# Violations of Bell inequalities for light in the turbulent atmosphere 

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## Violations of Bell inequalities

## A. Fedrizzi et al., Nature Physics 5, 389 (2009)



$$
\mathcal{B}=2.612 \pm 0.114
$$

In 2008 Zeilinger with his group managed to transmit the entangled pair of photons over a distance of 144 km between the observatories on the islands of Palma and Tenerife.

$$
B=\left|\mathrm{E}\left(\theta_{A}^{(1)}, \theta_{B}^{(1)}\right)-\mathrm{E}\left(\theta_{A}^{(1)}, \theta_{B}^{(2)}\right)\right|+\left|\mathrm{E}\left(\theta_{A}^{(2)}, \theta_{B}^{(2)}\right)-\mathrm{E}\left(\theta_{A}^{(2)}, \theta_{B}^{(1)}\right)\right| \leq 2
$$

## Experimental schemes


(a) - copropagation scenario, (b) - conterpropagation scenario

## Double-click events

$$
\begin{aligned}
P_{i_{A}, i_{B}} & \left(\theta_{A}, \theta_{B}\right)=\operatorname{Tr}\left(\hat{\Pi}_{i_{A}}^{(c)} \hat{\Pi}_{i_{B}}^{(c)} \hat{\Pi}_{j_{A}}^{(0)} \hat{\Pi}_{j_{B}}^{(0)} \hat{\rho}\right)+ \\
& +\frac{1}{2} \operatorname{Tr}\left(\hat{\Pi}_{i_{A}}^{(c)} \hat{\Pi}_{i_{B}}^{(c)} \hat{\Pi}_{j_{A}}^{(c)} \hat{\Pi}_{j_{B}}^{(0)} \hat{\rho}\right)+ \\
& +\frac{1}{2} \operatorname{Tr}\left(\hat{\Pi}_{i_{A}}^{(c)} \hat{\Pi}_{i_{B}}^{(c)} \hat{\Pi}_{j_{A}}^{(0)} \hat{\Pi}_{j_{B}}^{(c)} \hat{\rho}\right)+ \\
& +\frac{1}{4} \operatorname{Tr}\left(\hat{\Pi}_{i_{A}}^{(c)} \hat{\Pi}_{i_{B}}^{(c)} \hat{\Pi}_{j_{A}}^{(c)} \hat{\Pi} \hat{\Pi}_{B}^{(c)} \hat{\rho}\right),
\end{aligned}
$$

where

$$
\begin{gathered}
\hat{\Pi}_{i_{A(B)}}^{(0)}=: \exp \left(-\eta \hat{a}_{i_{A(B)}}^{\dagger} \hat{a}_{i_{A(B)}}-v\right): \\
\hat{\Pi}_{i_{A(B)}}^{(c)}=1-: \exp \left(-\eta \hat{a}_{i_{A(B)}}^{\dagger} \hat{a}_{i_{A(B)}}-v\right):
\end{gathered}
$$

are the positive operator-valued measures for the detector $i_{A(\mathrm{~B})}, \eta$ i $\quad v$ are the efficiency and the mean values of noise counts(originating from internal dark counts and background), and :: means normal ordering.

## Bell states and parametric down-conversion(PDC) states

$$
|P D C\rangle=(\cosh \xi)^{-2} \sum_{n=0}^{+\infty} \sqrt{n+1} \tanh ^{n} \xi\left|\Phi_{n}\right\rangle
$$

where $\quad\left|\Phi_{n}\right\rangle=\frac{1}{\sqrt{n+1}} \sum_{m=0}^{n}(-1)^{m}|n-m\rangle_{H_{A}}|m\rangle_{V_{A}}|m\rangle_{H_{B}}|n-m\rangle_{V_{B}}$

При

$$
n=1
$$



$$
\leadsto \quad\left|\Phi_{1}\right\rangle=\frac{1}{\sqrt{2}}\left(|1\rangle_{H_{A}}|0\rangle_{V_{A}}|0\rangle_{H_{B}}|1\rangle_{V_{B}}-|0\rangle_{H_{A}}|1\rangle_{V_{A}}|1\rangle_{H_{B}}|0\rangle_{V_{B}}\right)
$$

- Bell state.

$$
\begin{gathered}
\hat{\rho}=|P D C\rangle\langle P D C|=f(\xi) \\
P_{\text {same }}\left(\theta_{A}, \theta_{B}\right)=P_{T_{A}, T_{B}}\left(\theta_{A}, \theta_{B}\right)+P_{R_{A}, \mathrm{R}_{B}}\left(\theta_{A}, \theta_{B}\right) \\
P_{\text {different }}\left(\theta_{A}, \theta_{B}\right)=P_{T_{A}, \mathrm{R}_{B}}\left(\theta_{A}, \theta_{B}\right)+P_{R_{A}, T_{B}}\left(\theta_{A}, \theta_{B}\right) \\
\mathrm{E}\left(\theta_{A}, \theta_{B}\right)=\frac{P_{\text {same }}\left(\theta_{A}, \theta_{B}\right)-P_{\text {different }}\left(\theta_{A}, \theta_{B}\right)}{P_{\text {same }}\left(\theta_{A}, \theta_{B}\right)+P_{\text {different }}\left(\theta_{A}, \theta_{B}\right)} \\
B=\left|\mathrm{E}\left(\theta_{A}^{(1)}, \theta_{B}^{(1)}\right)-\mathrm{E}\left(\theta_{A}^{(1)}, \theta_{B}^{(2)}\right)\right|+\left|\mathrm{E}\left(\theta_{A}^{(2)}, \theta_{B}^{(2)}\right)-\mathrm{E}\left(\theta_{A}^{(2)}, \theta_{B}^{(1)}\right)\right|=B(\xi)
\end{gathered}
$$

## Correlated fading channels



The Bell parameter $B$ with the mean values of noise ${ }^{v}$ for Bell states: (a) deterministic attenuation, (b) - with and without consideration of doubleclick events.


The Bell parameter $B$ with the squeezing parameter $\xi$ for PDC states: (a) deterministic attenuation, (b), (c) - with and without consideration of double-click events.

## Uncorrelated fading channels



The Bell parameter $B$ with the squeezing parameter $\xi$ for PDC states: (a) deterministic attenuation, (b), (c) - with and without consideration of double-click events.

## Adaptive correlation of uncorrelated channels



Intense classical-light pulses are sent before nonclassical pulses in order to test the transmittances of the channels. The time $\tau$ is much less than the characteristic time for which the atmosphere is changed..

$$
\begin{array}{ll}
\text { If } & T_{A}<T_{B}, \text { then } \\
\text { If } & \mathrm{T} T_{B}=T_{A} . \\
& T_{A}>T_{B}, \text { then } \\
\mathrm{T} T_{A}=T_{B} .
\end{array}
$$



The Bell parameter $B$ with the squeezing parameter $\xi$ for PDC states: (a) deterministic attenuation, (b), (c) - scenario of counter-propagation with and without the application of adaptive protocol and consideration of double-click events..

## Summary and conclusions

1. Double-click events do not affect sufficiently on Bell-parameter values in the case of co-propagation. However, with increasing the part of multiphoton pairs from the PDC source the corresponding Bell parameter diminishes much faster comparing to one for which double-click events have not been considered;
2. A different behavior takes a place in the scenario of counterpropagation, when fading channels are uncorrelated. The presence of multi-photon pairs leads to a relatively better result for fading channels comparing to the standard attenuation also for the case with double-click events;
3. Adaptive protocol may improve the result also in the case with doubleclick events for some optimal number of multi-photon pairs. Therefore, in the case of counter-propagation we have a possibility to explore advantages of fading in order to improve characteristics of quantum channels.

## Thank you for attention!

