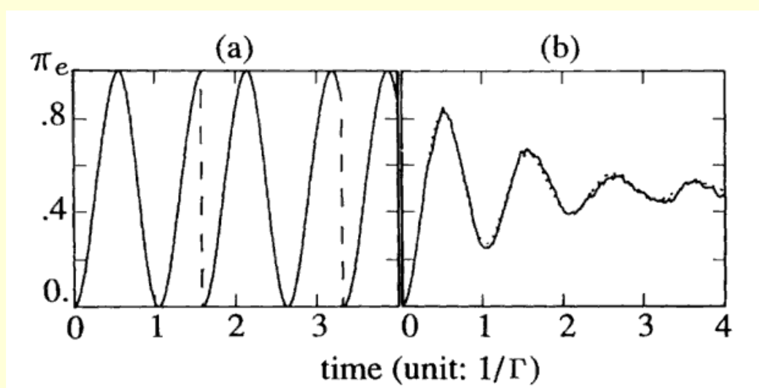


If the emission is *detected*, we *learn* something about the atomic state:

Projection postulate implies that the atom jumps into the ground state

→ Monte Carlo Wave Functions (J. Dalibard, Y. Castin, KM, 1991)

Single trajectory

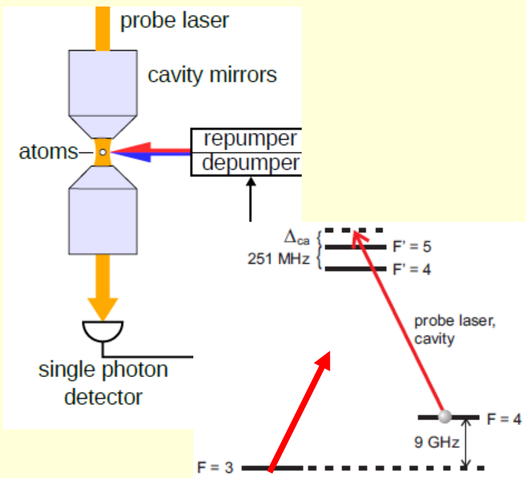


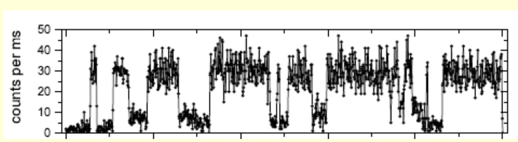
Density matrix &

<100 trajectories>

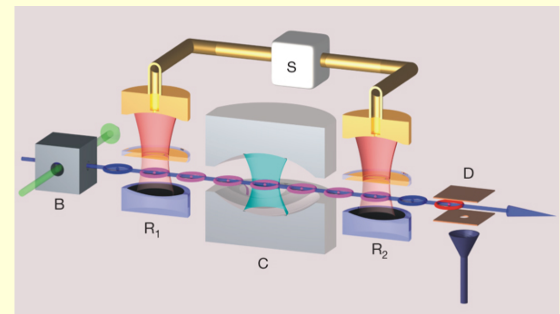
Probed quantum systems: two examples

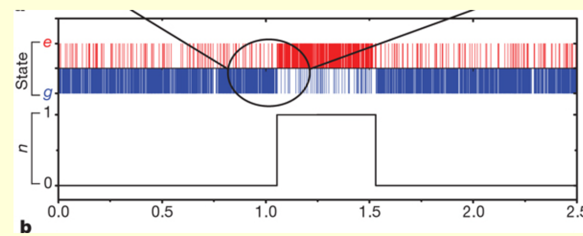
Optical transmission
probing (Bonn):





Atomic transmission
probing (ENS):



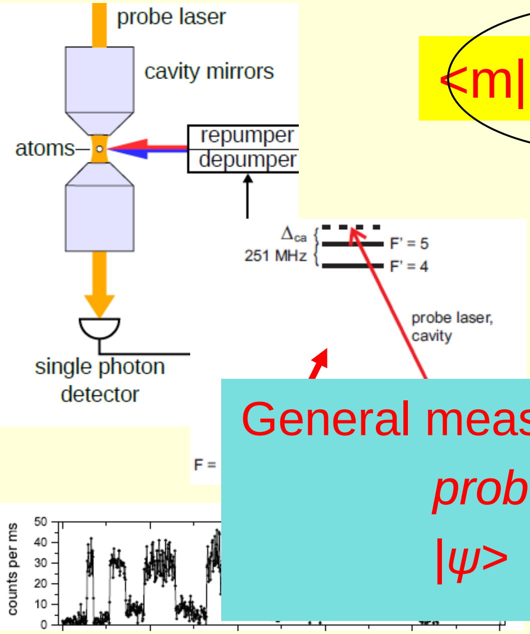


Detection signal → infer quantum state

A ”quantum trajectory”

Probed quantum systems: two examples

Optical transmission
probing (Bonn):



Ω_m

$\langle m|\ U_{S+P}\ |\varphi_P\rangle|\psi\rangle$

Atomic transmission
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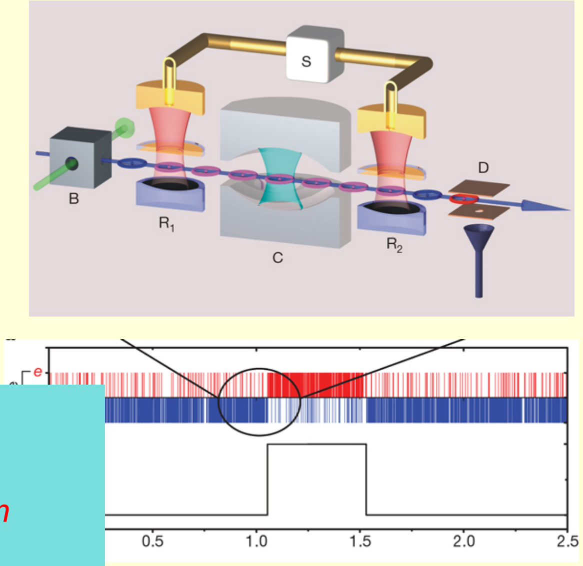


General measurements:

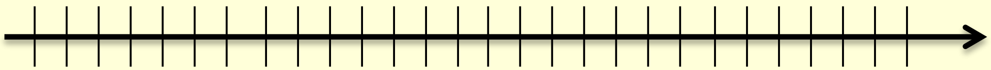
probe outcome m $\rightarrow \Omega_m$

$|\psi\rangle \rightarrow \Omega_m\ |\psi\rangle$

Repeated/continuous measurements

$\psi_c(t\ |m1, m2 \dots mt)$, state conditioned on measurements until time *t*

$$\overbrace{\Omega_{mt}\ \dots\ \Omega_{m2}\, \Omega_{m1}\ |\psi>}$$



time (events or bins)

Stochastic Schrödinger Equation

The quantum state *ψ(t)* or *ρ(t)* depends on measurements until time *t*

"**Can Quantum-Mechanical Description of Physical Reality be Considered Complete?**"
A. Einstein, B Podolsky, N Rosen,
Phys. Rev. **47**, 777-780 (1935)

"$|\psi\rangle \rightarrow \Omega_m\,|\psi\rangle$ implies spooky action at a distance"

"**Can Quantum-Mechanical Description of Physical Reality be Considered Complete?**"
N. Bohr, Phys. Rev. **48**, 696-702 (1935)

" …not a mechanical influence …
… an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system."

An influence on ρ is an influence on

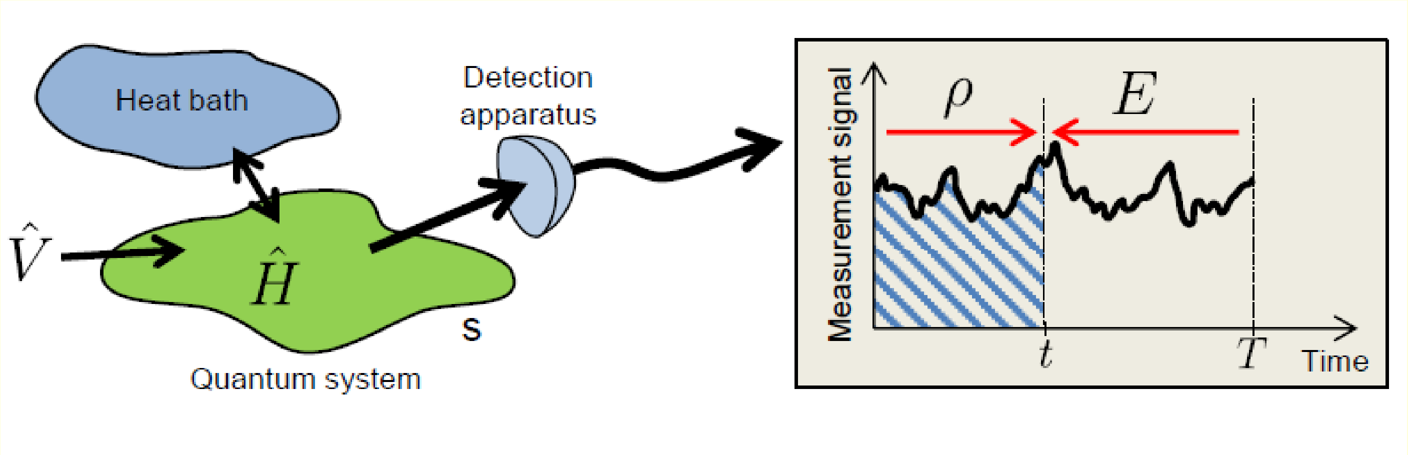
" … the very conditions which define
the possible types of predictions regarding
the future behavior of the system."

$$p(m) = \mathrm{Tr}(\hat{\Omega}_m \rho(t) \hat{\Omega}_m^\dagger) / \mathrm{Tr}(\rho(t))$$

Ill. Sidse Damgaard Hansen

Søren Kierkegaard
1813-1855

“Life can only be understood backwards; but it must be lived forwards."



Do I, at time *T*, know more about the *past state* at time *t*, than I already did at that time *t* ?

Do measurements cause ”spooky action in the past” ?

By the (past) quantum state, I will refer to …

” … the very conditions which define
the possible types of predictions regarding
the ~~future~~ behavior of the system.”

”the state” = our description of the state = our ”knowledge”

How are these ”conditions” determined and represented ?

How do we verify predictions about the past ?

For what purposes may past knowledge be applied ?

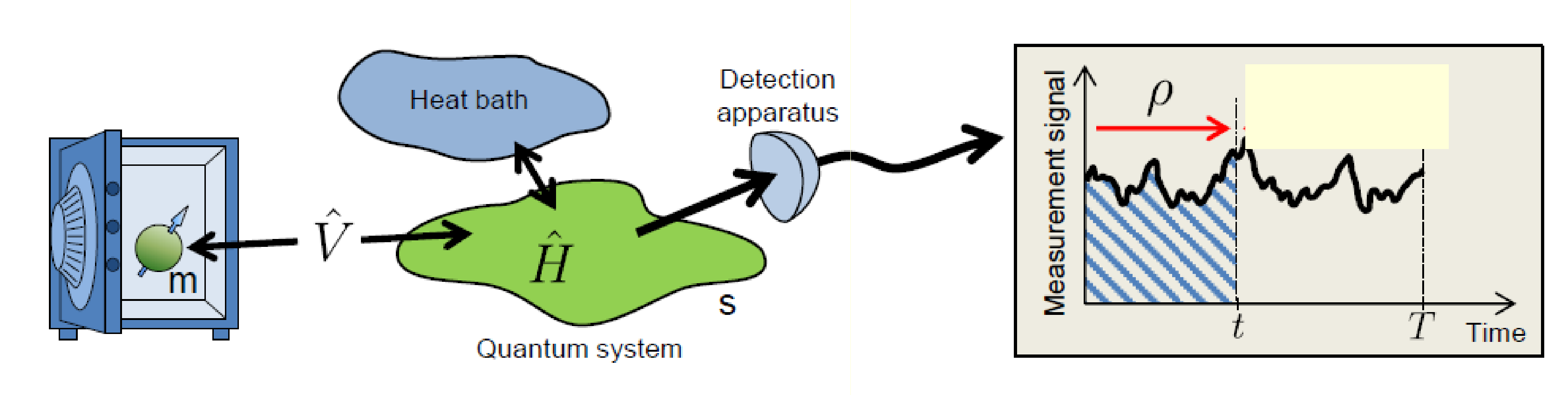
Classical example: ”Where are my car keys ?”

Having found the keys, at time T,
I know precisely where they were at time t !

Non trivial prediction/verification:
where did my wife see our car keys at time t ?

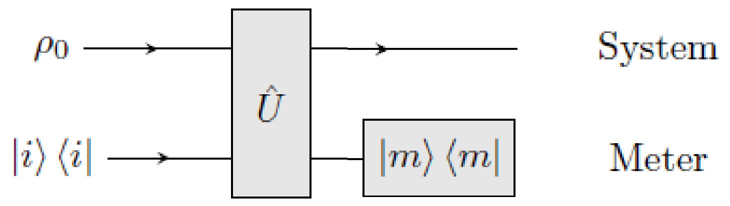
Now, replace ”keys” by”cat” → non-trivial dynamics !

Past *quantum* state - definition



Any - strong or weak - measurement of any observable, can be implemented by coupling to - and projective read-out of - a meter system.

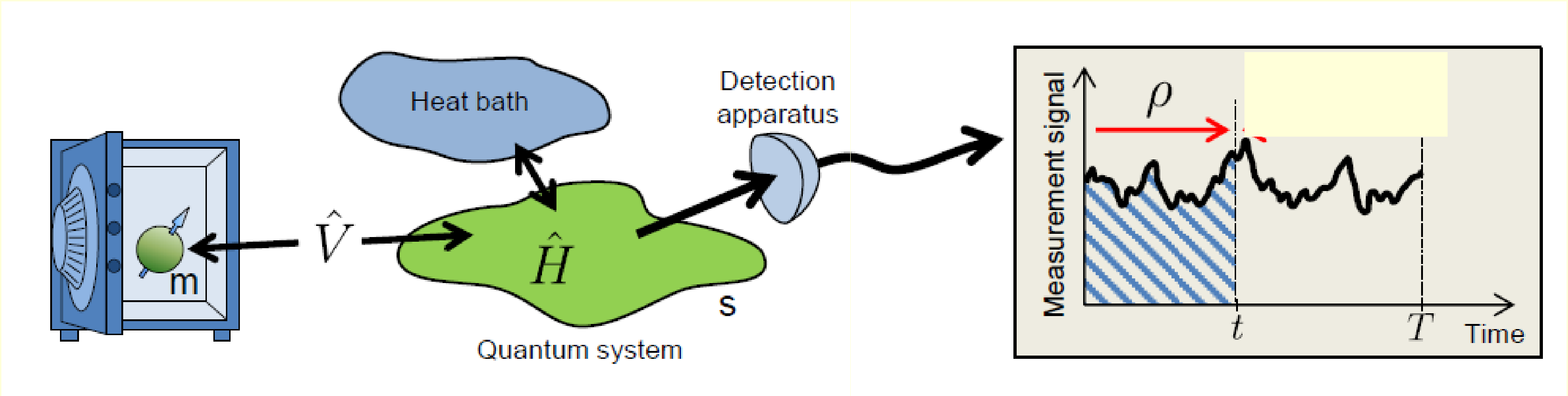
time t



$p(m)$

$$p(m) = \mathrm{Tr}(\hat{\Omega}_m \rho(t) \hat{\Omega}_m^\dagger)/\mathrm{Tr}(\rho(t))$$

Past *quantum* state - definition



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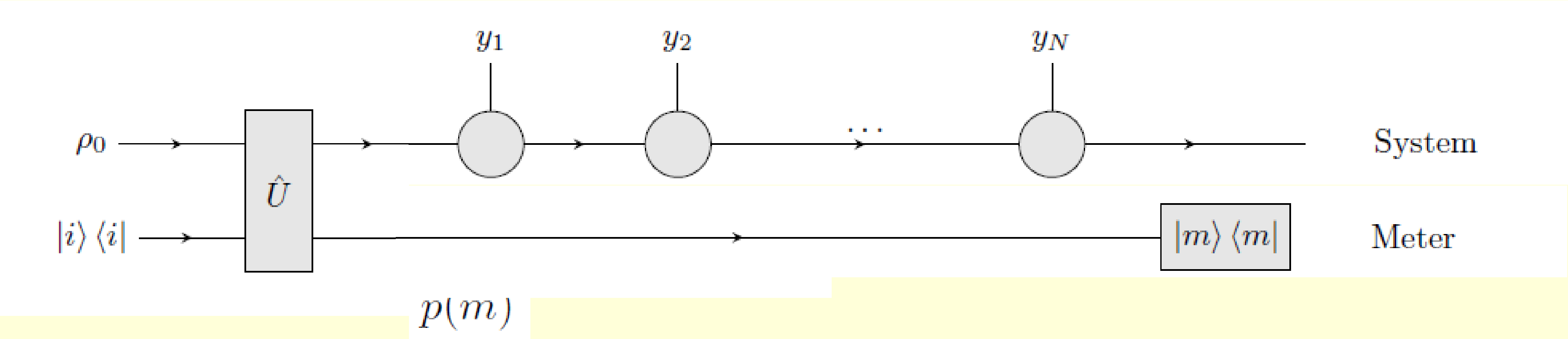
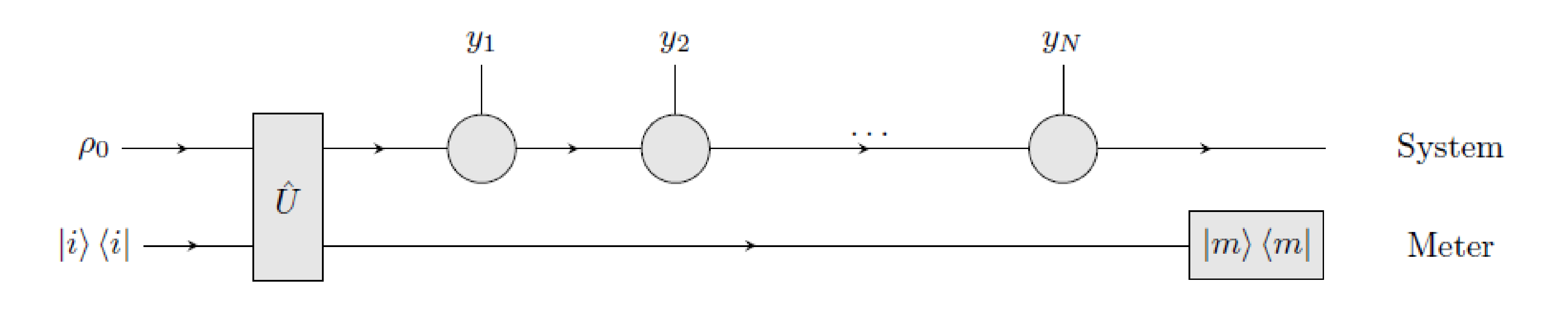


Past *quantum* state – heuristic derivation



$|m\rangle\langle m|\, U(|i\rangle|\psi\rangle)$ $\qquad$ $\mathrm{Tr}\big(\, |m\rangle\langle m|\, U(\rho\, |i\rangle\langle i|)U^{+}\, |m\rangle\langle m| \,\big)$

$$\mathrm{Tr}\big(\, (|m\rangle\langle m|) M_N \ldots M_2 M_1\, U(\rho\, |i\rangle\langle i|)U^{+} M_1^{+} M_2^{+} \ldots M_N^{+} (|m\rangle\langle m|) \,\big)$$

$$= \mathrm{Tr}\big(\, M_N \ldots M_2 M_1 (|m\rangle\langle m|)\, U(\rho\, |i\rangle\langle i|)U^{+}\, (|m\rangle\langle m|)\, M_1^{+} M_2^{+} \ldots M_N^{+} \,\big)$$

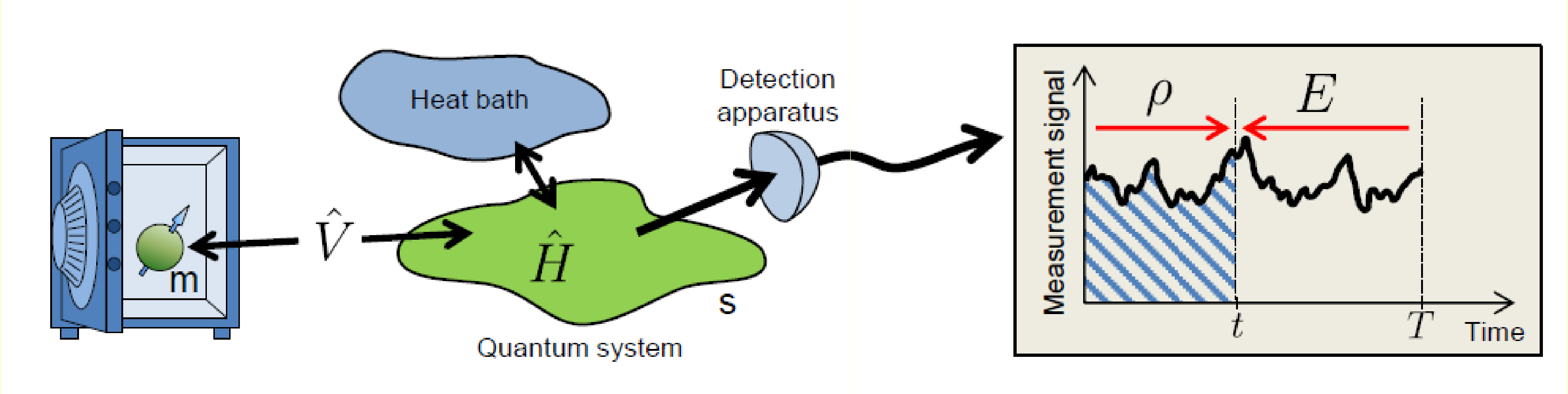
$$= \mathrm{Tr}\big(\, (|m\rangle\langle m|)\, U(\rho\, |i\rangle\langle i|)U^{+}\, (|m\rangle\langle m|)\, M_1^{+} M_2^{+} \ldots M_N^{+} M_N \ldots M_2 M_1 \,\big)$$

$$= \mathrm{Tr}\big(\, \Omega_m\, \rho\, \Omega_m^{+}\, E \,\big)$$

I

E(t) solves adjoint SME

Past *quantum* state - consistent definition



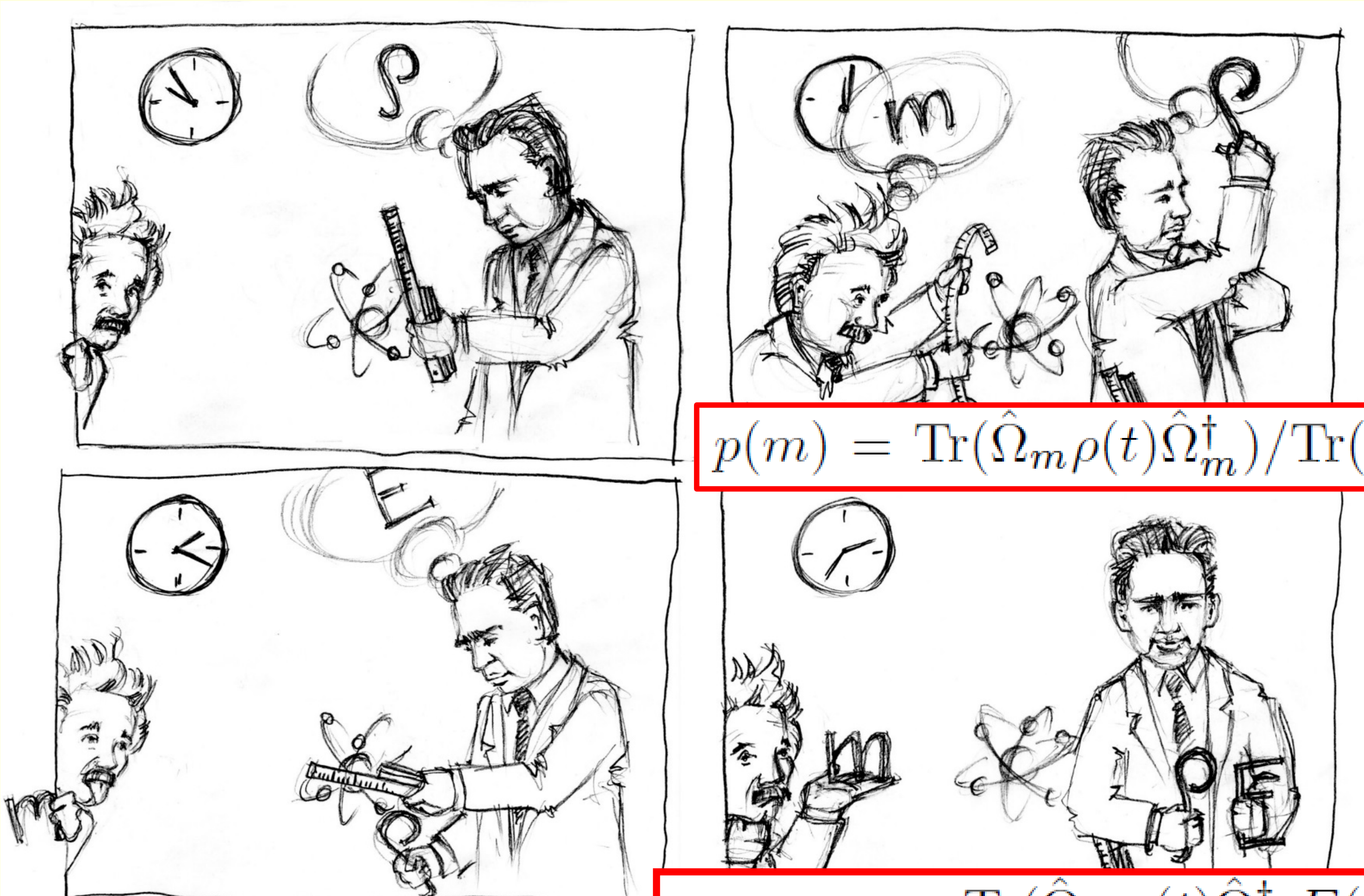
$$\Xi(t) = (\rho(t),\ E(t))$$

ρ(t) solution to SME

E(t) solution to adjoint SME

$$p(m) = \mathrm{Tr}(\hat{\Omega}_m \rho(t) \hat{\Omega}_m^\dagger)/\mathrm{Tr}(\rho(t))$$

$$p_{\mathrm{p}}(m) = \frac{\mathrm{Tr}(\hat{\Omega}_m \rho(t) \hat{\Omega}_m^\dagger E(t))}{\sum_m \mathrm{Tr}(\hat{\Omega}_m \rho(t) \hat{\Omega}_m^\dagger E(t))}$$



Ill. Sidse Damgaard Hansen

$$p(m) = \mathrm{Tr}(\hat{\Omega}_m \rho(t) \hat{\Omega}_m^\dagger)/\mathrm{Tr}(\rho(t))$$

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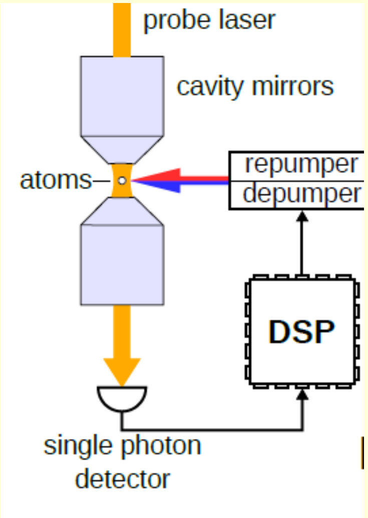
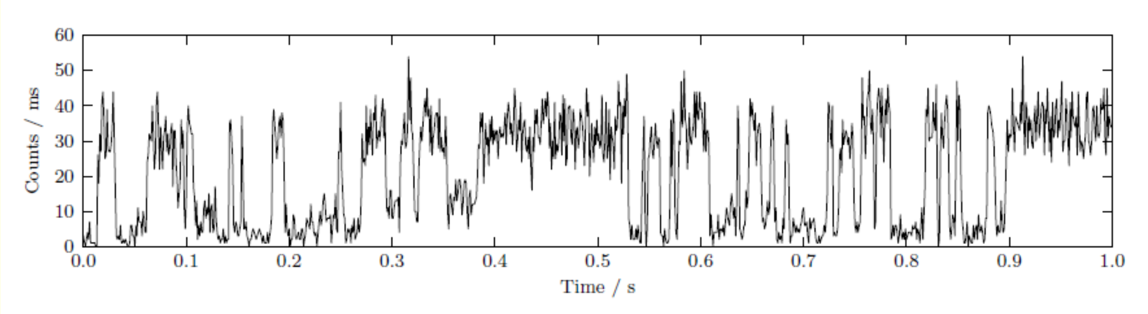
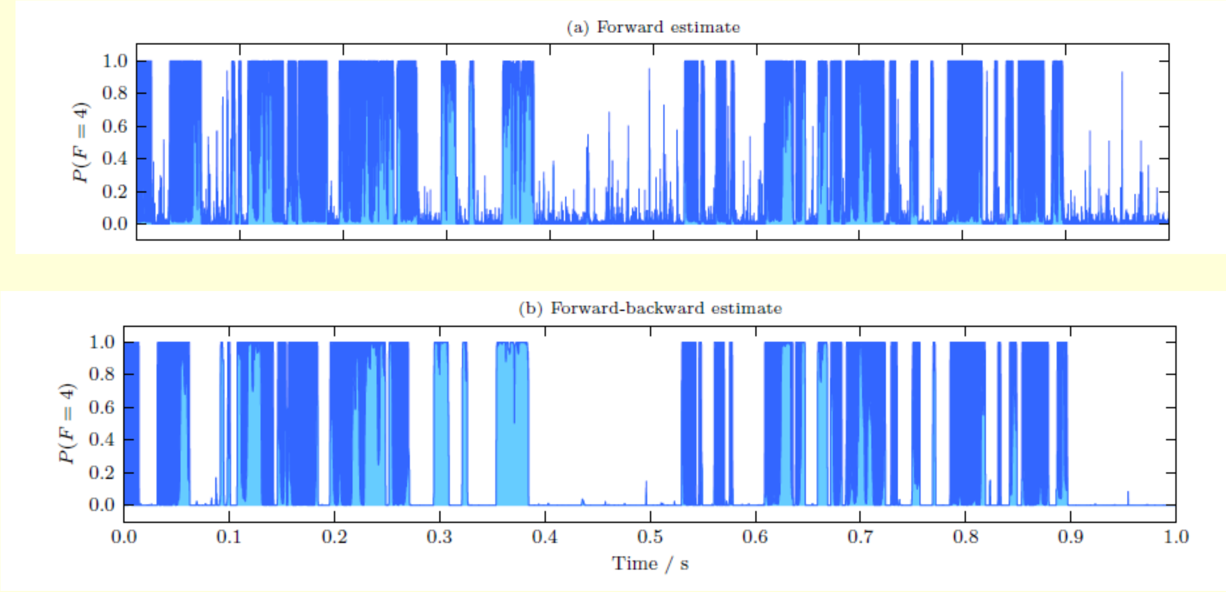
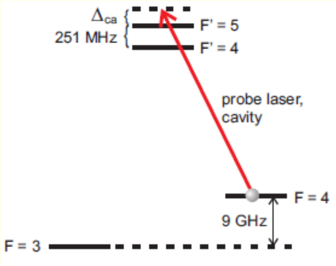
Analysis of Bonn experiments (Meschede)





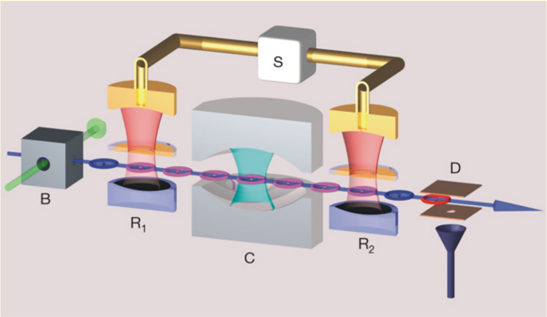




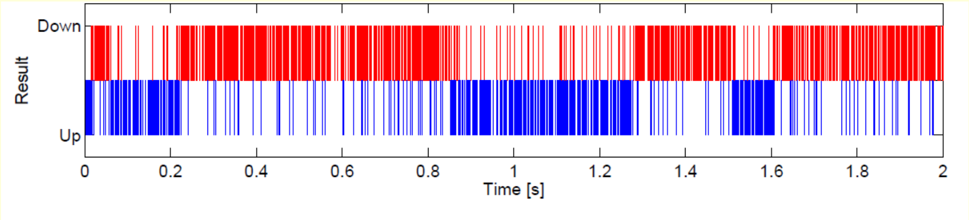
Two effects: ”improve signal-to-noise”

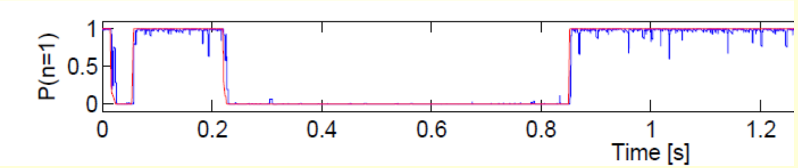
”do not overreact on spikes”

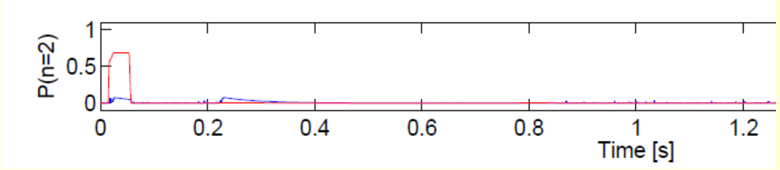
Analysis of a simulated ENS experiment



Simulated field dynamics and atom detection







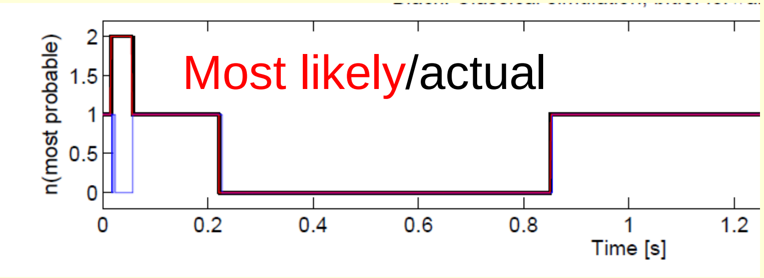
Usual Bayes:

"If the photon number is odd, it is most likely 1."

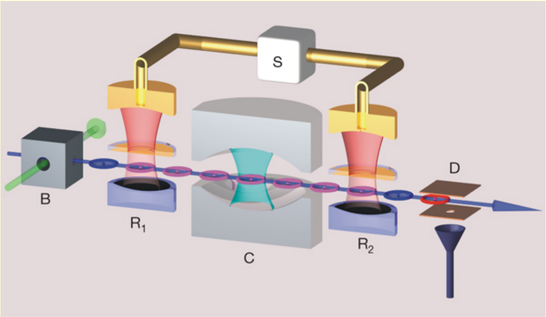
"If the photon number is even, it is most likely 0."

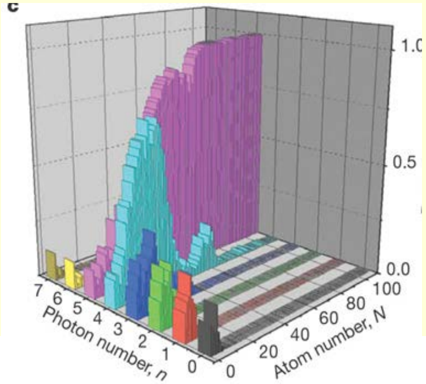
In Hindsight:

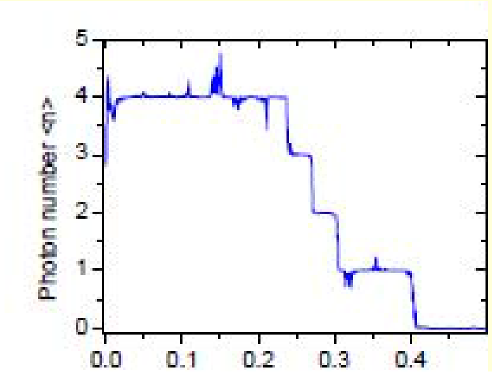
"If the photon number is even for only a very short time, it is probably 2 rather than 0."

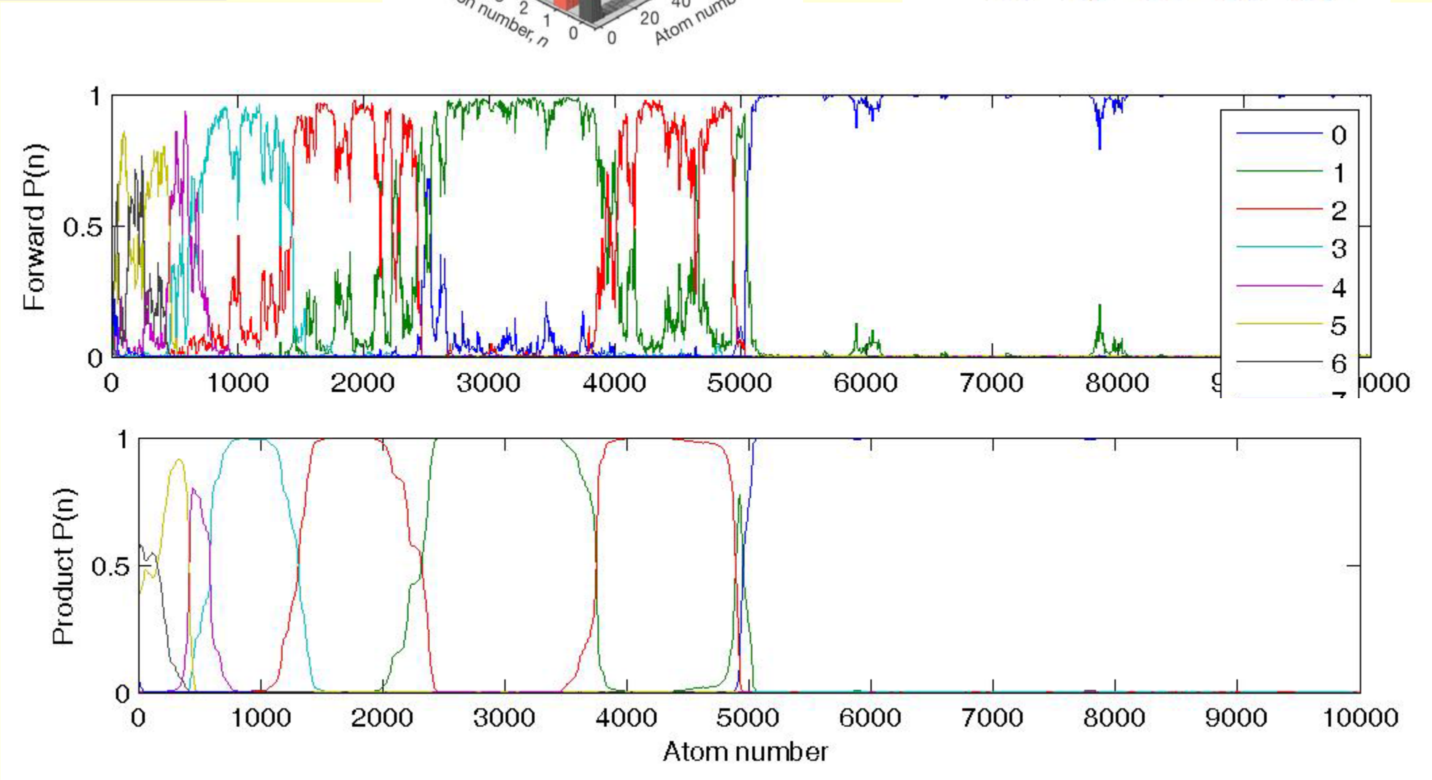


Analysis of a *real* ENS experiment (I. Dotsenko)









Summary

The state of a quantum system is conditioned on the outcome of probing measurements.

States in the past are (now) conditioned on measurements until the present – the past quantum state.

Natural quantum extension of classical Bayes/HMM theory.

Natural generalization and extension of Aharonov and Vaidmans ideas of ”weak value measurements” with pre- and post selection.

Past states make more accurate predictions, e.g., for:

state assignment

guessing games

counterfactual paradoxes

parameter estimation and metrology

… and for publication !!!

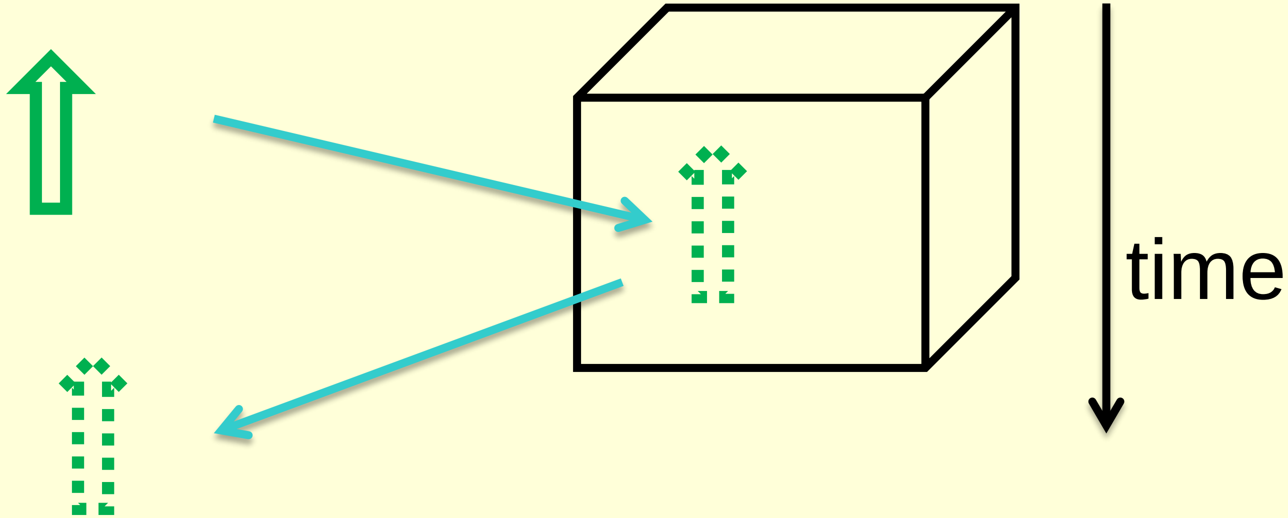
I hope you will be looking backward to this talk ;-)

A trivial example

Spin ½ particle

Measure S_x : m_x

Measure S_y : m_y



I can tell you both the value of S_x and S_y

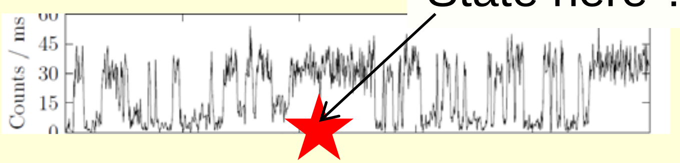
if you measured S_x, you got m_x

if you measured S_y, you got m_y

Past states: classical case

An exercise in Bayesian reasoning,

hidden Markov models.



actual data

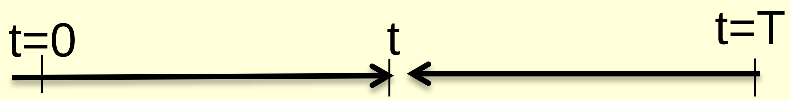
$$P(X_t = i | s_1, \dots s_N) = \frac{P(X_t = i, s_1, \dots s_N)}{P(s_1, \dots s_N)}$$

$$P(s_1, \dots s_t, X_t = i) \; P(s_{t+1}, \dots s_N | X_t = i)$$

$$P(X_t | s_1, \dots s_t)$$

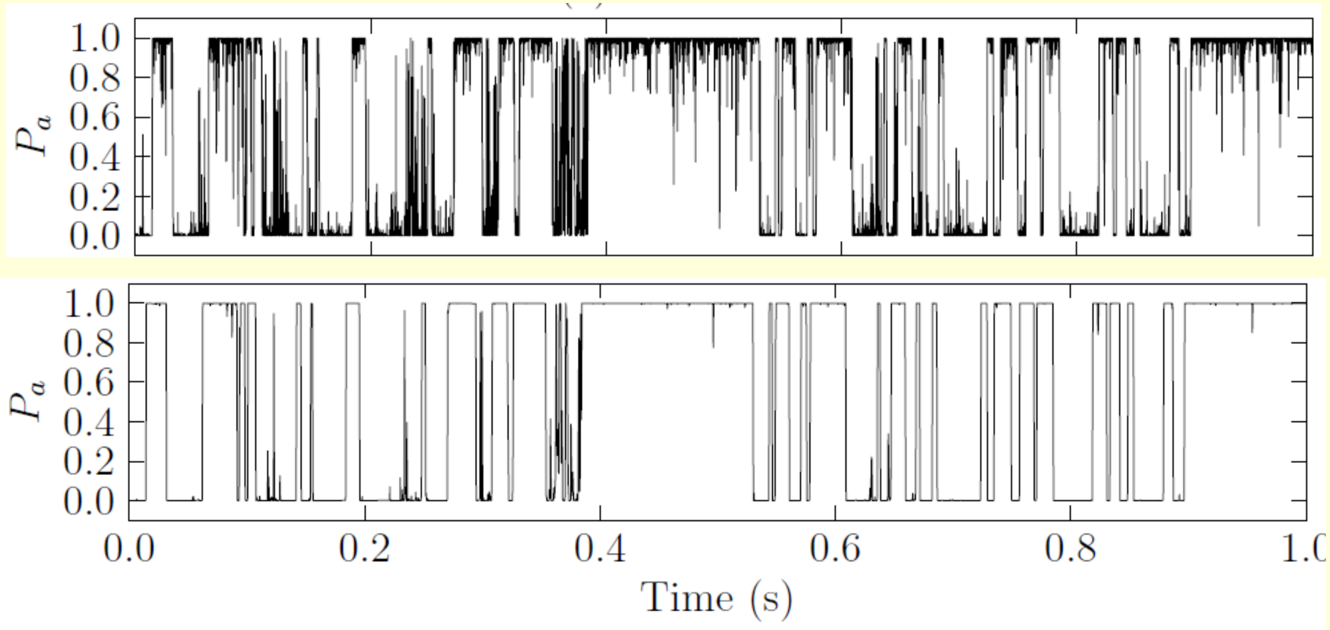
"hindsight-factor"

Bayes



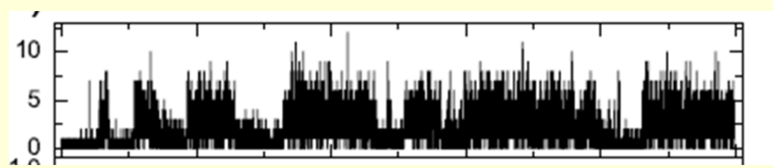
W. H. Press, S. A. Teukolsky, W. T. Vetterling, B. P. Flannery, and S. A. Teukovsky, *Numerical recipes: the art of scientific computing*, 3rd ed. (Cambridge University Press, 2007) p. 1235.

Past quantum states and parameter re-estimation



Better state estimate

→Better estimate of transition rates



→ Better estimate of signal rates (Baum-Welsch)