



INVESTMENTS IN EDUCATION DEVELOPMENT

## Scientific stay

Universidad de Malaga (20.4.-27.4.)

J. Konecny

# Research Visit – Jan Konečný – Malaga, Spain

## Universidad de Malaga

- established 1972, almost 40 000 students, 2 000 researchers,
- new campus 1000 m<sup>2</sup>
- 23 schools (50 departments)

## Matemática Aplicada

- 30 researchers
- various topics
- multiadjoint framework, functional dependencies, ...

# A recap of our previous work

J. Medina and M. Ojeda-Aciego.

On multi-adjoint concept lattices based on heterogeneous conjunctors.

*Fuzzy Sets and Systems*, 2012.

- wide variety of concept-forming operators

R. Belohlavek and V. Vychodil.

Formal concept analysis and linguistic hedges.

*Int. J. General Systems*, 41(5):503–532, 2012.

- parametric way to reduce size of (fuzzy) concept lattice

We studied

- What is  $\square \vee \blacksquare$ ?
- What is  $\square \wedge \blacksquare$ ?

## Main Topic: implementation of S. Kuznetsov's idea

... as we discussed it after the talk of Jan Konecny, it would be very interesting to relate your approach with hedges to standard “projections” of contexts (by taking subsets of attributes, as described in Ganter & Wille) and pattern structures, as described in

- B. Ganter and S.O. Kuznetsov, Pattern Structures and Their Projections. In: Proc. G. Stumme and H. Delugach, Eds., 9th International Conference on Conceptual Structures (ICCS 2001), Lecture Notes in Artificial Intelligence (Springer), Vol. 2120, pp. 129-142, 2001
- Sergei O. Kuznetsov, Pattern Structures for Analyzing Complex Data. In: H.Sakai et al., Eds., Proc. 12th International Conference on Rough Sets, Fuzzy Sets, Data Mining and Granular Computing (RSFDGrC 2009), Lecture Notes in Artificial Intelligence (Springer), Vol. 5908, pp. 33-44, 2009.

# Pattern structures

$G$  - set of objects,  $\langle D, \wedge \rangle$  be meet-semilattice

$\delta : G \rightarrow D$  be a mapping.

$\langle G, \underline{D}, \delta \rangle$  with  $\underline{D} = \langle D, \wedge \rangle$  is called **pattern structure** provided that the set

$$\delta(G) := \{\delta(g) \mid g \in G\}.$$

generates a complete subsemilattice of  $\langle D, \wedge \rangle$ .

Derivation operators:

$$A^\square = \bigwedge_{g \in A} \delta(g) \quad \text{for } A \subseteq G$$

$$d^\square = \{g \in G \mid d \leq \delta(g)\} \quad \text{for } d \subseteq D$$

(as in FCA) their fixpoints are **concepts**.

# Multi-adjoint framework

## Definition

A multi-adjoint frame is a tuple  $(L_1, L_2, P, \&_1, \swarrow_1, \nwarrow_1, \dots, \&_n, \swarrow_n, \nwarrow_n)$  where  $L_i$ s are complete lattices and  $P$  is a poset, such that  $(\&_i, \swarrow_i, \nwarrow_i)$  is an adjoint triple with respect to  $L_1, L_2, P$  for all  $i = 1, \dots, n$ .

## Definition

Let  $(L_1, L_2, P, \&_1, \dots, \&_n)$  be a multi-adjoint frame, a *multi-adjoint context* is a tuple  $(A, B, R, \sigma)$  such that  $A$  and  $B$  are non-empty sets (usually interpreted as attributes and objects, respectively),  $R$  is a  $P$ -fuzzy relation  $R: A \times B \rightarrow P$  and  $\sigma: B \rightarrow \{1, \dots, n\}$  is a mapping which associates any element in  $B$  with some particular adjoint triple in the frame.

Given a complete lattice  $(L, \preceq)$  such that  $L_1$  and  $L_2$  are  $L$ -connected, a multi-adjoint frame  $(L_1, L_2, P, \&_1, \dots, \&_n)$ , and a context  $(A, B, R, \sigma)$ , we can define the mappings  $\uparrow^{c\sigma} : L^B \rightarrow L^A$  and  $\downarrow^{c\sigma} : L^A \rightarrow L^B$  defined for all  $g \in L^B$  and  $f \in L^A$  as follows:

$$g^{\uparrow^{c\sigma}}(a) = \psi_1(\inf\{R(a, b) \swarrow^{\sigma(b)} \phi_2(g(b)) \mid b \in B\}) \quad (1)$$

$$f^{\downarrow^{c\sigma}}(b) = \psi_2(\inf\{R(a, b) \nwarrow_{\sigma(b)} \phi_1(f(a)) \mid a \in A\}) \quad (2)$$

# The trick

basic idea: have one attribute in sense of FCA, with  $D = \mathbf{P}_2$   
plus use a special hedge \*

selection of \* leads to various projections in pattern structure

The basic idea naturally leads to:

- extension to multiple attributes
- fuzzy setting

# Conclusions of the discussion

- application of the idea is possible,
- with particular changes of the framework

