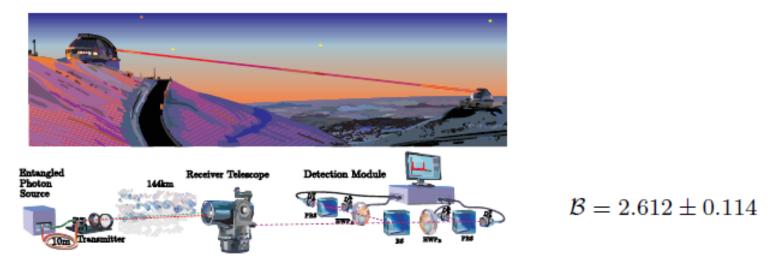
Violations of Bell inequalities for light in the turbulent atmosphere

made by Gumberidze Mariia

Supervisor Candidate of Sciences in Physics and Mathematics Semenov Andrew

Violations of Bell inequalities

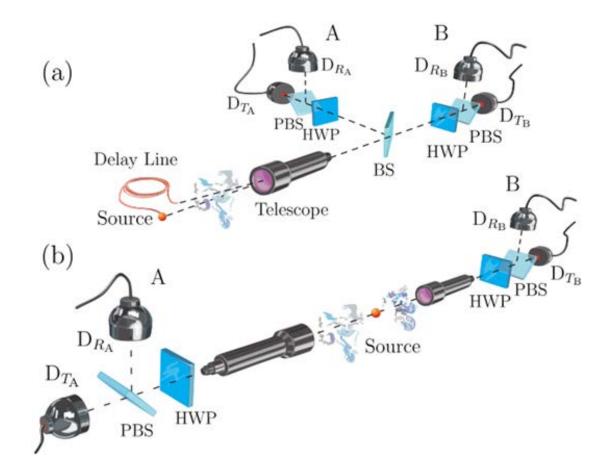
A. Fedrizzi et al., Nature Physics 5, 389 (2009)



In 2008 Zeilinger with his group managed to transmit the entangled pair of photons over a distance of 144 km between the observatories on the islands of Palma and Tenerife.

$$B = \left| \mathrm{E}(\theta_{A}^{(1)}, \theta_{B}^{(1)}) - \mathrm{E}(\theta_{A}^{(1)}, \theta_{B}^{(2)}) \right| + \left| \mathrm{E}(\theta_{A}^{(2)}, \theta_{B}^{(2)}) - \mathrm{E}(\theta_{A}^{(2)}, \theta_{B}^{(1)}) \right| \le 2$$

Experimental schemes



(a) – copropagation scenario, (b) – conterpropagation scenario

Double-click events

$$\begin{split} P_{i_{A},i_{B}}(\theta_{A},\theta_{B}) &= Tr(\hat{\Pi}_{i_{A}}^{(c)}\hat{\Pi}_{i_{B}}^{(c)}\hat{\Pi}_{j_{A}}^{(0)}\hat{\Pi}_{j_{B}}^{(0)}\hat{\rho}) + \\ &+ \frac{1}{2}Tr(\hat{\Pi}_{i_{A}}^{(c)}\hat{\Pi}_{i_{B}}^{(c)}\hat{\Pi}_{j_{A}}^{(c)}\hat{\Pi}_{j_{B}}^{(0)}\hat{\rho}) + \\ &+ \frac{1}{2}Tr(\hat{\Pi}_{i_{A}}^{(c)}\hat{\Pi}_{i_{B}}^{(c)}\hat{\Pi}_{j_{A}}^{(0)}\hat{\Pi}_{j_{B}}^{(c)}\hat{\rho}) + \\ &+ \frac{1}{4}Tr(\hat{\Pi}_{i_{A}}^{(c)}\hat{\Pi}_{i_{B}}^{(c)}\hat{\Pi}_{j_{A}}^{(c)}\hat{\Pi}_{j_{B}}^{(c)}\hat{\rho}), \end{split}$$

where

$$\hat{\Pi}_{i_{A(B)}}^{(0)} =: \exp(-\eta \hat{a}_{i_{A(B)}}^{\dagger} \hat{a}_{i_{A(B)}} - \nu):$$
$$\hat{\Pi}_{i_{A(B)}}^{(c)} = 1 -: \exp(-\eta \hat{a}_{i_{A(B)}}^{\dagger} \hat{a}_{i_{A(B)}} - \nu):$$

are the positive operator-valued measures for the detector $i_{A(B)}$, η i ν are the efficiency and the mean values of noise counts(originating from internal dark counts and background), and :: means normal ordering.

Bell states and parametric down-conversion(PDC) states

$$|PDC\rangle = (\cosh \xi)^{-2} \sum_{n=0}^{+\infty} \sqrt{n+1} \tanh^n \xi |\Phi_n\rangle,$$

where
$$\left|\Phi_{n}\right\rangle = \frac{1}{\sqrt{n+1}}\sum_{m=0}^{n}(-1)^{m}\left|n-m\right\rangle_{H_{A}}\left|m\right\rangle_{V_{A}}\left|m\right\rangle_{H_{B}}\left|n-m\right\rangle_{V_{B}}$$

При
$$n=1$$

$$\square \qquad |\Phi_1\rangle = \frac{1}{\sqrt{2}} \left(|1\rangle_{H_A} |0\rangle_{V_A} |0\rangle_{H_B} |1\rangle_{V_B} - |0\rangle_{H_A} |1\rangle_{V_A} |1\rangle_{H_B} |0\rangle_{V_B} \right)$$

- Bell state.

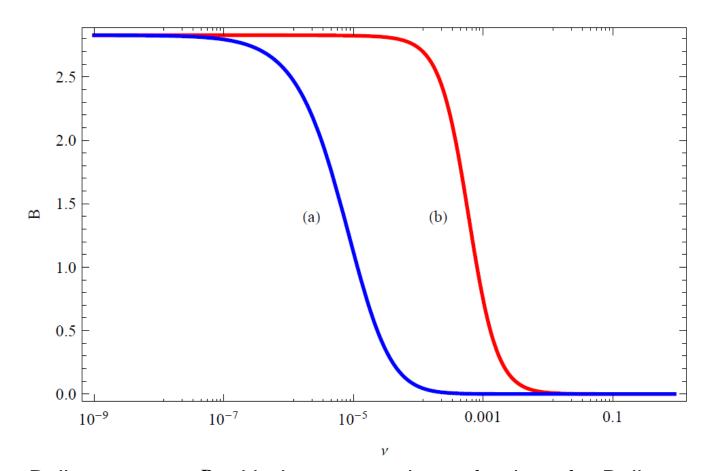
$$P_{same}(\theta_A, \theta_B) = P_{T_A, T_B}(\theta_A, \theta_B) + P_{R_A, R_B}(\theta_A, \theta_B)$$

$$P_{different}(\theta_A, \theta_B) = P_{T_A, R_B}(\theta_A, \theta_B) + P_{R_A, T_B}(\theta_A, \theta_B)$$

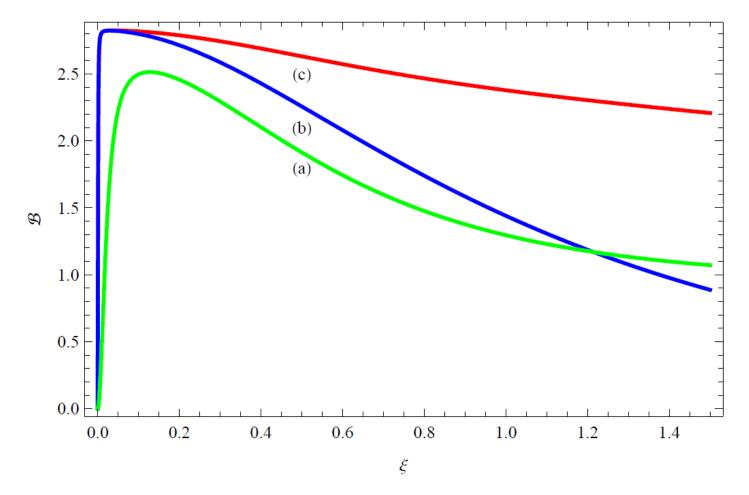
$$E(\theta_{A}, \theta_{B}) = \frac{P_{same}(\theta_{A}, \theta_{B}) - P_{different}(\theta_{A}, \theta_{B})}{P_{same}(\theta_{A}, \theta_{B}) + P_{different}(\theta_{A}, \theta_{B})}$$

$$B = \left| E(\theta_A^{(1)}, \theta_B^{(1)}) - E(\theta_A^{(1)}, \theta_B^{(2)}) \right| + \left| E(\theta_A^{(2)}, \theta_B^{(2)}) - E(\theta_A^{(2)}, \theta_B^{(1)}) \right| = B(\xi)$$

Correlated fading channels

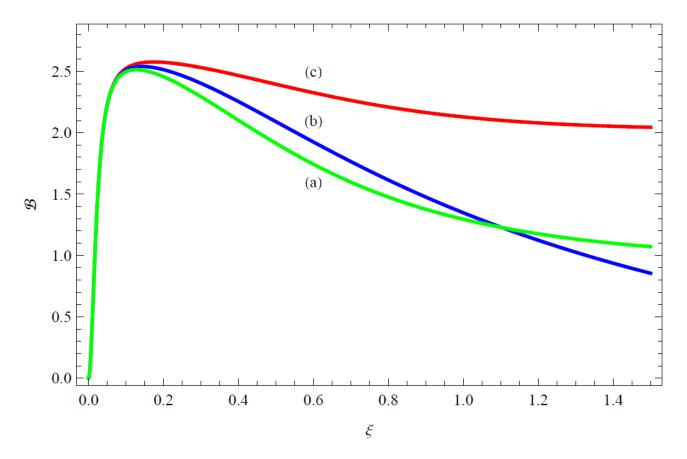


The Bell parameter *B* with the mean values of noise v for Bell states: (a) – deterministic attenuation, (b) – with and without consideration of double-click events.



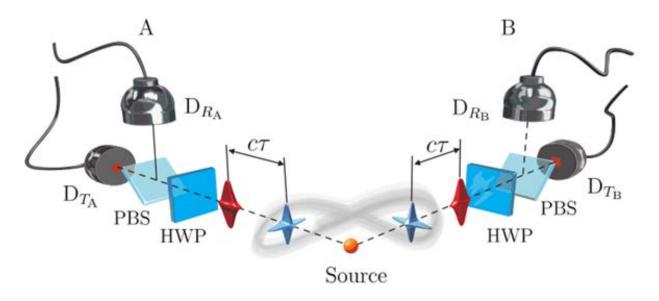
The Bell parameter *B* with the squeezing parameter ξ for PDC states: (a) – deterministic attenuation, (b), (c) – with and without consideration of double-click events.

Uncorrelated fading channels



The Bell parameter *B* with the squeezing parameter ξ for PDC states: (a) – deterministic attenuation, (b), (c) – with and without consideration of double-click events.

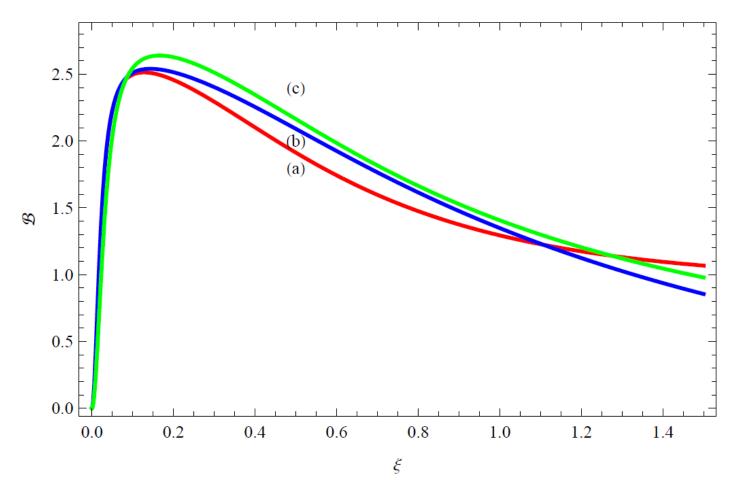
Adaptive correlation of uncorrelated channels



Intense classical-light pulses are sent before nonclassical pulses in order to test the transmittances of the channels. The time τ is much less than the characteristic time for which the atmosphere is changed.

If
$$T_A < T_B$$
, then $TT_B = T_A$.

If $T_A > T_B$, then $TT_A = T_B$.



The Bell parameter *B* with the squeezing parameter ξ for PDC states: (a) – deterministic attenuation, (b), (c) – scenario of counter-propagation with and without the application of adaptive protocol and consideration of double-click events..

Summary and conclusions

1. Double-click events do not affect sufficiently on Bell-parameter values in the case of co-propagation. However, with increasing the part of multiphoton pairs from the PDC source the corresponding Bell parameter diminishes much faster comparing to one for which double-click events have not been considered;

2. A different behavior takes a place in the scenario of counterpropagation, when fading channels are uncorrelated. The presence of multi-photon pairs leads to a relatively better result for fading channels comparing to the standard attenuation also for the case with double-click events;

3. Adaptive protocol may improve the result also in the case with doubleclick events for some optimal number of multi-photon pairs. Therefore, in the case of counter-propagation we have a possibility to explore advantages of fading in order to improve characteristics of quantum channels.

Thank you for attention!