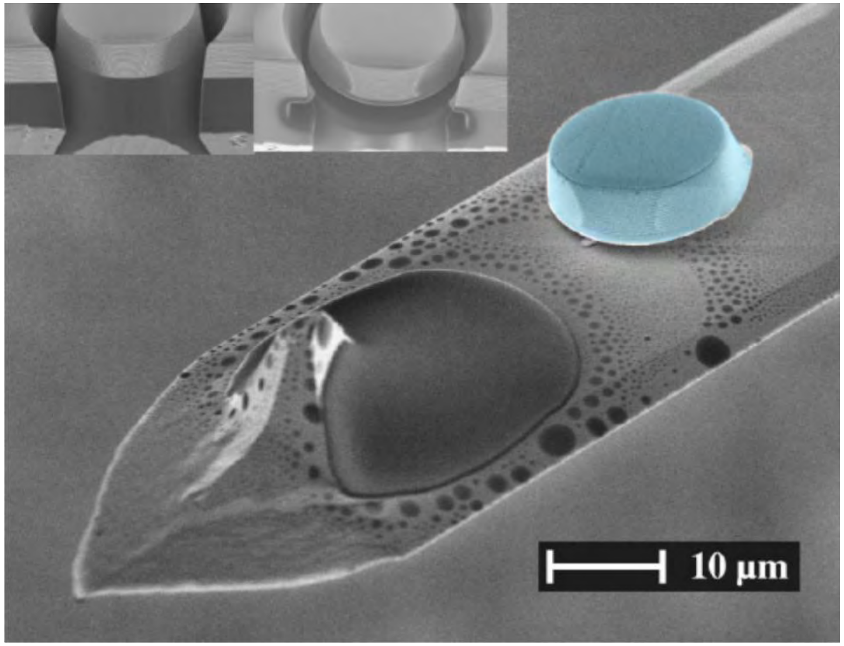
Bell inequality violation with EPR states in optomechanics

Klemens Hammerer

Leibniz University Hannover
Institute for Theoretical Physics
Institute for Gravitational Physics (Albert-Einstein-Institute)

Olomouc – Sep 2015

Optomechanical systems



D. Bowmeester, Santa Barbara/Leiden

LIGO – Laser Interferometer Gravitational Wave Observatory

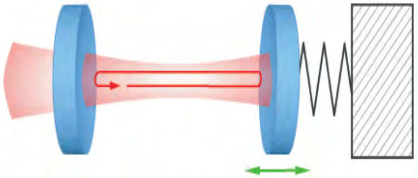
$$\omega_m \simeq \mathrm{Hz} \ldots 10\,\mathrm{GHz}$$

$$m_{eff} \simeq 1\,\mathrm{pg} \ldots 1\,\mathrm{kg}$$

$$Q = \frac{\omega_m}{\gamma} \quad \text{up to} \quad 10^7$$

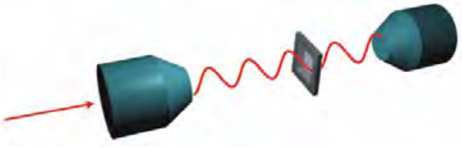
Optomechanical systems

Micromirrors

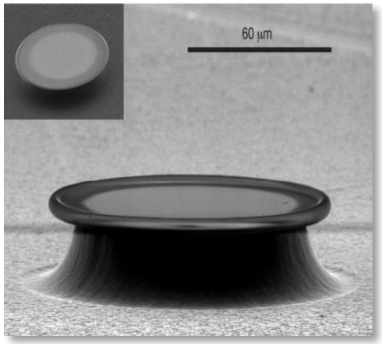


Aspelmeyer (Vienna)
Heidmann (Paris)
Bouwmeester (St Barbara, Leiden)
…

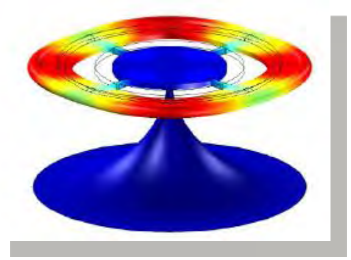
Micromembranes



Harris (Yale)
Kimble (Caltech)
Vengalattore (Cornell)
Treutlein (Basel)
Polzik (Copenhagen)
…

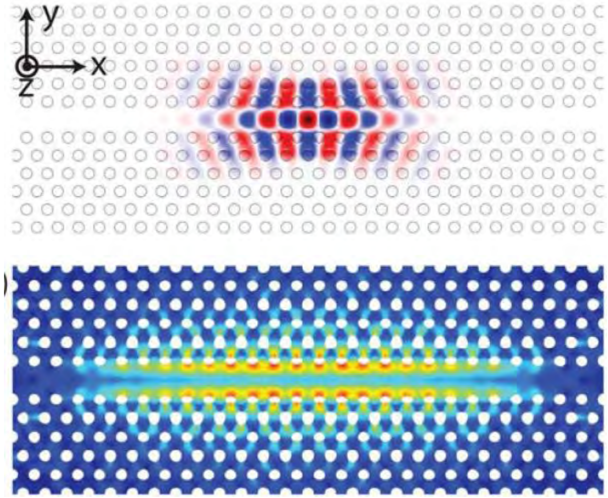


Microtoroids



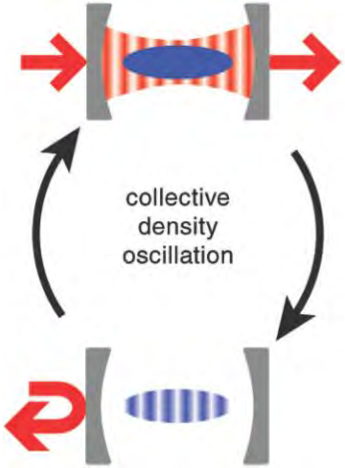
Kippenberg (MPQ)
Weig (LMU)
Vahala (Caltech)
Bowen (UQ)
…

Optomechanical systems



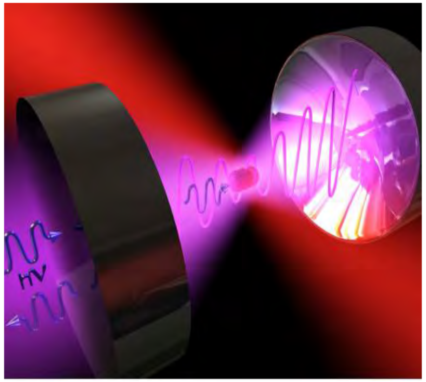
Optomechanical Crystals

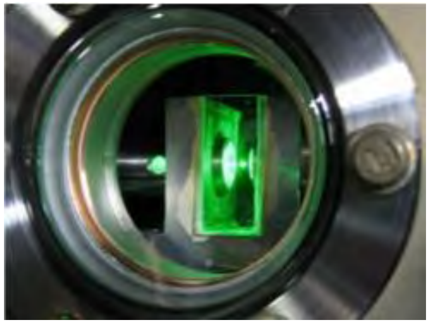
Painter (Caltech)
Tang (Yale)
Pernice (KIST)
Groeblacher (Delft)
…



Optomechanics with BEC

Esslinger (Caltech)
Stamper-Kurn (Berkeley)
…

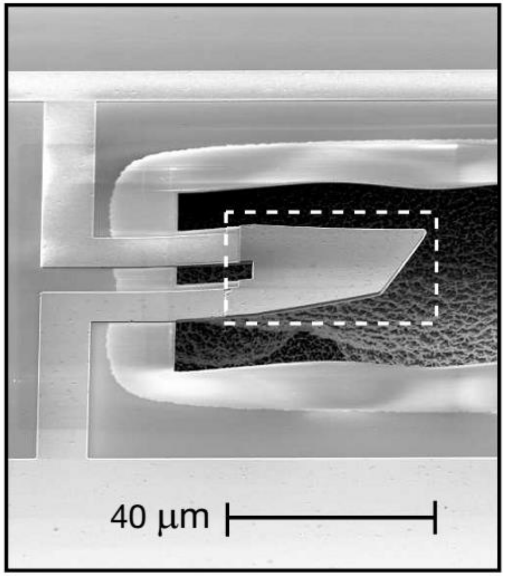


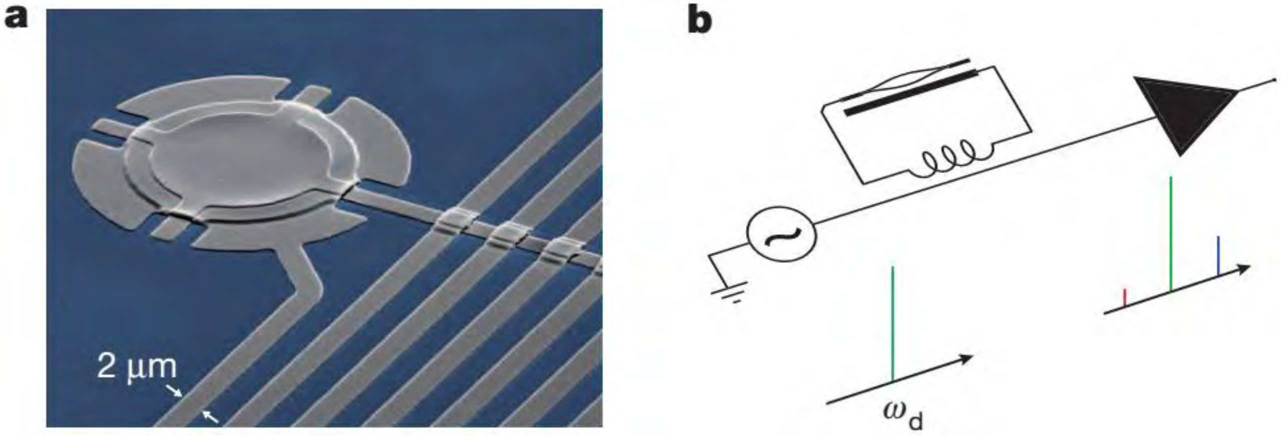


Levitated Nanoobjects

Aspelmeyer/Arndt (Wien)
Raizen (Austin)
Novotny (ICFO, ETH)
…

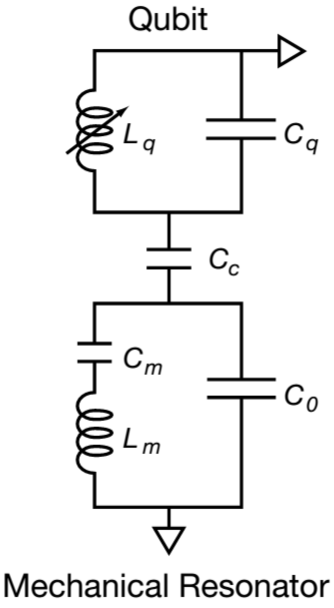
Electromechanics





Cleland
Lehnert
Teufel
Schwab
…

Hybrid system:



O'Connell et al Nature 464, 697 (2010).

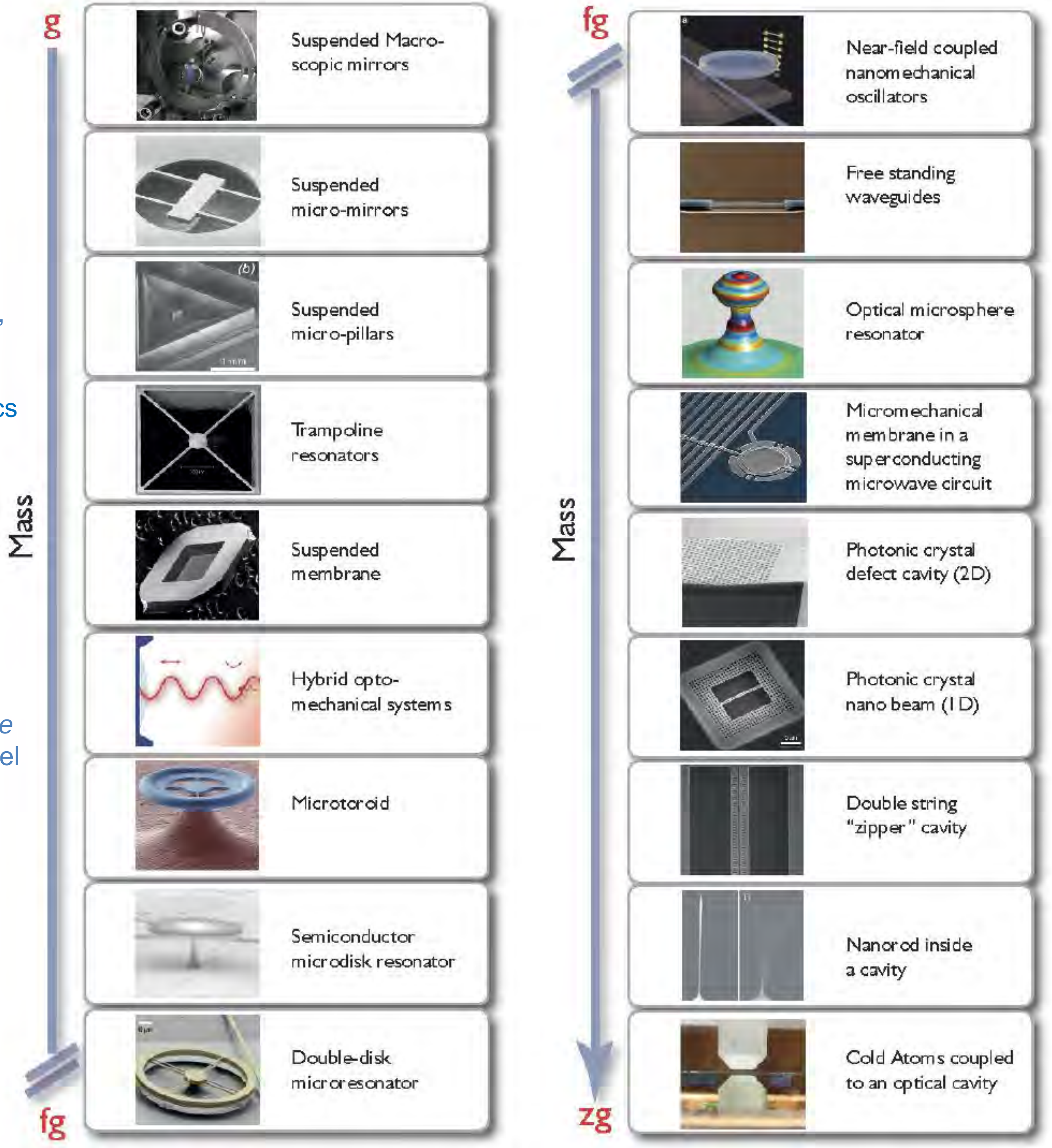
Mechanical Systems Coupled to Light

Quantum Optomechanics
M. Aspelmeyer, F. Marquardt, T. Kippenberg,
RMP, arXiv:1303.0733

A short walk through quantum optomechanics
P. Meystre
Annalen der Physik 525, 215 (2013).

Macroscopic Quantum Mechanics:
Theory and Experimental Concepts
of Optomechanics
Y. Chen
J Phys. B 46 104001 (2013)

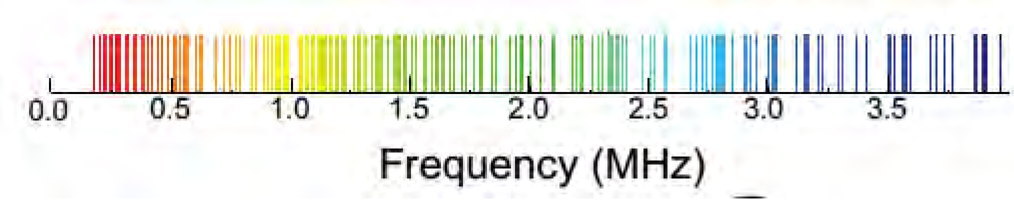
Quantum Optomechanics - throwing a glance
M. Aspelmeyer, S. Groeblacher, KH, N. Kiesel
JOSA B 27, A189 (2010)

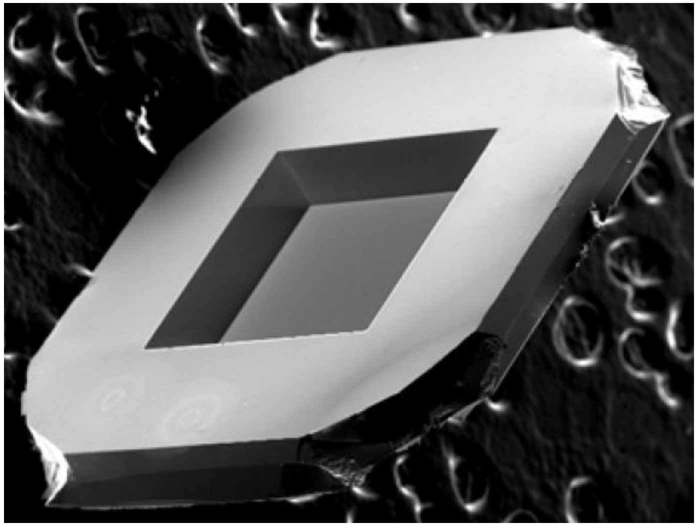


Mechanical oscillators

Small vibrations described by:

ω_n



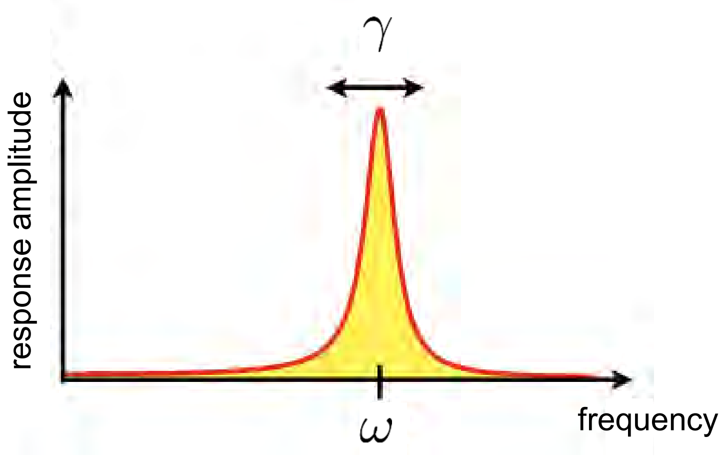


Eigenmodes $\vec{u}_n(\vec{r})$

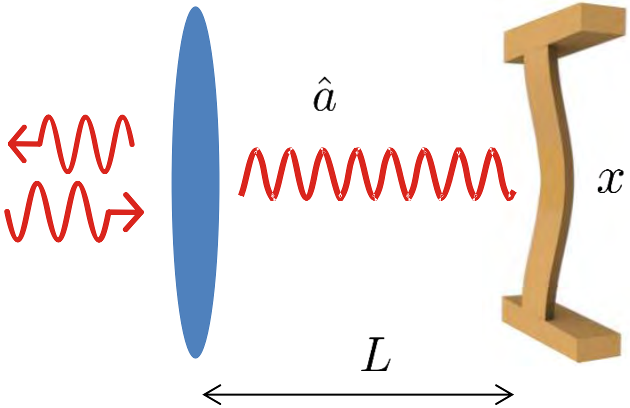
Displacement field $\vec{u}(\vec{r}) = \sum_n x_n(t)\vec{u}_n(\vec{r})$

Each eigenmode is a harmonic oscillator

$$\underset{\underset{\text{effective mass}}{\uparrow}}{m}\frac{d^2x}{dt^2} = -m\omega^2 x - m\underset{\underset{\text{damping}}{\uparrow}}{\gamma}\frac{dx}{dt} + \underset{\underset{\text{external force}}{\uparrow}}{F(t)}$$



Optomechanical systems



$$x = x_{\mathrm{ZPF}}\, x_m$$

$$x_m = \hat{b} + \hat{b}^\dagger$$

$$x_{\mathrm{ZPF}} = \sqrt{\frac{\hbar}{2m\omega_{\mathrm{mec}}}}$$

$$H = \hbar\omega_{\mathrm{cav}}(x)\hat{a}^\dagger\hat{a} + \hbar\omega_{\mathrm{mec}}\hat{b}^\dagger\hat{b} + \ldots$$

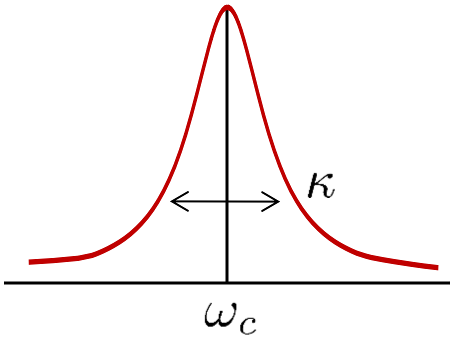
$$\omega_{\mathrm{cav}}(x) = \omega_{\mathrm{cav}} + \frac{\partial\omega_{\mathrm{cav}}}{\partial x}x \quad = \omega_{\mathrm{cav}} - \omega_{\mathrm{cav}}\frac{x}{L}$$

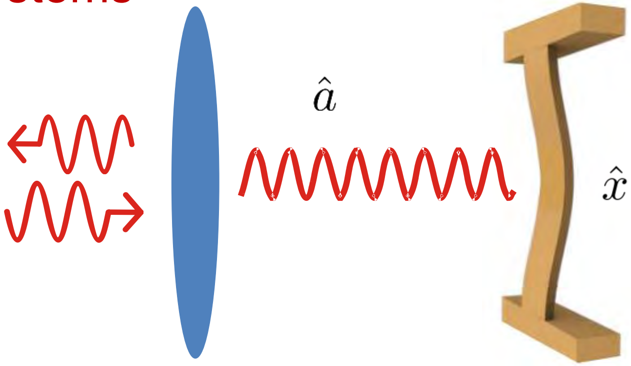
$$H = \hbar\omega_{\mathrm{cav}}\hat{a}^\dagger\hat{a} + \hbar\omega_{\mathrm{mec}}\hat{b}^\dagger\hat{b} - \hbar g_0 x_m \hat{a}^\dagger\hat{a} + \ldots$$

radiation pressure interaction

$$g_0 = \omega_{\mathrm{cav}}\frac{x_{\mathrm{ZPF}}}{L}$$

Optomechanical systems



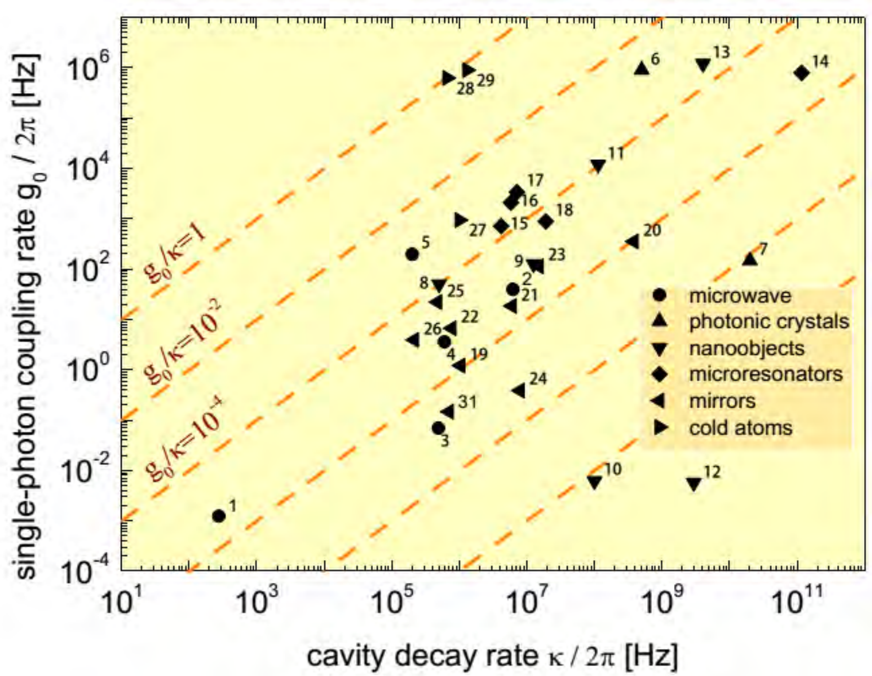


$$\hat{x} = x_{\mathrm{ZPF}} \left(\hat{b} + \hat{b}^\dagger \right)$$

$$x_{\mathrm{ZPF}} = \sqrt{\frac{\hbar}{2m\omega_{\mathrm{mec}}}}$$

$$H = \hbar\omega_{\mathrm{cav}}\hat{a}^\dagger\hat{a} + \hbar\omega_{\mathrm{mec}}\hat{b}^\dagger\hat{b} - \underline{\hbar g_0 \left(\hat{b} + \hat{b}^\dagger \right) \hat{a}^\dagger\hat{a}} + \dots$$

radiation pressure interaction



M. Aspelmeyer, F. Marquardt, T. Kippenberg, RMP, arXiv:1303.0733

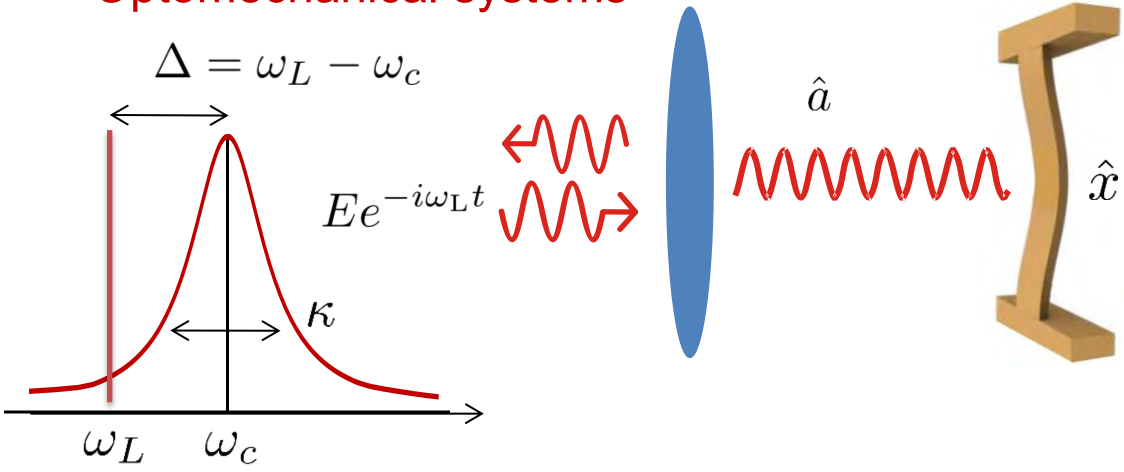
example:

$$g_0 = \omega_{\mathrm{cav}} \frac{x_{\mathrm{ZPF}}}{L} \simeq 100 \text{ Hz}$$

for $L \simeq 10 \times \lambda$

$$x_{ZPF} \simeq \text{fm for 1 ng, 1 MHZ}$$

Optomechanical systems



$$\hat{a}^\dagger \hat{a} \rightarrow (\hat{a}^\dagger + \alpha)(\hat{a} + \alpha) \simeq \alpha^2 + \alpha(\hat{a} + \hat{a}^\dagger)$$

$$H = -\hbar\Delta \hat{a}^\dagger \hat{a} + \hbar\omega_{\rm mec}\hat{b}^\dagger \hat{b} - \underline{\hbar g_0 \alpha \left(\hat{b} + \hat{b}^\dagger\right)\left(\hat{a} + \hat{a}^\dagger\right)}$$

linearized radiation pressure interaction

effective coupling strength

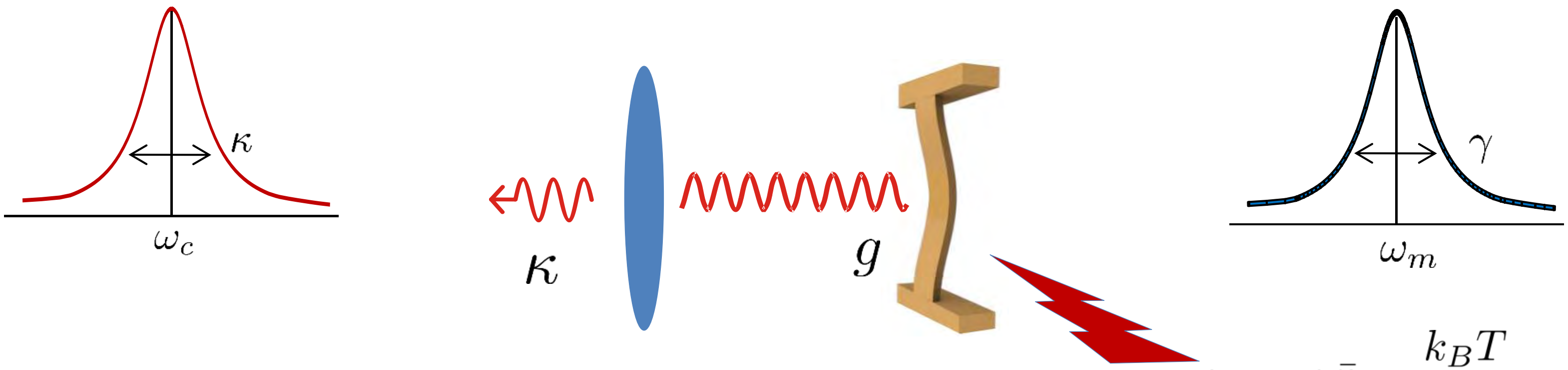
$$g = g_0 \alpha \gg g_0$$

number of circulating photons $|\alpha|^2 = \dfrac{P/\hbar\omega_L}{\kappa}$ for

$|\alpha|^2 \simeq 10^{11}$ for 1mW and 1 MHz line width

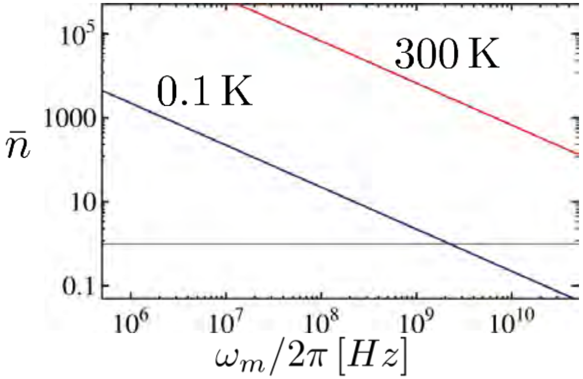
$g \simeq \omega_m, \kappa$ strong coupling & normal mode splitting

Optomechanical Cooperativity



$$\gamma_T = \gamma \bar{n} = \frac{k_B T}{\hbar Q}$$

$$H = -\hbar \Delta \hat{a}^\dagger \hat{a} + \hbar \omega_{\mathrm{mec}} \hat{b}^\dagger \hat{b} - \hbar g \big(\hat{b} + \hat{b}^\dagger\big)\big(\hat{a} + \hat{a}^\dagger\big)$$



condition for quantum coherent dynamics:

large optomechanical cooperativity

$$\boxed{\mathcal{C} = \frac{4g^2}{\kappa \gamma_T} > 1}$$

Putting Mechanics into Quantum Mechanics

PHYSICS TODAY

Roukes, Schwab (2005)

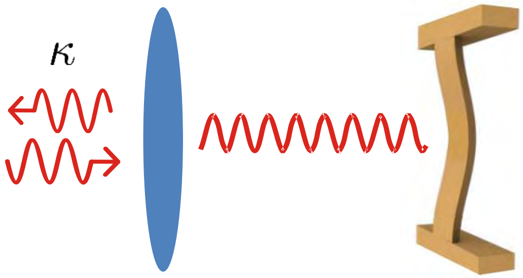
Quantum effects so far in optomechanics (incl. μw electromechanics)

» ground state cooling	Chan Nature 478, 89 (2011). Teufel, Nature 475, 359 (2011).
» Quantum coherent coupling	Verhagen, Nature 482, 63 (2012).
» ponderomotive squeezing	Safavi-Naeini, Nature 500, 185 (2013). Brooks, Nature 488, 476 (2012).
» back action noise in position sensing	Purdy, Science 339, 801 (2013).
» quantum coherent state transfer	O'Connell, Nature 464, 697 (2010) Palomaki, Nature 495, 210 (2013)
» optomechanical entanglement	Palomaki, Science 342, 710 (2013)
» feedback control within decoherence time	Wilson, arXiv:1410.6191 (2014)
» Quantum squeezing of motion	Wollman, arXiv:1507.01662 (2015) Lecocq, arXiv:1509.01629 (2015)

$\mathcal{C} > 1$

KH, Science 342 702 (2013)

Quantum Optomechanics



quantum effects so far…

- demonstrate large cooperativity $\mathcal{C} = \frac{4g^2}{\kappa\gamma\bar{n}} \geq 1$

$$H = g(a + a^\dagger)(b + b^\dagger) \quad \gamma\bar{n}$$

- rely on linear dynamics
- use homodyne detections
- preserve Gaussian states
- therefore, have an equivalent classical interpreation (with some level of noise)

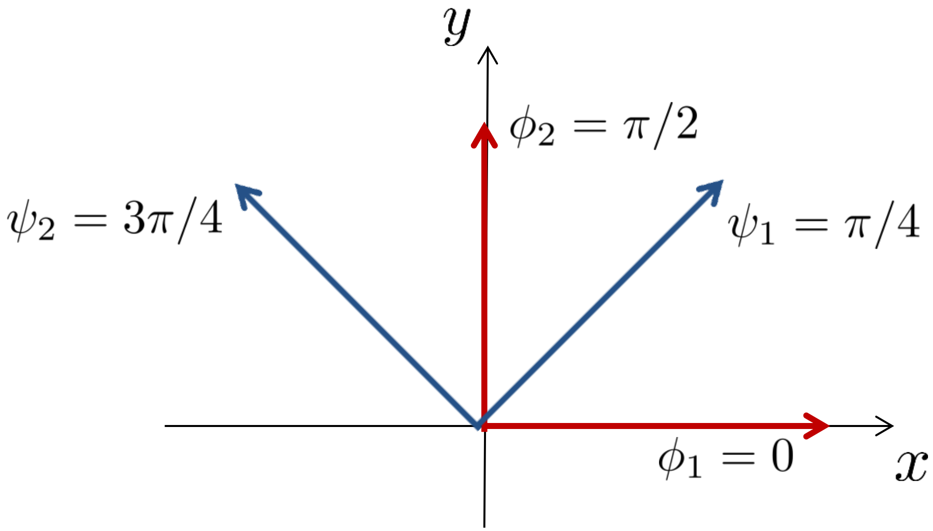
Bell (CHSH) Inequality

CHSH Inequality

$$S = \Big| \langle \sigma_A(\phi_1)\sigma_B(\psi_1)\rangle + \langle \sigma_A(\phi_1)\sigma_B(\psi_2)\rangle + \langle \sigma_A(\phi_2)\sigma_B(\psi_1)\rangle - \langle \sigma_A(\phi_2)\sigma_B(\psi_2)\rangle \Big| \leq 2$$

For two (effective) spin ½ systems in singlet state:

$$\sigma(\phi) = \cos(\phi)\sigma_x + \sin(\phi)\sigma_y$$



$$\langle \sigma_A(\phi)\sigma_B(\psi)\rangle = -\cos(\phi - \psi)$$

$$\boxed{S = 2\sqrt{2} > 2}$$

rules out (realism and locality)

Bell Inequality in Optomechanics

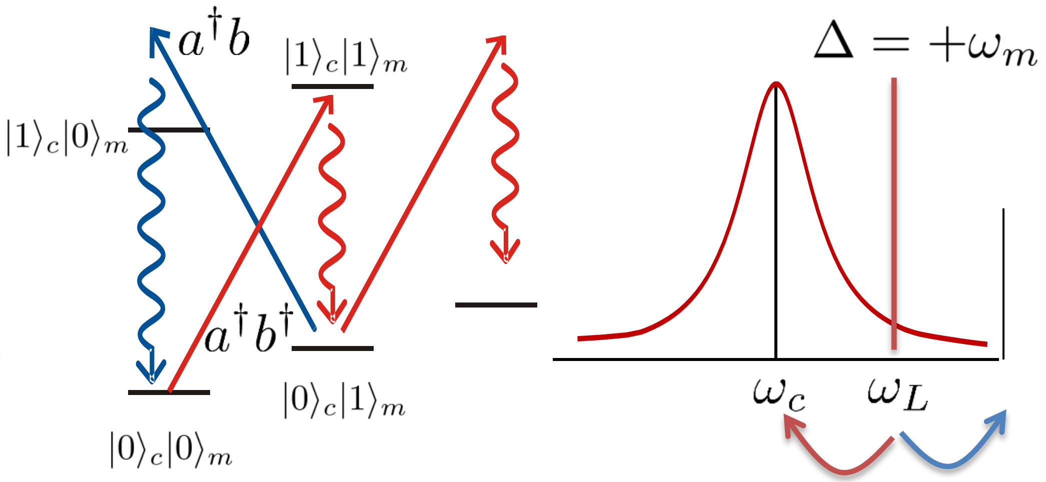
We need

- optomechanical entanglement
- measurement of binary observables $\sigma(\phi) = \cos(\phi)\sigma_x + \sin(\phi)\sigma_y$

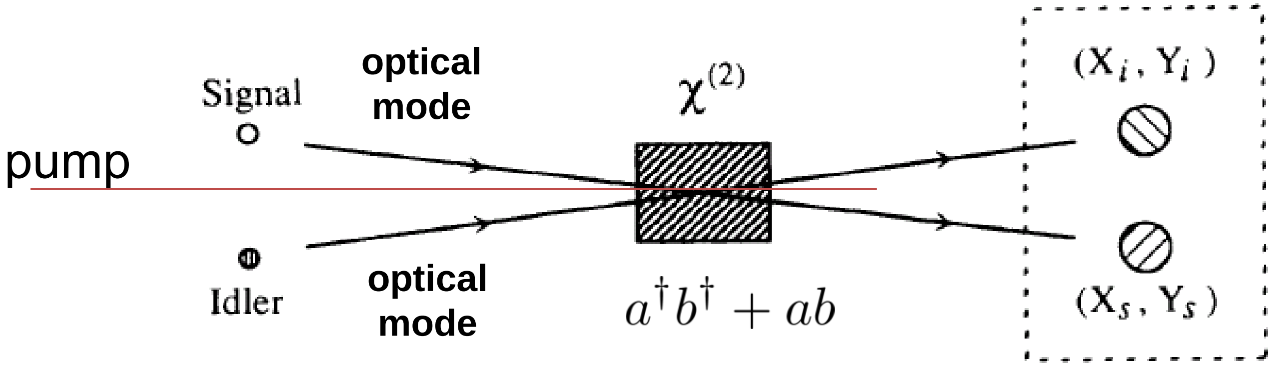
Drive on first blue sideband

Resonant interaction is entangling

$$H = \underbrace{g(a^\dagger b + a b^\dagger)} + \underbrace{g(a^\dagger b^\dagger + ab)}$$



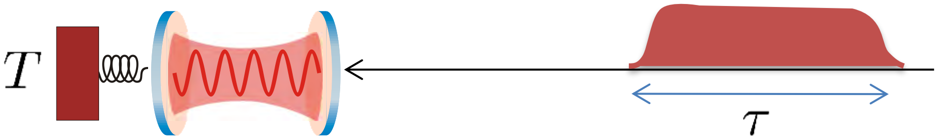
Compare to parametric down-conversion in nonlinear optics:



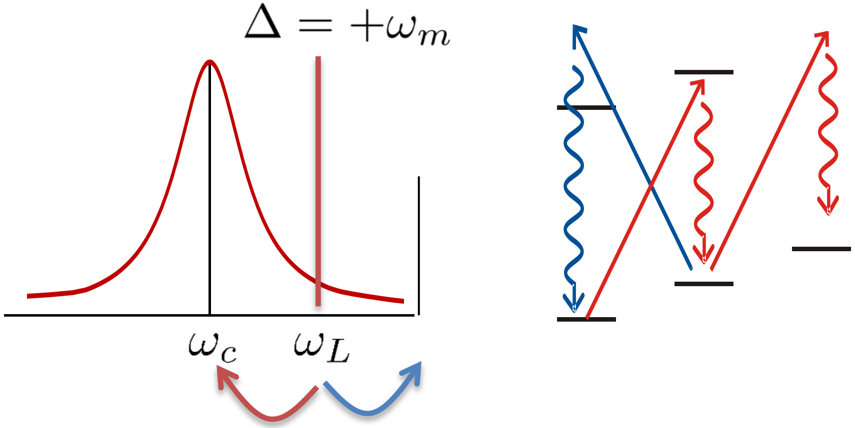
Ou, Pereira, Kimble, Peng, PRL 68, 3663 (1992)

Pulsed Generation of Entanglement

integrate for pulse suration τ



central frequency at upper sideband



assuming

weak thermal decoherence

$$\gamma \bar{n} \tau \ll 1$$

sideband resolved limit for suppression of Anti-Stokes scattering

$$\kappa \ll \omega_m$$

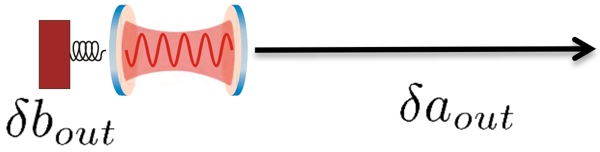
weak coupling: adiabatic elimination of cavity mode (avoid memory effects)

$$g \ll \kappa$$

Pulsed Generation of Entanglement

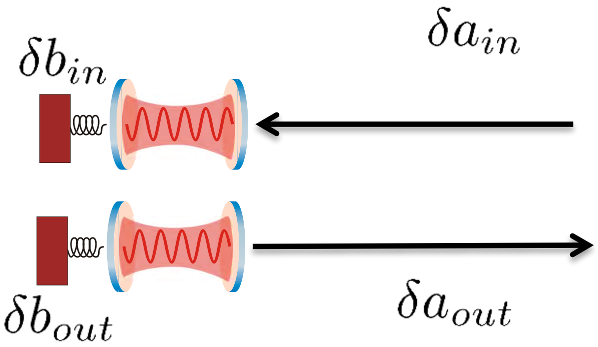
will generate photons at cavity frequency in precise temporal mode

mode profile $\sim \exp\left[\frac{g^2}{\kappa}t\right]$



$H \simeq g(a^\dagger b^\dagger + ab)$

input-output relations for scattered pulse (neglecting thermal noise, in RWA)



$$a_{out} = e^r a_{in} + i\sqrt{e^{2r} - 1}b_{in}^\dagger$$

$$b_{out} = e^r b_{in} - i\sqrt{e^{2r} - 1}a_{in}^\dagger$$

squeezing parameter $r = \frac{g^2\tau}{\kappa}$

two mode squeezed state!

Pulsed Generation of Entanglement

EPR variance, taking into account initial thermal occupation of mirror

entanglement!

$$\Delta EPR = \Delta(x_a + x_b)^2 + \Delta(p_a - p_b)^2$$

$$= 2(\bar{n}+1)\left(e^r - \sqrt{e^{2r}-1}\right) \simeq (\bar{n}+1)e^{-r} \quad < 1$$

large EPR squeezing requires large cooperativity:

for pulse length $\tau < (\gamma_{\mathrm{m}}\bar{n})^{-1}$

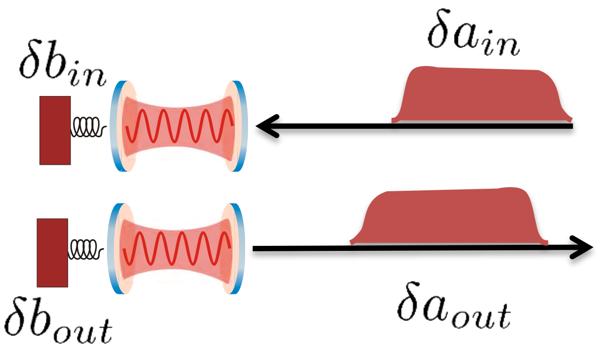
squeezing parameter $r = \dfrac{g^2\tau}{\kappa} < \dfrac{g^2}{\kappa\gamma_{\mathrm{m}}\bar{n}} \propto \mathcal{C}$

Verification of entanglement

drive system on first red sideband:

$$a_{out} = e^{-r} a_{in} + i\sqrt{1-e^{-2r}} b_{in}$$

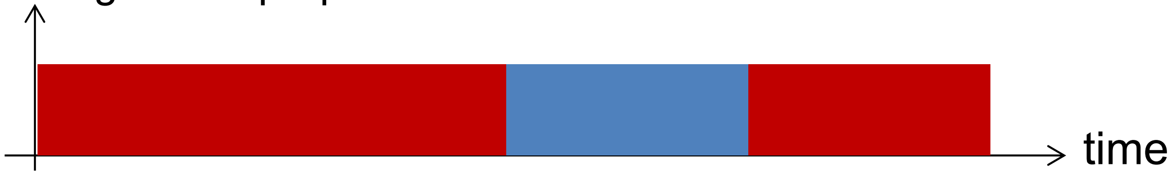
$$= i b_{in} \qquad r \gg 1$$



Palomaki, Nature 495, 210 (2013)

mechanical state is swapped to light

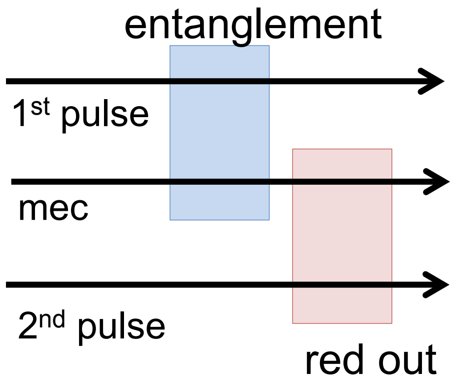
entanglement preparation and verification:



Precooling
on red sideband

entangling pulse
on blue sideband

readout pulse
on red sideband



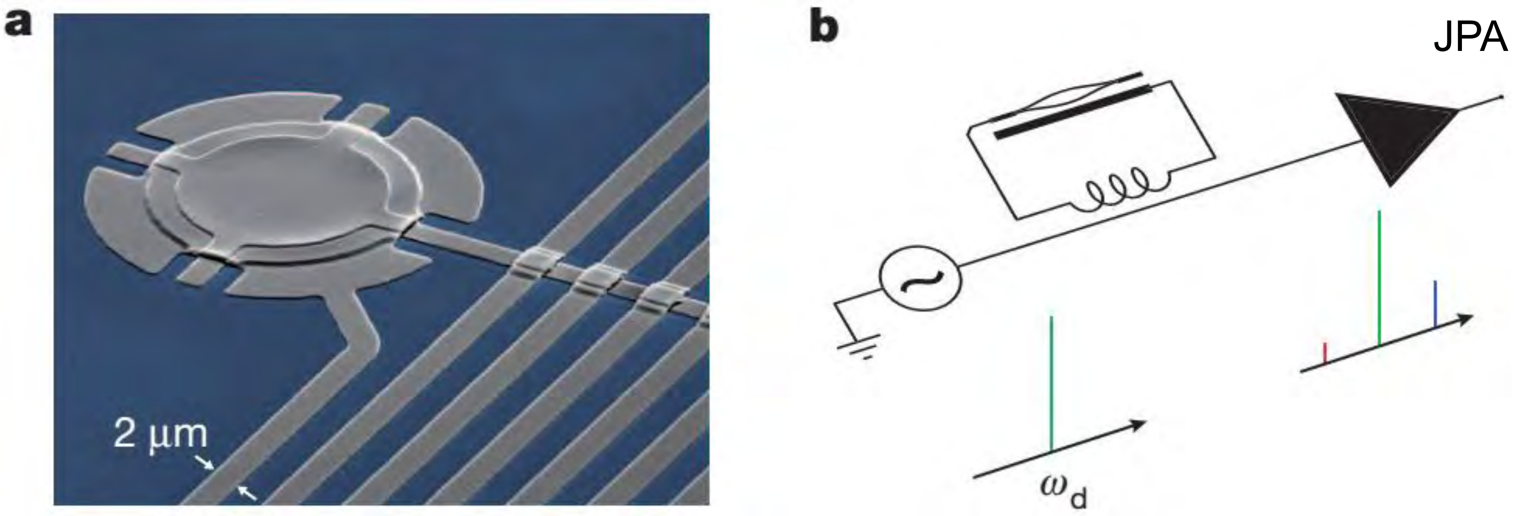
measure EPR quadratures of 1st and 2nd pulse and correlate

Sebastian G. Hofer, Witlef Wieczorek, Markus Aspelmeyer, KH
Phys. Rev. A 84, 052327 (2011)

T. A. Palomaki, J. D. Teufel, R. W. Simmonds, and K. W. Lehnert
Entangling mechanical motion with microwave fields
Nature (2013)

Experiment by Lehnert group

mw optomechanical system:



T. A. Palomaki, J. D. Teufel, R. W. Simmonds, and K. W. Lehnert
Entangling mechanical motion with microwave fields
Nature (2013)

Experiment by Lehnert group

w/o entangling drive

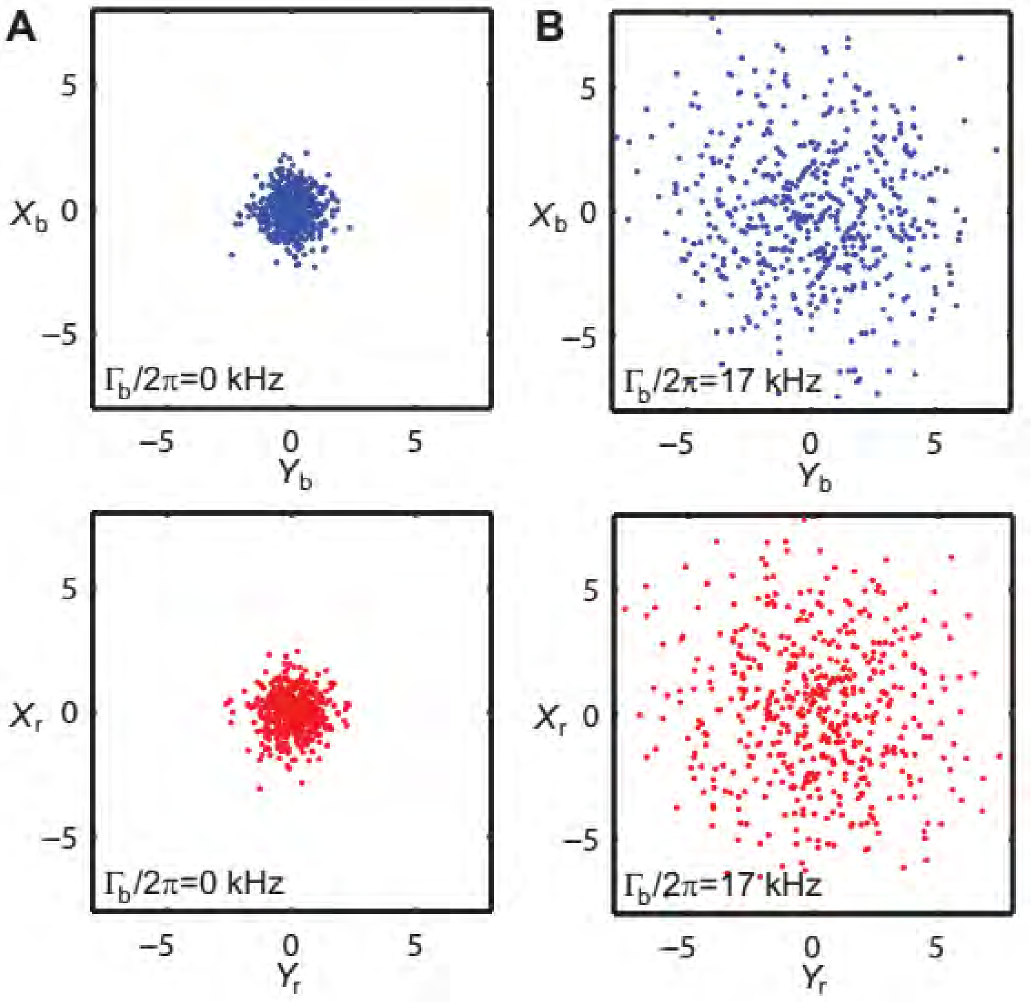
$$g^2/\kappa = 0$$

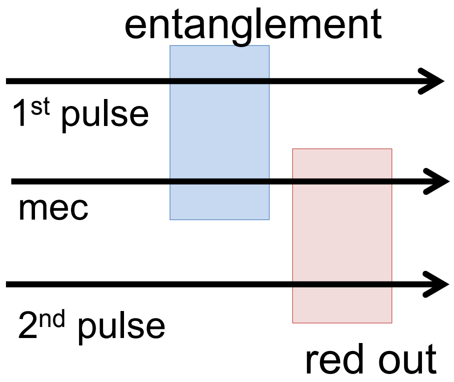
with entangling drive

$$g^2/\kappa = 2\pi 17\text{kHz}$$

entangling pulse

read-out pulse (= mechanics)




entanglement

1st pulse

mec

2nd pulse

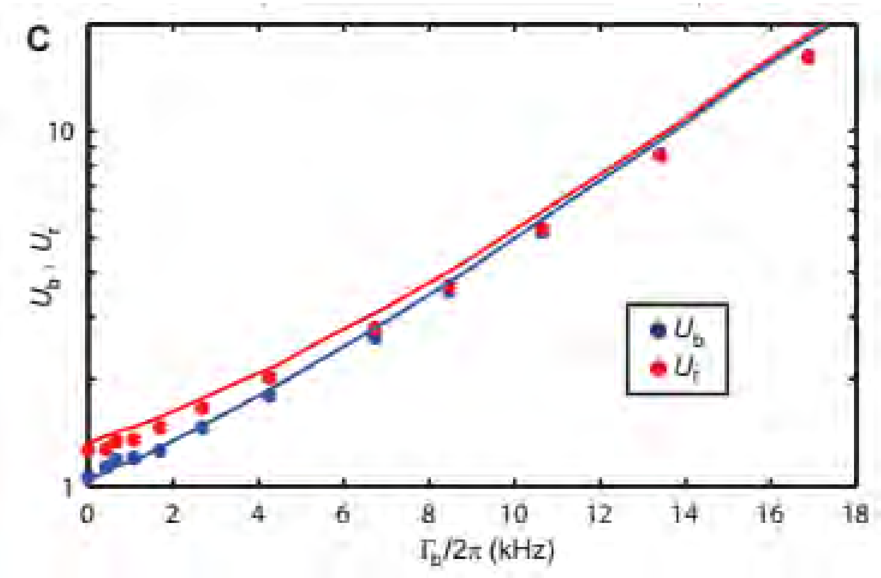
red out

T. A. Palomaki, J. D. Teufel, R. W. Simmonds, and K. W. Lehnert
Entangling mechanical motion with microwave fields
Nature (2013)

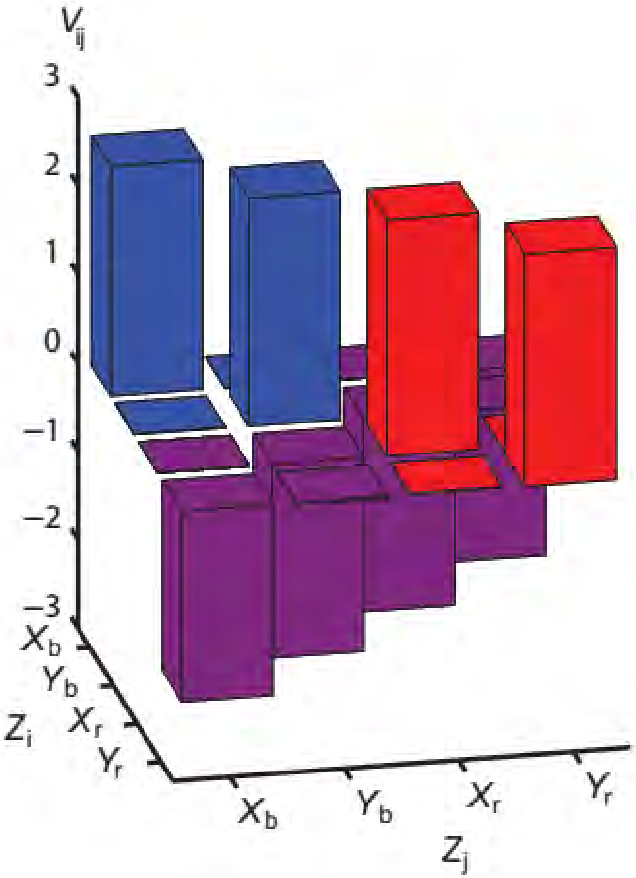
Experiment by Lehnert group

variances

Variances



$$g^2/\kappa$$



$$\bar{n}_b = \langle b^\dagger b \rangle = \sinh^2(2r) \simeq e^{2r}/2$$

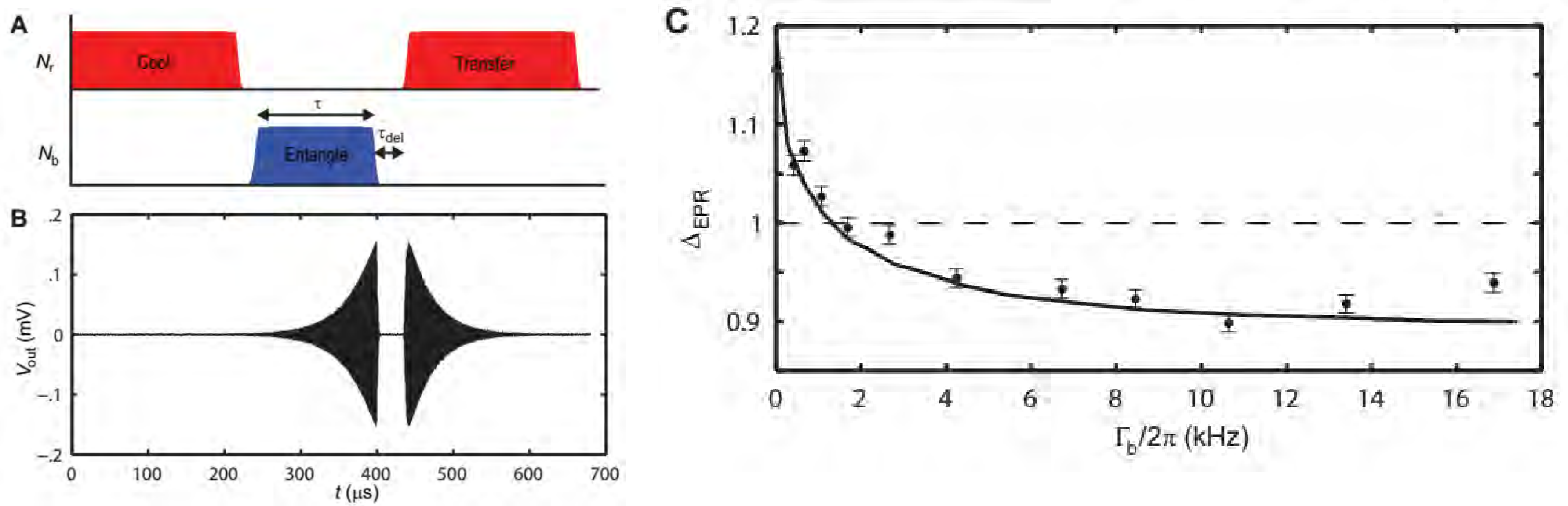
covariances

e.g. $\langle X_b Y_r \rangle$

T. A. Palomaki, J. D. Teufel, R. W. Simmonds, and K. W. Lehnert
Entangling mechanical motion with microwave fields
Nature (2013)

Experiment by Lehnert group

$$\Delta EPR = \Delta(x_a + x_b)^2 + \Delta(p_a - p_b)^2 < 1$$



$$g^2/\kappa$$

Optomechanical Entanglement

Two-mode squeezed (Gaussian), entangled state of a macroscopic (micron-sized) mechanical oscillator and a travelling pulse of (mw) light

$$|\Psi\rangle \sim |00\rangle + \epsilon|11\rangle + \epsilon^2|22\rangle + \cdots \neq |\psi\rangle_{\mathrm{mec}} \otimes |\phi\rangle_{\mathrm{cav}}$$

Bell Inequality in Optomechanics

We need

- optomechanical entanglement (Gaussian)

Palomaki, Science 342, 710 (2013)

- measurement of binary observables $\sigma(\phi) = \cos(\phi)\sigma_x + \sin(\phi)\sigma_y$

homodyne detection is not sufficient:

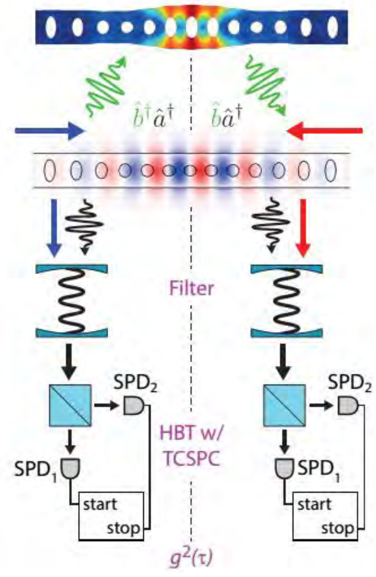
Gaussian state & Gaussian measurement have realistic description in Wigner function

- requires Non-Gaussian measurement:

counting of photons/phonons

Cohen, arXiv:1410.1047

Lecocq, arXiv: 1409.0872

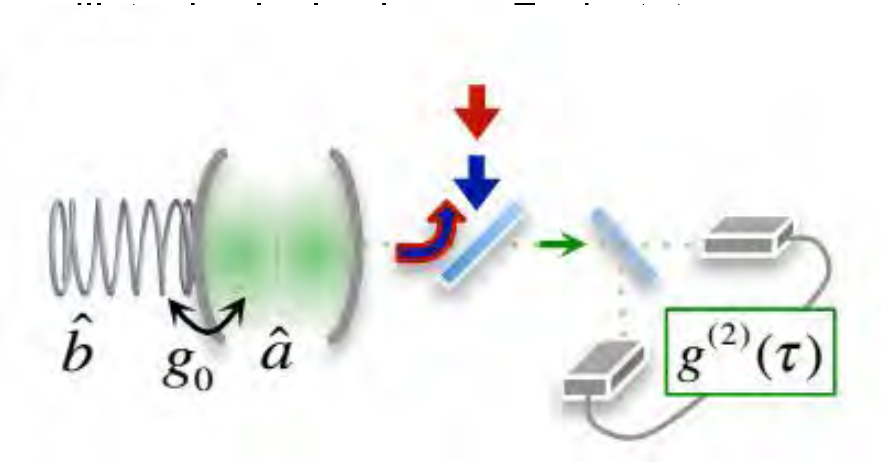


Optomechanical Entanglement

$$|\Psi\rangle \sim |00\rangle + \epsilon|11\rangle + \epsilon^2|22\rangle + \cdots \neq |\psi\rangle_{\mathrm{mec}} \otimes |\phi\rangle_{\mathrm{cav}}$$

detection of single photon projects mechanical

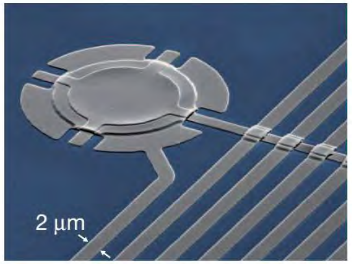
$${}_{\mathrm{photon}}\langle 1|\Psi\rangle \propto |1\rangle_{\mathrm{phonon}}$$



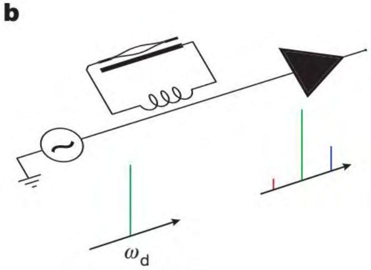
Galland, Sangouard, Piro, Gisin, Kippenberg, PRL 112, 143602 (2014)

Photon-Counting in Electro-Mechanics

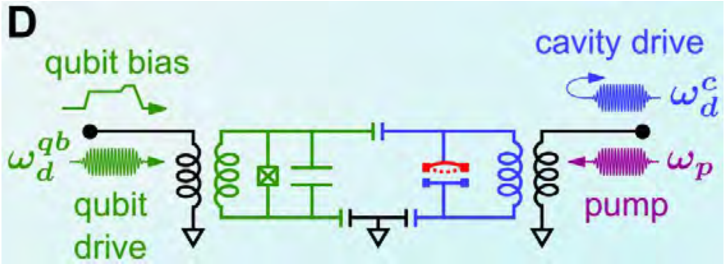
a



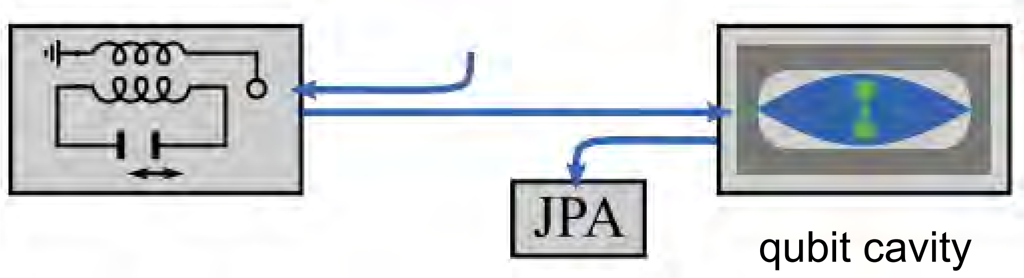
b

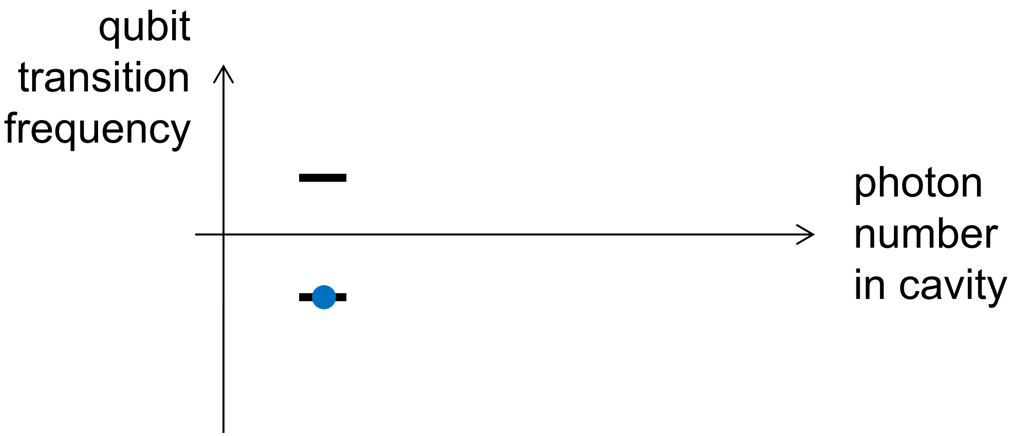


D

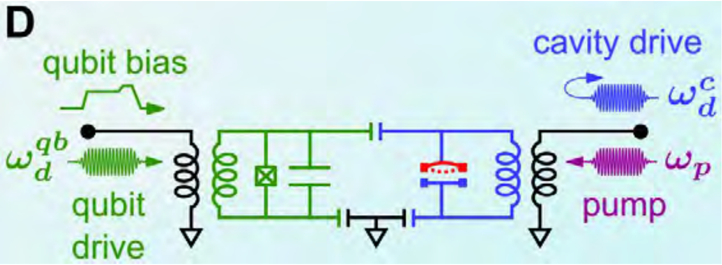


Lecocq, arXiv: 1409.0872

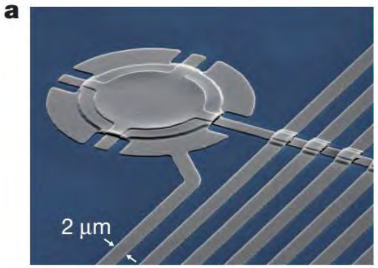


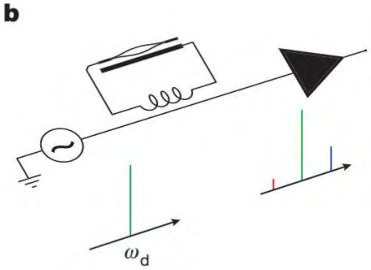


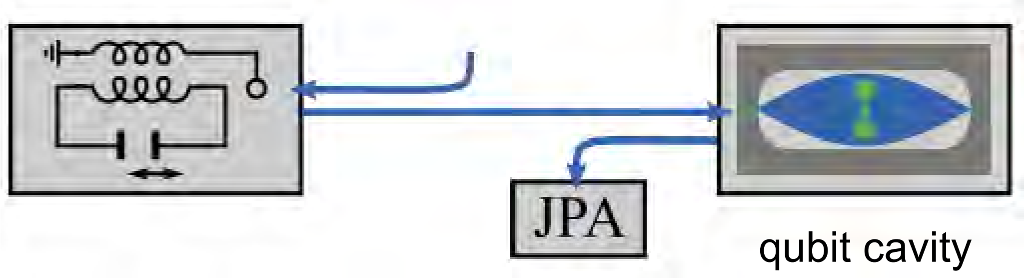
Photon-Counting in Electro-Mechanics



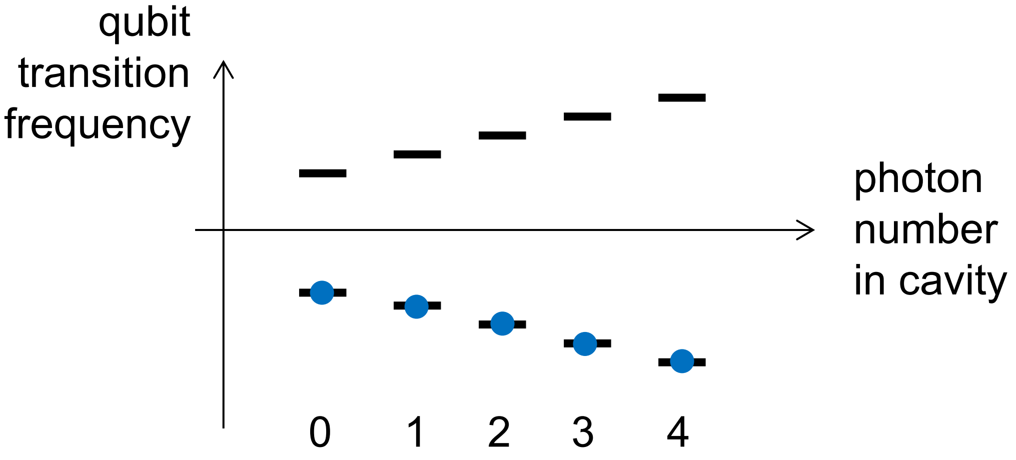
Lecocq, arXiv: 1409.0872



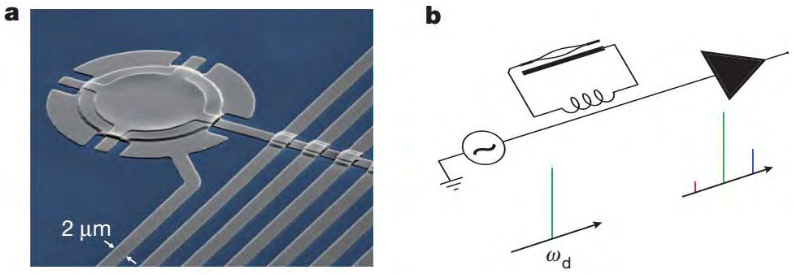


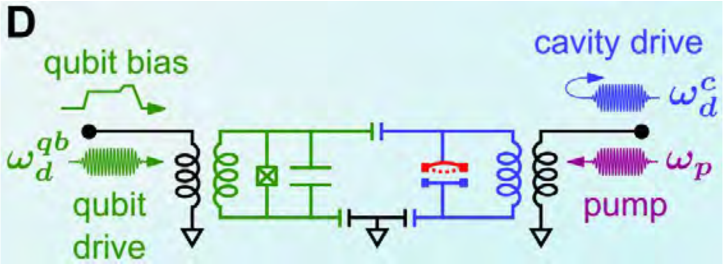


$$H_{qb} = G a^{\dagger} a \sigma_z$$

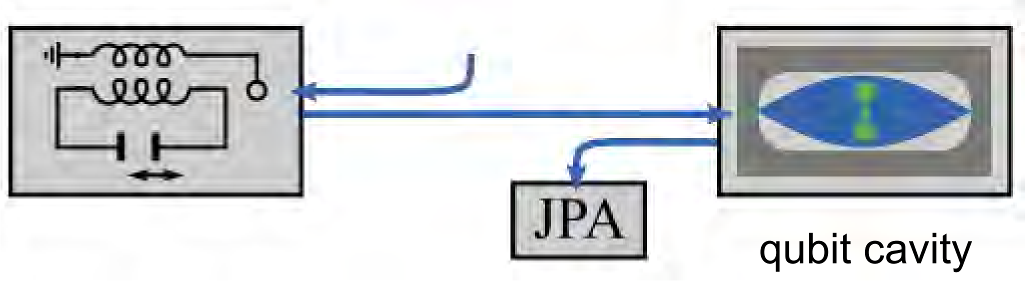


Photon-Counting in Electro-Mechanics

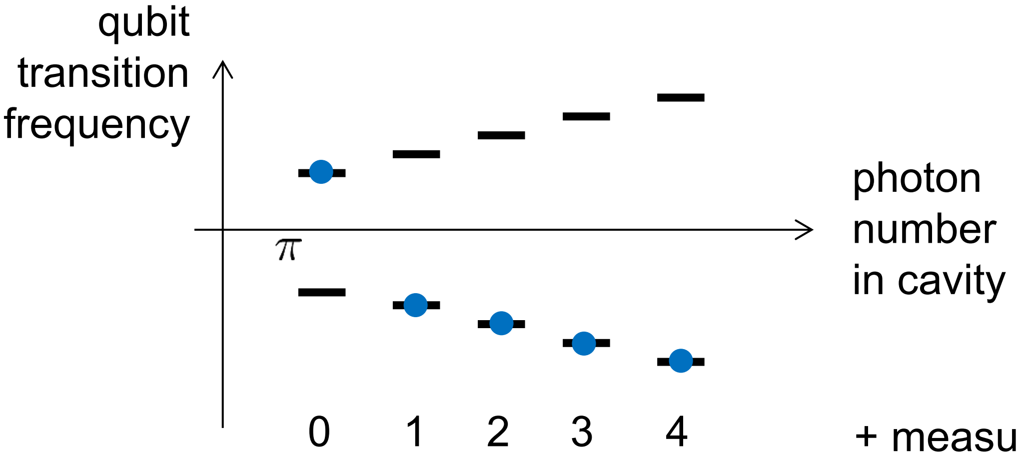




Lecocq, arXiv: 1409.0872



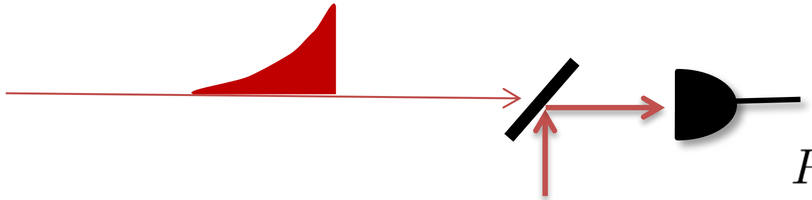
$$H_{qb} = G a^\dagger a \sigma_z$$



+ measurement of qubit in *e*

Binary Observable for Bosonic Modes

Add coherent amplitude before detection

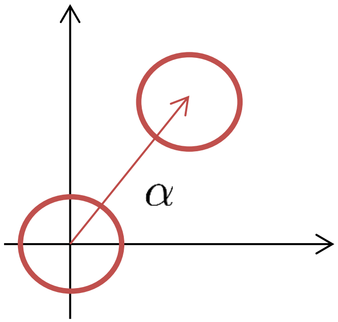


$$P_0(\alpha) \propto |\langle 0|D(\alpha)|EPR\rangle|^2$$
$$\propto |\langle \alpha|EPR\rangle|^2$$

$$|EPR\rangle \propto \sum_n |n,n\rangle$$

$$D(\alpha)|EPR\rangle$$

$$D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$$



define the binary observable

$$\sigma(\alpha) = \underbrace{(+1)|\alpha\rangle\langle\alpha|}_{\text{no click}} + \underbrace{(-1)\left[\mathbf{1} - |\alpha\rangle\langle\alpha|\right]}_{\text{click!}}$$

Banaszek, PRL 82, 2009 (1999)

Bell Inequality in Optomechanics

We need

- optomechanical entanglement (Gaussian)

Palomaki, Science 342, 710 (2013)

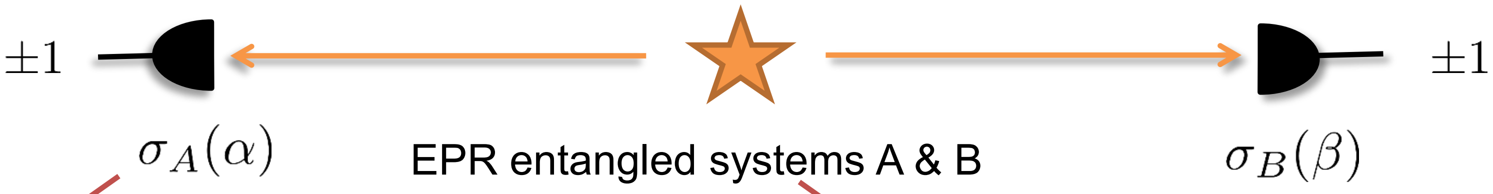
- Non-Gaussian measurement:

  counting of photons/phonons

- measurement of binary observables $\sigma(\phi) = \cos(\phi)\sigma_x + \sin(\phi)\sigma_y$

  requires a controllable phase

BI violation with ideal EPR state



$$\sigma(\alpha) = (+1)|\alpha\rangle\langle\alpha| + (-1)\left[\mathbf{1} - |\alpha\rangle\langle\alpha|\right]$$

$$|EPR\rangle = \mathrm{ch}(r)^{-1} \sum_n \mathrm{th}(r)^n |n, n\rangle$$

CHSH Inequality

$$S = \langle\sigma_A(\alpha_1)\sigma_B(\beta_1)\rangle + \langle\sigma_A(\alpha_1)\sigma_B(\beta_2)\rangle + \langle\sigma_A(\alpha_2)\sigma_B(\beta_1)\rangle - \langle\sigma_A(\alpha_2)\sigma_B(\beta_2)\rangle$$

BI originally introduced in discussion regarding the nonlocality/entanglement of single photon states

K. Banaszek, K. Wodkiewicz, PRL 82, 2009 (1999)

$$|0, 1\rangle + |1, 0\rangle$$

application to EPR entangled states

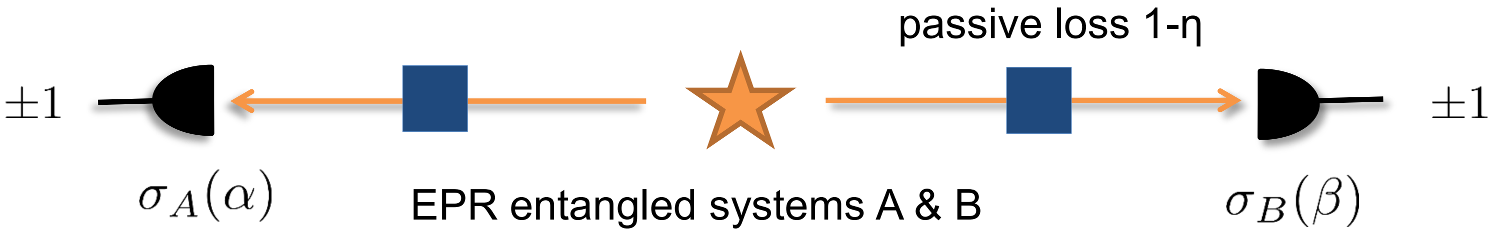
theory

K. Banaszek, quant-ph/9904071v1
S.-W. Lee, H. Jeong, D. Jaksch, PRA 80, 022104 (2009)
J. Bohr-Brask, R. Chaves, PRA 86, 010103 (2012)

experiment
with squ light

Kuzmich et al,
PRL 85, 1349 (2000)

BI violation with ideal EPR state


passive loss 1-η

± 1 $\quad \sigma_A(\alpha)$

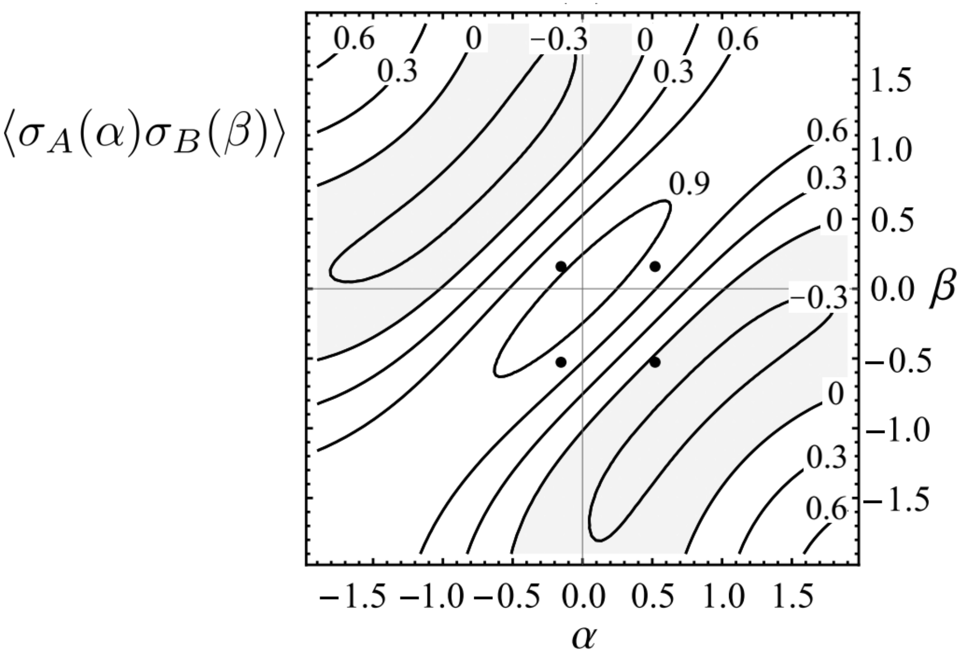
EPR entangled systems A & B

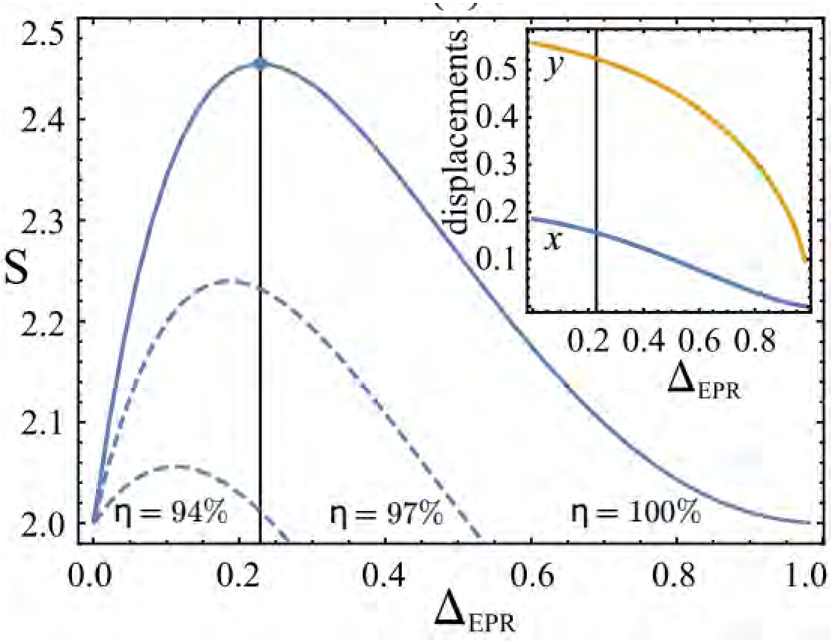
$\sigma_B(\beta)$ $\quad \pm 1$

CHSH Inequality

$$S = \langle \sigma_A(\alpha_1)\sigma_B(\beta_1)\rangle + \langle \sigma_A(\alpha_1)\sigma_B(\beta_2)\rangle + \langle \sigma_A(\alpha_2)\sigma_B(\beta_1)\rangle - \langle \sigma_A(\alpha_2)\sigma_B(\beta_2)\rangle$$

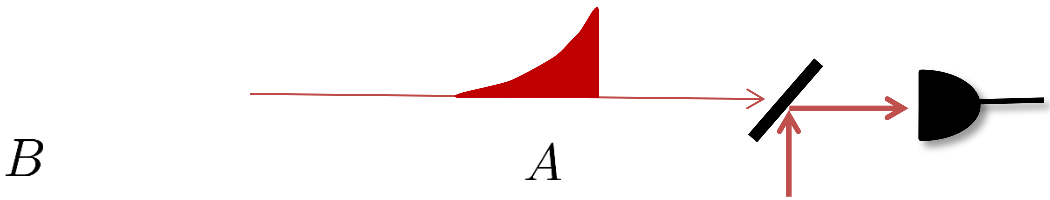
≤ 2.48 for real amplitudes α and β and finite EPR squeezing





Banaszek, Bohr-Brask, Jaksch…

Bell Inequality in Optomechanics



0) Initialization of mechanics in ground state by red sideband cooling

1) Optomechanical entanglement by blue sideband pulse

2) Displacement by amplitude α & photon counting $\sigma_A(\alpha)$

3) Swap of mechanical state to photons by red sideband pulse

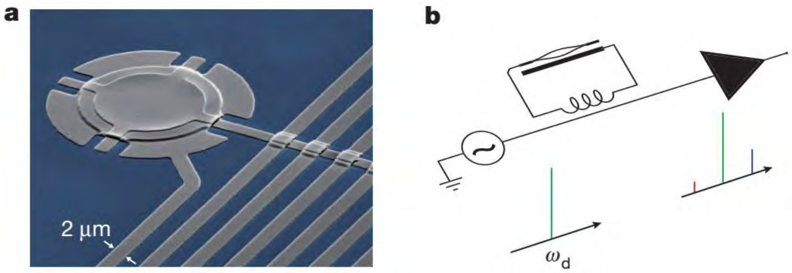
4) Displacement by amplitude β & photon counting $\sigma_B(\beta)$

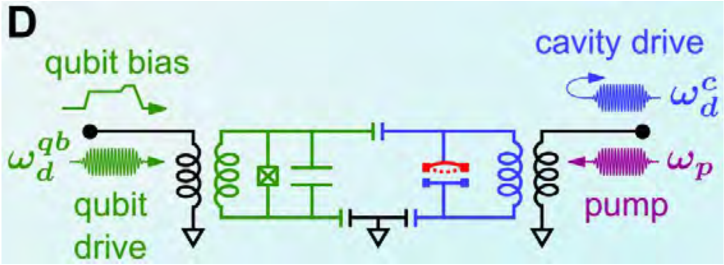
Repeat for various measurement setting α and β and infer

$$S = \left| \langle \sigma_A(\alpha_1)\sigma_B(\beta_1)\rangle + \langle \sigma_A(\alpha_1)\sigma_B(\beta_2)\rangle + \langle \sigma_A(\alpha_2)\sigma_B(\beta_1)\rangle - \langle \sigma_A(\alpha_2)\sigma_B(\beta_2)\rangle \right|$$

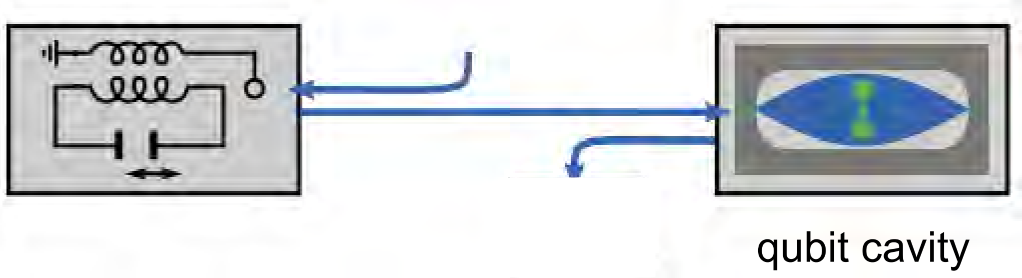
S.G. Hofer, K.W. Lehnert, KH, arXiv:1506.08097
V. Caprara Vivoli, T. Barnea, C. Galland, N. Sangouard, arXiv:1506.06116

Realization in Electro-Mechanics





Lecocq, arXiv: 1409.0872



optomechanical master equation for cascaded cavity setup:

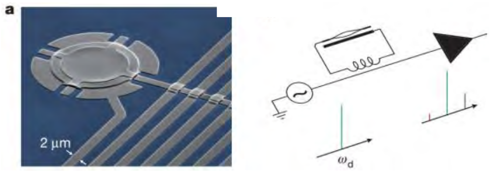
$$\begin{aligned}\dot{\rho}(t) = &-i[\omega_m b^\dagger b - \Delta c^\dagger c + (g(t)c + g^*(t)c^\dagger)(b + b^\dagger), \rho(t)] \\ &+ \left\{\gamma_m(\bar{n}+1)\mathcal{D}[b] + \gamma_m \bar{n}\mathcal{D}[b^\dagger] + \kappa_c \mathcal{D}[c] + \kappa_a(t)\mathcal{D}[a]\right\}\rho(t) \\ &- \sqrt{\lambda_t \kappa_c \kappa_a(t)/4}\left\{[a^\dagger, c\rho(t)] + [\rho(t)c^\dagger, a]\right\},\end{aligned}$$

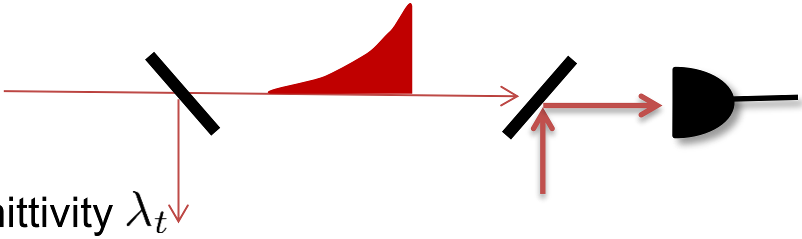
technical aspects:

- Gaussian dynamics can be integrated analytically including losses & thermalisation
- optimal control required for state readout
- Q functions of global and reduced state determines Bell parameter S(α,β)

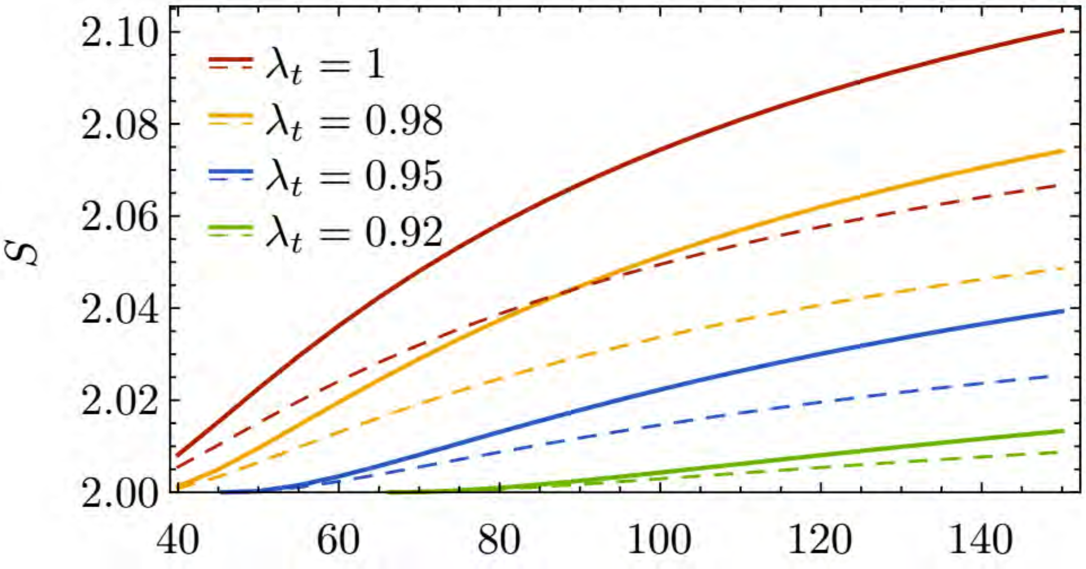
S.G. Hofer, K.W. Lehnert, KH, arXiv:1506.08097

BI Violation in Optomechanics





transmittivity λ_t



Bell violation

cooperativity C

For parameters of: T. A. Palomaki, Science 342 6159 (2013) with $\bar{n}_0 = 0.1(0.25)$ solid (dashed)

Includes thermal decoherence, finite detection efficiency, transmission losses nonperturbatively

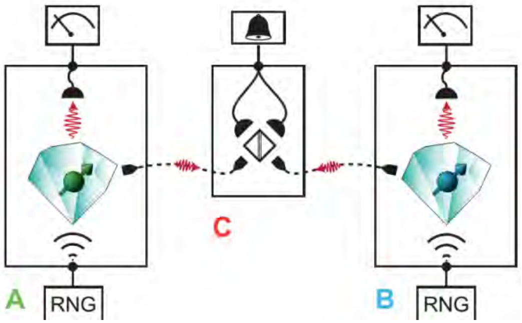
S.G. Hofer, K.W. Lehnert, KH, arXiv:1506.08097

implementation in optics V. Caprara Vivoli, T. Barnea, C. Galland, N. Sangouard, arXiv:1506.06116

Bell test with mechanical dofs

first loophole free Bell test with electron spins in NV centers entangled over 1.3 km

Experimental loophole-free violation of a Bell inequality using entangled electron spins separated by 1.3 km
Hensen et al, arXiv:1508:05949



Bell tests with quantum *mechanical* dofs?

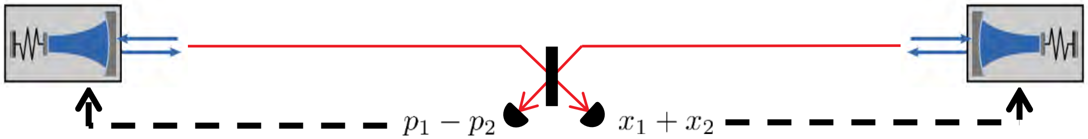
Putting Mechanics into Quantum Mechanics PHYSICS TODAY

Roukes, Schwab (2005)

entangled states of mechanical degrees of freedom with trapped ions in one trap

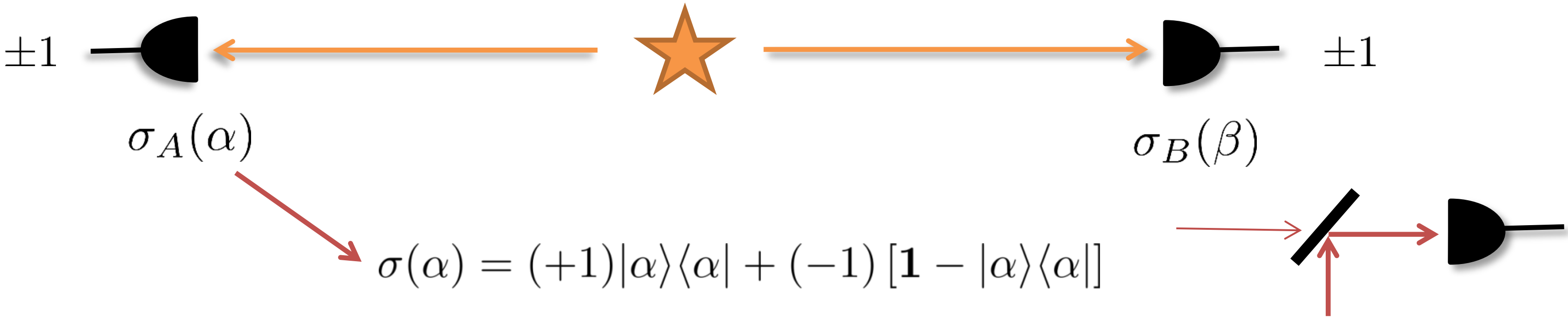
Entangled mechanical oscillators, Jost, Nature 459, 683 (2009)

present scheme can be extended to long distance with entanglement swapping



Hofer, Vasilyev, Aspelmeyer, KH, PRL 111, 170404 (2013)
Hofer, KH PRA 91, 033822 (2015)

Bell test with “weak field homodyning”



$$\sigma(\alpha) = (+1)|\alpha\rangle\langle\alpha| + (-1)\,[\mathbf{1} - |\alpha\rangle\langle\alpha|]$$

CHSH Inequality

$$S = \langle\sigma_A(\alpha_1)\sigma_B(\beta_1)\rangle + \langle\sigma_A(\alpha_1)\sigma_B(\beta_2)\rangle + \langle\sigma_A(\alpha_2)\sigma_B(\beta_1)\rangle - \langle\sigma_A(\alpha_2)\sigma_B(\beta_2)\rangle$$

violation with EPR states is limited to $S \leq 2.48 < 2\sqrt{2} = 2.83$

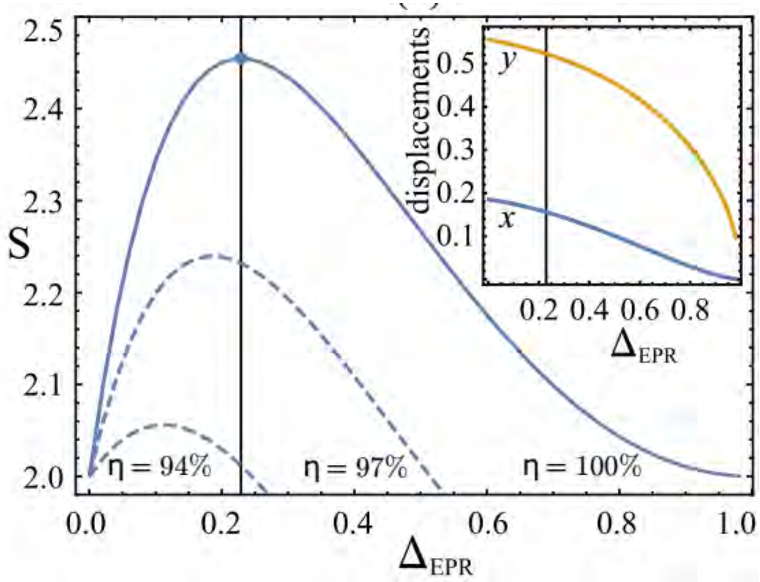
maximal violation possible with entangled “Schrödinger citten states”

$$|\alpha\rangle + |-\alpha\rangle$$

Kiukas, Werner, J. Math. Phys. 51, 072105 (2010)

EPR states are pretty good approximations

Oudot et al, arXiv:1410.8421



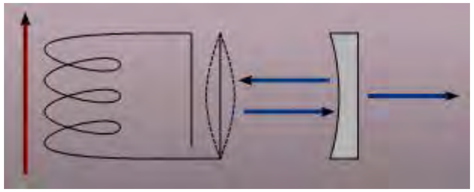
Leibniz
Universität
Hannover

Group:

- **Sebastian Hofer**
- Ondrej Cernotik
- Alexander Roth
- Jonas Lammers
- Marius Schulte
- Hashem Zoubi
- Klemens Hammerer

former members:

- Niels Loerch (Univ Basel)
- Sergey Tarabrin
- Andre Xuereb (Univ Malta)



optomechanical transducer

Thank you!

Collaborators on optomechanics

- **Konrad Lehnert (JILA)**
- Philipp Treutlein (Basel)
- Peter Zoller (Innsbruck)
- Eugene Polzik (Copenhagen)
- Florian Marquardt (Erlangen)
- Ash Clerk (McGill)
- Roman Schnabel (Hamburg)
- Farit Khalili (Moscow)
- Markus Aspelmeyer (Vienna)
- Klaus Hornberger (Duisburg)

Centre for Quantum Engineering and Space-Time Research

Institute for Theoretical Physics

Albert Einstein Institute

Support through:
DFG (QUEST, GRK 1991)
EC (MALICIA, iQUOEMS)
Vienna (WWTF)