Università degli Studi di Milano
Istituto Nazionale di Fisica Nucleare

CHARACTERIZATION OF HEAT IN NON-MARKOVIAN OPEN QUANTUM SYSTEMS

GIACOMO GUARNIERI

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- QUANTUM NON-MARKOVIANITY
- HEAT IN OPEN QUANTUM SYSTEMS
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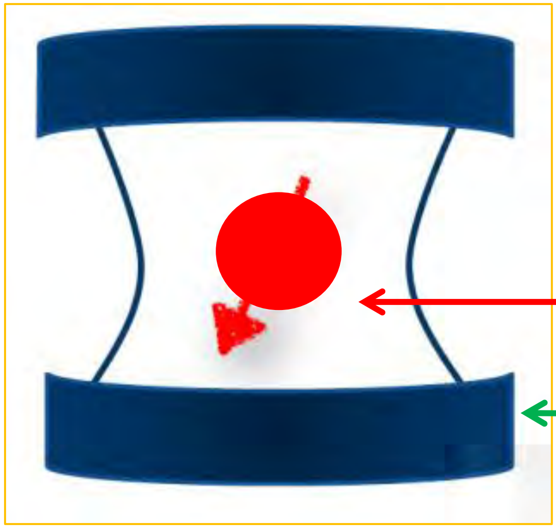
  - XX-COUPLED AND DRIVEN V-SYSTEM
- CONCLUSIONS

Open quantum systems



Composite system $\rho_{SE} \in \mathcal{S}(\mathscr{H}_{SE})$

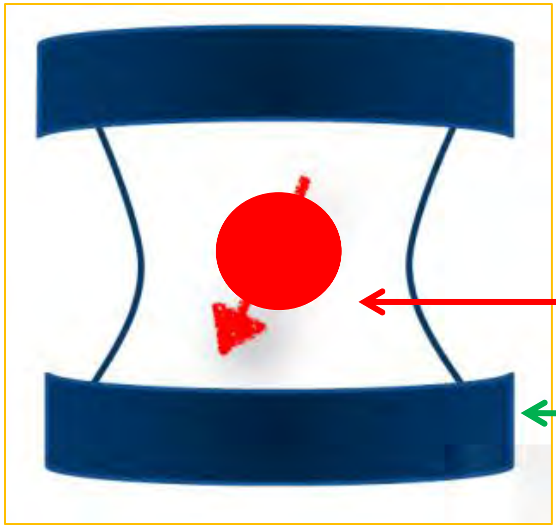
$$\mathcal{S}(\mathscr{H}) = \{\rho \in \mathcal{T}(\mathscr{H}) | \rho \geq 0, \| \rho \|_1 = 1\}$$

System of interest $\rho_S \equiv \mathrm{Tr}_E\,[\rho_{SE}] \in \mathcal{S}(\mathscr{H}_S)$

Environment $\rho_E \equiv \mathrm{Tr}_S\,[\rho_{SE}] \in \mathcal{S}(\mathscr{H}_E)$

$$\begin{tikzcd}
\rho_{SE}(t_0) \arrow[r, "\mathcal{U}(t,t_0)"] \arrow[d, "\mathrm{Tr}_E"] & \rho_{SE}(t) = \mathcal{U}(t,t_0)\rho_{SE}(t_0)\mathcal{U}^\dagger(t,t_0) \arrow[d, "\mathrm{Tr}_E"] \\
\rho_S(t_0) & \rho_S(t) = \mathrm{Tr}_E\left[\mathcal{U}(t,t_0)\rho_{SE}(t_0)\mathcal{U}^\dagger(t,t_0)\right]
\end{tikzcd}$$

Open quantum systems



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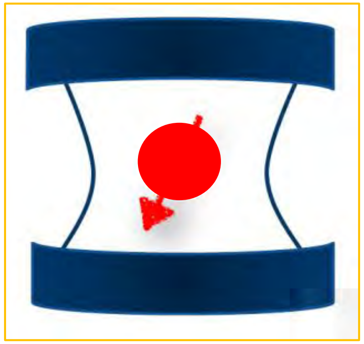
Environment $\rho_E \equiv \mathrm{Tr}_S [\rho_{SE}] \in \mathcal{S}(\mathscr{H}_E)$

$$\begin{tikzcd}
\rho_S(0) \otimes \rho_E \arrow[r, "\mathcal{U}(t,t_0)"] \arrow[d, "\mathrm{Tr}_E"] & \rho_{SE}(t) = \mathcal{U}(t,t_0)\, \rho_S(0) \otimes \rho_E\, \mathcal{U}^\dagger(t,t_0) \arrow[d, "\mathrm{Tr}_E"] \\
\rho_S(t_0) \arrow[r, "\Lambda(t,t_0)"] & \rho_S(t) = \mathrm{Tr}_E \left[\mathcal{U}(t,t_0)\, \rho_S(0) \otimes \rho_E\, \mathcal{U}^\dagger(t,t_0)\right]
\end{tikzcd}$$

Open quantum systems



$$\rho_S(t) = \Lambda(t,t_0)\rho_S(t_0), \quad \Lambda(t,t_0) : \mathcal{S}(\mathscr{H}_S) \to \mathcal{S}(\mathscr{H}_S)$$

- The dynamical map $\Lambda(t,t_0)$ has to be linear, trace preserving and completely positive (CP)

$$\frac{d}{dt}\rho_S(t) = -i\,[\mathcal{H}(t), \rho_S(t)] + \sum_{k=1}^{N^2-1} \boxed{\gamma_k(t)} \left(\sigma_k(t)\rho_S(t)\sigma_k^\dagger(t) - \frac{1}{2}\{\sigma_k^\dagger(t)\sigma_k(t), \rho_S(t)\} \right)$$

Time – dependent GKSL master equation

- No time-dependence and non-negative rates → QUANTUM DYNAMICAL SEMIGROUPS

Quantum non - Markovianity

NON – MARKOVIANITY IN QUANTUM DYNAMICS

- There is no immediate quantum parallel of the classical definition
- Non-Makovianity is related to the presence of memory effects in the dynamics
- Many sufficient conditions and estimators have been constructed which cope with the time-behavior of the statistical operator $\rho(t)$.

CP – divisibility based criterion

A´. Rivas, S.F. Huelga, M.B. Plenio, PRL **105**, 050403 (2010)

The time-evolution of a quantum system described in terms of a family of quantum dynamical maps $\{\Lambda(t,t_0)\}_{t\geq t_0}$ is Markovian if it is CP-divisible $\Lambda(t,t_0)=\Lambda(t,s)\Lambda(s,t_0)$

$$\frac{d}{dt}\rho_S(t)=-i\left[\mathcal{H}(t),\rho_S(t)\right]+\sum_{k=1}^{N^2-1}\boxed{\gamma_k(t)}\left(\sigma_k(t)\rho_S(t)\sigma_k^\dagger(t)-\frac{1}{2}\{\sigma_k^\dagger(t)\sigma_k(t),\rho_S(t)\}\right)$$

$\gamma_k(t)\geq 0$

Markovian dynamics

$\gamma_k(t)< 0$

Non-Markovian dynamics

Quantum non - Markovianity

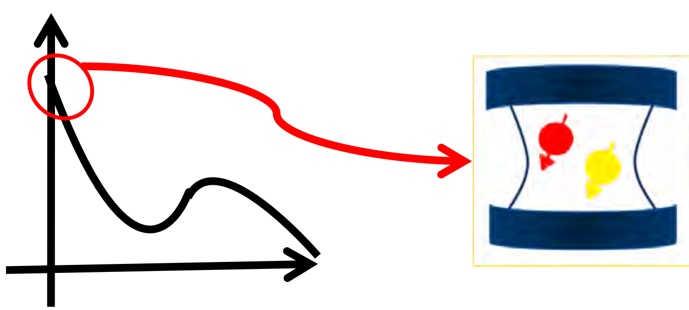
Trace-distance based criterion

$$D(\rho^1,\rho^2) = \frac{1}{2}\|\rho^1 - \rho^2\|_1 = \frac{1}{2}\sum_k |x_k|$$

It is a measure of the distinguishability between quantum states

It is a contraction under the action of PTP maps $D(t,\rho_S^{1,2}) \equiv D\big(\rho_S^1(t),\rho_S^2(t)\big), \quad \rho_S^k(t) = \Lambda(t)\rho_S^k$

Its can be employed to quantify the information flow between S and E



H. – P. Breuer, E. – M. Laine, J- Piilo, PRL **103** 210401 (2009)

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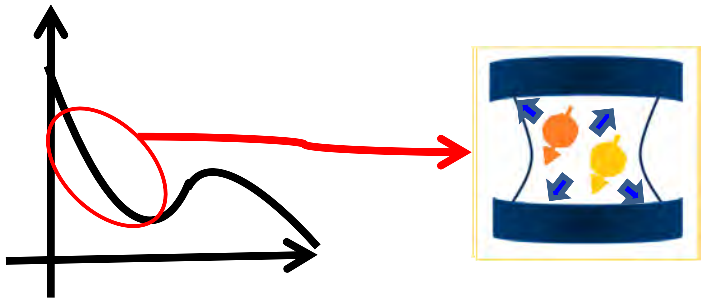

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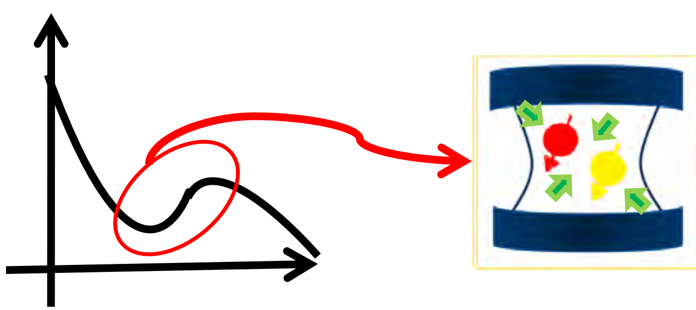

$$\sigma(t,\rho_S^{1,2}) = \frac{d}{dt}D(t,\rho_S^{1,2}) > 0$$

$$\mathcal{N} = \max_{\rho_S^{1,2}(0)} \frac{1}{2}\int_0^{+\infty} dt\, \left(\left|\sigma(t,\rho^{1,2})\right| + \sigma(t,\rho^{1,2})\right)$$

H. – P. Breuer, E. – M. Laine, J- Piilo, PRL **103** 210401 (2009)

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Closed quantum systems

Change in the internal energy: $\Delta U(t) = \mathrm{Tr}\left[\mathcal{H}(t)\rho(t)\right] - \mathrm{Tr}\left[\mathcal{H}(0)\rho(0)\right]$

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$$\begin{aligned}
\Delta U(t) &= \int_0^t d\tau \, \frac{d}{d\tau}\left(\mathrm{Tr}\left[\mathcal{H}(\tau)\rho(\tau)\right]\right) \\
&= \int_0^t d\tau \left(\mathrm{Tr}\left[\frac{d\mathcal{H}(\tau)}{d\tau}\rho(\tau)\right] + \mathrm{Tr}\left[\mathcal{H}(\tau)\frac{d\rho(\tau)}{d\tau}\right]\right) \\
&\equiv \int_0^t d\tau \left[\delta W(\tau) + \delta Q(\tau)\right],
\end{aligned}$$

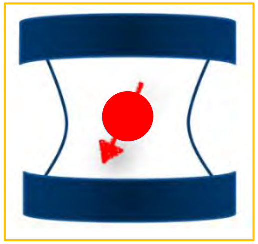
$$W(t) \equiv \int_{t_0}^t d\tau \, \delta W(\tau), \quad Q(t) \equiv \int_{t_0}^t d\tau \, \delta Q(\tau)$$

Work
(no change in system's entropy)

Heat
(no change in the Hamiltonian)

$$\Delta U(t) = W(t) + Q(t)$$

First law of thermodynamics

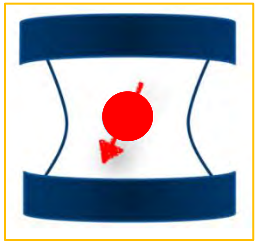


Open quantum systems

$$\mathcal{H}(t) = \mathcal{H}_S(t) + \mathcal{H}_E + \mathcal{H}_{SE}(t)$$

The eventual time dependence has to be thought as due to external driving fields, under the control of an eventual experimenter.

For this reason H_E is considered independent on time.



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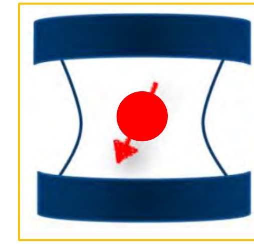
Work

$$W_E(t) = \int_0^t d\tau\, \mathrm{Tr}_E\left[\frac{d\mathcal{H}_E}{dt}\rho_E(t)\right] = 0$$

Heat

$$Q_E(t) \equiv \mathrm{Tr}_E\left[\mathcal{H}_E\left(\rho_E(t) - \rho_E(0)\right)\right] = \int_0^t d\tau\, \mathrm{Tr}_E\left[\mathcal{H}_E \frac{d\rho_E(t)}{dt}\right]$$

Heat in open quantum systems



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Work

$$W_E(t) = \int_0^t d\tau \, \mathrm{Tr}_E \left[\frac{d\mathcal{H}_E}{dt} \rho_E(t) \right] = 0$$

Heat

$$\boxed{Q_E(t)} \equiv \mathrm{Tr}_E \left[\mathcal{H}_E \left(\boxed{\rho_E(t)} - \rho_E(0) \right) \right] = \int_0^t d\tau \, \mathrm{Tr}_E \left[\mathcal{H}_E \frac{d\rho_E(t)}{dt} \right]$$

The full-counting statistics allows to access all the cumulants of the probability distribution

$p_t(\Delta a)$ of a change $\Delta a \equiv a_t - a_0$

in the eigenvalues of a self-adjoint operator

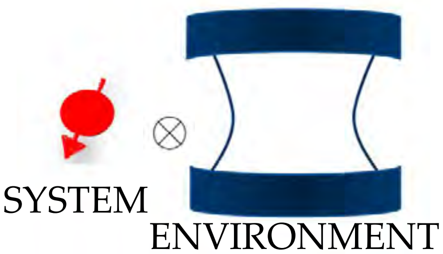
$$\hat{A}(t) = \sum_{a_t} a_t \hat{\Pi}_{a_t}$$

whose eventual time-dependence is due to the action of external driving fields

Full – Counting Statistics

Two – time measurement protocol

1

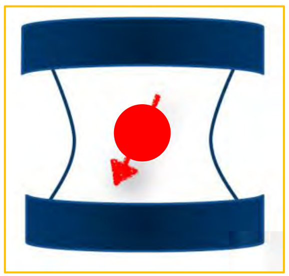


Consider a composite system starting in a product state and such a selected observable A

$$\rho_{SE}(0) = \rho_S(0) \otimes \rho_E$$

$$\boxed{\left[\hat{A}(0), \rho_{SE}(0)\right] = 0}$$

2



Perform a projective measurement of the observable A at initial time

$$\rho'_{SE}(0) = \frac{\Pi_{a_0} \rho_{SE} \Pi_{a_0}}{\mathrm{Tr}_{SE}\left\{\rho_{SE} \Pi_{a_0}\right\}}$$

3

Let the overall system undergo the coupled evolution $U(t,0)$

$$\rho'_{SE}(t) = U(t,0)\rho'_{SE}(0)U^{\dagger}(t,0)$$

4

Detatch S and E and perform another measurement of the same observable A at time t

$$\rho''_{SE}(t) = \frac{\Pi_{a_t} \rho'_{SE}(t) \Pi_{a_t}}{\mathrm{Tr}_{SE}\left\{\rho'_{SE}(t) \Pi_{a_t}\right\}}$$

Esposito, Harbola, Mukamel, Rev. Mod. Phys. **81**, 1665 (2009)

Full – Counting Statistics

- The probability distribution $p_t(\Delta a)$ for a change $\Delta a \equiv a_t - a_0$ to occur between time 0 and time t is given by

$$p_t(\Delta a) = \sum_{a_0, a_t} \mathbb{P}_t\,[a_t; a_0]\,\delta(\Delta a - a_t + a_0)$$

where $\mathbb{P}_t\,[a_t; a_0] = \mathrm{Tr}\left[\hat{\Pi}_{a_t}\hat{U}(t,0)\hat{\Pi}_{a_0}\rho(0)\hat{\Pi}_{a_0}\hat{U}^\dagger(t,0)\hat{\Pi}_{a_t}\right]$

- Upon introducing the cumulant generating function

$$\Theta(\eta, t) \equiv \ln\langle e^{i\eta\Delta a}\rangle_t = \ln\int d(\Delta a)\, p_t(\Delta a) e^{i\eta\Delta a}$$

the cumulants of Δa are obtained by derivation as

$$\langle(\Delta a)^n\rangle_t = (-i)^n \frac{\partial^n}{\partial\eta^n}\Theta(\eta, t)|_{\eta=0}$$

Full – Counting Statistics

- Under the assumption $\left[\hat{A}(0), \rho(0)\right] = 0$,

  the cumulant generating can be re-expressed as

$$\boxed{\Theta(\eta, t) = \ln \mathrm{Tr}_S\left[\rho_S(\eta, t)\right]}$$

System's operator

where

$$\rho_S(\eta, t) \equiv \mathrm{Tr}_E\left\{U_{\eta/2}(t,0)\rho_{SE}(0)U^{\dagger}_{-\eta/2}(t,0)\right\}$$

with $$\hat{U}_\eta(t,0) = e^{i\eta\hat{A}(t)}\hat{U}(t,0)e^{-i\eta\hat{A}(0)}$$

Modified evolution operator: usual evolution conditioned on two 'rotations' induced by the observable A

MAIN POINT: USEFULNESS OF FULL-COUNTING STATISTICS

- Under the same approximations and employing the same techniques (Nakajima-Zwanzig projectors, perturbative expansion...) used to derive a master equation for the evolution of $\rho_S(t)$, one obtains a master equation for $\rho_S(\eta, t)$, usually called generalized master equation (GME).

- The cumulant generating function $\Theta(\eta, t) = \ln \mathrm{Tr}_S \left[\rho_S(\eta, t)\right]$ is then obtained by solving the GME and, simply by derivation, it gives the cumulants of the probability distribution for the change in the selected observable.

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- QUANTUM REGRESSION THEOREM
  - THE PURE-DEPHASING SPIN BOSON

PART 1

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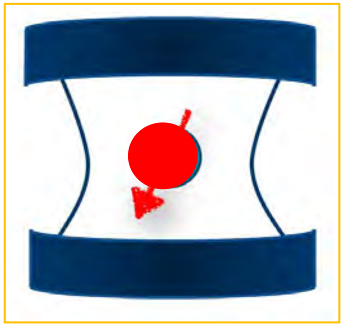
PART 2

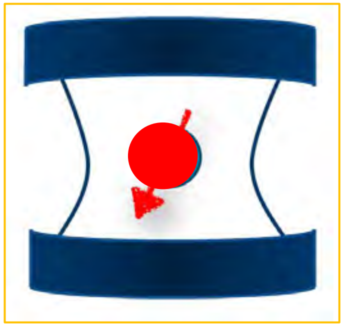
Heat Backflow: occurrence and quantifier



$$\Theta(\eta, t) = \ln \mathrm{Tr}_S \left[\rho_S(\eta, t)\right]$$

- $\rho_{SE}(0) = \rho_S(0) \otimes \rho_E$
- Weak coupling between S and E
- $\frac{d}{dt}\rho_S(\eta, t) = \Xi^\eta(t)\rho_S(\eta, t)$
- ρ_E Gibbs state

Heat Backflow: occurrence and quantifier



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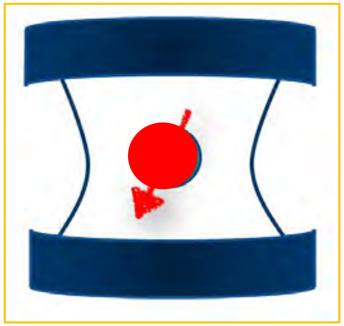
$$\Theta(\eta,t) = \ln \mathrm{Tr}_S\left[\rho_S(\eta,t)\right]$$

Projection – operator technique and second-order time-convolutionless expansion

$$\Xi^{(\eta)}(t)\left[\omega\right] = -i\left[\mathcal{H}_S,\omega\right] - \int_0^t d\tau\, \mathrm{Tr}_E\left\{\left[\mathcal{H}_{int},\, \left[\mathcal{H}_{int}(-\tau),\, \omega\otimes\rho_E(0)\right]_{\eta}\right]_{\eta}\right\}$$

$$\left[\mathcal{H}_{int}(t)\,,\,B\right]_{\eta} \equiv \mathcal{H}^{\eta}_{int}(t)B - B\mathcal{H}^{-\eta}_{int}(t) \qquad \mathcal{H}^{\eta}_{int}(t) = e^{(i/2)\eta\mathcal{H}_E}\mathcal{H}_{int}(t)e^{-(i/2)\eta\mathcal{H}_E}$$

Heat Backflow: occurrence and quantifier



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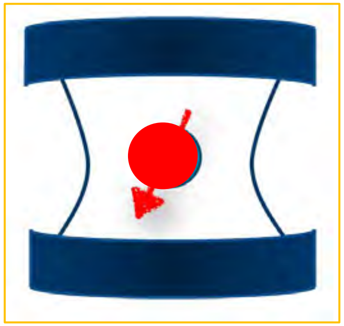
FINITE DIMENSIONAL SYSTEMS

$$\rho_S(\eta, t) = T_+ \exp\left[\int_0^t d\tau \Xi^\eta(\tau)\right]\rho_S(0) \rightarrow |\rho_S(\eta, t)\rangle = T_+ \exp\left[\int_0^t d\tau \boldsymbol{\Xi}^\eta(\tau)\right]|\rho_S(0)\rangle \equiv \boldsymbol{\Lambda}^\eta(t, 0)\,|\rho_S(0)\rangle$$

$$\rightarrow \quad \langle \Delta q \rangle_t = \int_0^t d\tau \theta(\tau) \qquad \theta(t) \equiv \langle \mathbb{1}| \left.\frac{\partial \boldsymbol{\Xi}^\eta(t)}{\partial(i\eta)}\right|_{\eta=0} |\rho_S(t)\rangle$$

Heat Backflow: occurrence and quantifier



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INFINITE DIMENSIONAL SYSTEMS

$$\chi\left[\rho_S(\eta,t)\right](\lambda,\lambda^*) \equiv \chi^{(\eta)}(\lambda,\lambda^*,t) = \mathrm{Tr}_S\left[\rho_S(\eta,t)e^{\lambda a^\dagger - \lambda^* a}\right]$$

$$\Theta(\eta,t) = \ln \mathrm{Tr}_S\left[\rho_S(\eta,t)\right] = \ln \chi^{(\eta)}(0,0,t)$$

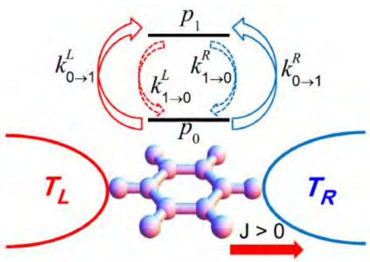
$$\rightarrow \quad \langle \Delta q\rangle_t = \frac{\partial \chi^{(\eta)}(0,0,t)}{\partial(i\eta)}\Big|_{\eta=0} \qquad \theta(t) = \frac{\partial \dot{\chi}^{(\eta)}(0,0,t)}{\partial(i\eta)}\Big|_{\eta=0}$$

Heat Backflow: occurrence and quantifier

Born-Markov and RWA approximations

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = \mathcal{L}\ \rho(t) = -i[H\,,\rho(t)] + \sum_m \Delta_m \left[C_m\,\rho C_m^\dagger - \frac{1}{2}\left\{ C_m^\dagger C_m\,,\rho \right\} \right]$$

Time-independent GKSL master equation



$$\vartheta(\eta) = lim_{t\to+\infty}\Theta(\eta,t)/t \longrightarrow \langle\Delta q\rangle_t \approx \langle\Delta q\rangle t$$

Temperature-induced **steady** heat flow from hot to cold subsystem

Esposito *et al.*, RMP **81**, 1665 (2009); Ren *et al.*, PRL **104**, 170601 (2010); C. Uchiyama PRE **89**, 052108 (2014)

Heat Backflow: occurrence and quantifier

QUANTUM DYNAMICAL SEMIGROUPS

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t)=\mathcal{L}\;\rho(t)=-i[H\,,\rho(t)]+\sum_m\Delta_m\left[C_m\,\rho C_m^\dagger-\frac{1}{2}\left\{C_m^\dagger C_m\,,\rho\right\}\right]$$

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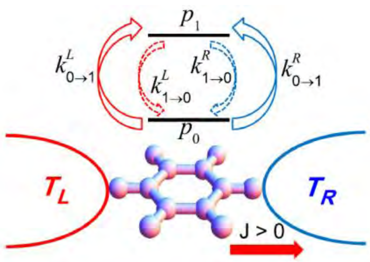

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Beyond semigroup dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t)=\mathcal{L}_t\rho(t)=-i[H(t),\rho(t)]+\sum_m\Delta_m(t)\left[C_m(t)\rho C_m^\dagger(t)-\frac{1}{2}\left\{C_m^\dagger(t)C_m(t),\rho\right\}\right]$$

$$\langle\Delta q\rangle_t=\int_0^t d\tau\theta(\tau)$$

Heat Backflow: occurrence and quantifier

- Given a system S weakly coupled to an environment E, we speak of time regions of *heat backflow from E to S* whenever, considering dynamical situations which in the Born-Markov semigroup approximation would lead to a non-negative steady energy transfer from system to environment, we have that at some time t

$$\theta(t) < 0.$$

Building on this condition, a measure for the total amount of energy which has flown back from the environment to the system during the evolution is naturally introduced as

$$\langle \Delta q \rangle_{back} = \max_{\rho_S(0)} \frac{1}{2} \int_0^{+\infty} dt \, \left(|\theta(t)| - \theta(t) \right),$$

G. Guarnieri, C. Uchiyama, B. Vacchini, PRA **93**, 012118 (2016)

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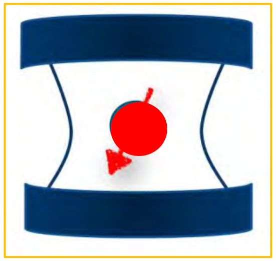
- After the maximization over the possible initial states of the reduced system, it becomes a property of the dynamical map only.

Application to the Spin – boson model



- The two – level system is coupled to an environment consisting of an infinite number of bosonic modes.
- The Hamiltonian is $\mathcal{H} = \frac{\omega_0}{2}\sigma_z + \sum_k \omega_k b_k^\dagger b_k + \sigma_x \otimes B_E$

$$B_E \equiv \sum_k \left(g_k b_k^\dagger + g_k^* b_k \right)$$

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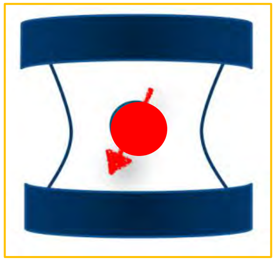

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- The energy flow per unit of time has the form $\theta(t) \equiv \langle 1 | \frac{\partial \Xi^{(\eta)}(t)}{\partial (i\eta)} | \rho_S(t) \rangle_{|\eta=0}$

$$\Xi^\eta(t) = -\int_0^t d\tau \begin{pmatrix} V_+(\tau) & 0 & 0 & W_+^\eta(\tau) \\ 0 & Y_+(\tau) & Z_+^\eta(\tau) & 0 \\ 0 & Z_-^\eta(\tau) & Y_-(\tau) & 0 \\ W_-^\eta(\tau) & 0 & 0 & V_-(\tau) \end{pmatrix}$$

$$V_\pm(\tau) = \Phi(\tau) e^{\mp i\omega_0 \tau} + \Phi(-\tau) e^{\pm i\omega_0\tau},$$
$$W_\pm^\chi(\tau) = -\left[\Phi(\tau-\chi) e^{\pm i\omega_0\tau} + \Phi(-\tau-\chi) e^{\mp i\omega_0\tau} \right],$$
$$Y_\pm(\tau) = 2Re\left[\Phi(\tau)\right] e^{\mp i\omega_0\tau},$$
$$Z_\pm^\chi(\tau) = -\left[\Phi(\tau-\chi) + \Phi(-\tau-\chi)\right] e^{\pm i\omega_0\tau}.$$

Environmental correlation function

$$\Phi(\tau) = \int_0^{+\infty} d\omega\, J(\omega) \left[\coth\left(\frac{\omega}{2T_E}\right) \cos(\omega\tau) - i\sin(\omega\tau) \right]$$

Application to the Spin – boson model

- Assuming the spetral density to be of the form $J(\omega) = \lambda\omega e^{-\frac{\omega}{\Omega}}$
- $\rho_S(0) = Z^{-1}\left(|0\rangle\langle 0| + e^{-\omega_0/T_S}|1\rangle\langle 1|\right)\,, \quad Z = 1+e^{-\omega_0/T_S}$
- $T_S \geq T_E$ Condition which guarantees, in the Born-Markov limit, a steady heat flow from the system to the environment

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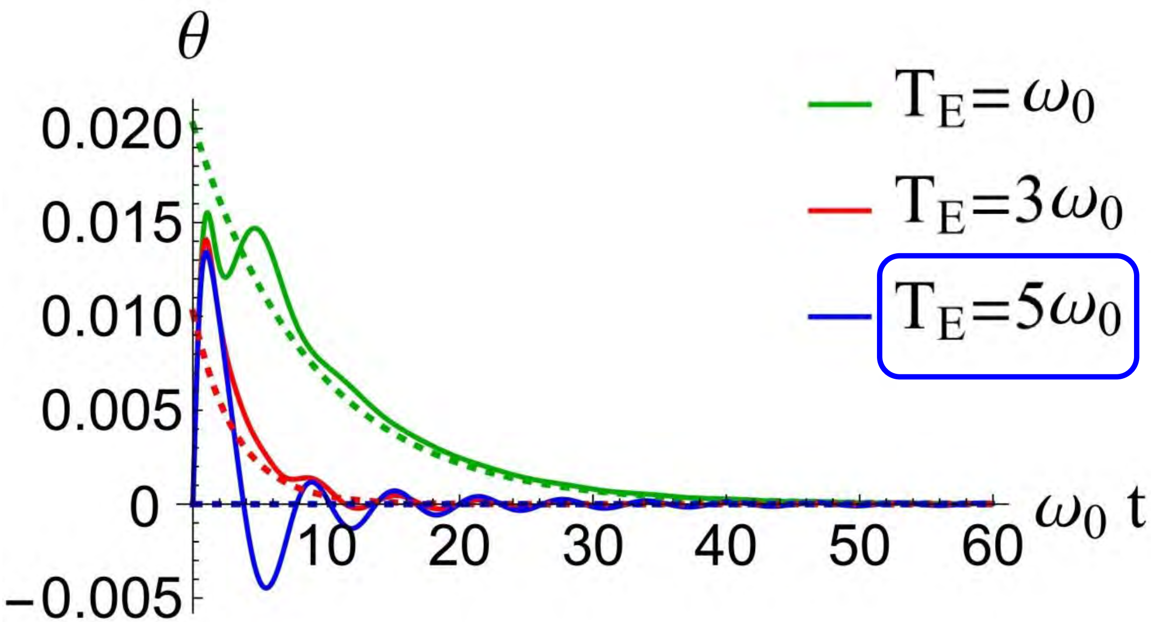


$\Omega = 0.4\,\omega_0$

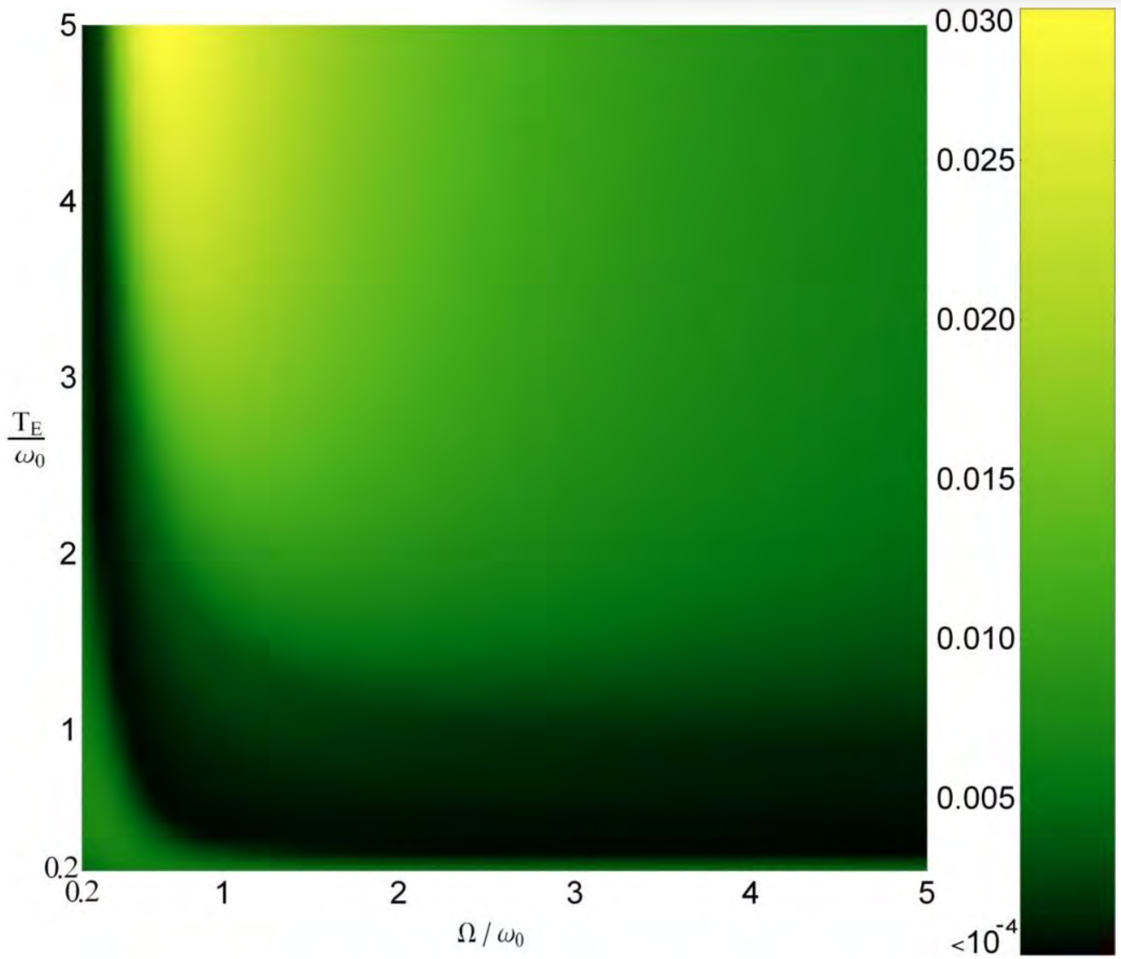
$\lambda = 0.1$

$\boxed{T_S = 5\omega_0}$

For every value of λ, Ω and T_E the heat backflow is maximized by the choice $T_S = T_E$

Application to the Spin – boson model

$$\langle \Delta q \rangle_{back} = \max_{\rho_S(0)} \frac{1}{2} \int_0^{+\infty} dt \, (|\theta(t)| - \theta(t))$$

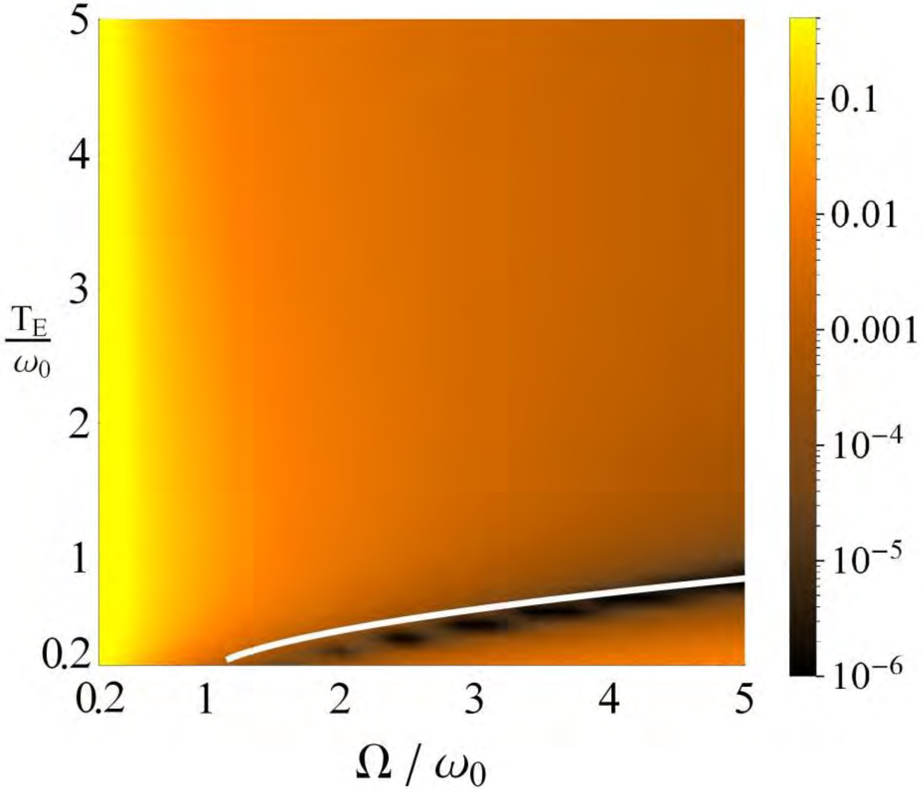


- $\lambda = 0.1$
- $T_S = T_E$
- Region of max heat backflow
- Region of absent heat backflow

G. Guarnieri, C. Uchiyama, B. Vacchini, PRA **93**, 012118 (2016)

Relationship with the non-Markovanity

Spin – boson model



- $\lambda = 0.1$
- The reduced dynamics is always non-Markovian except on the resonance curve, defined by the condition

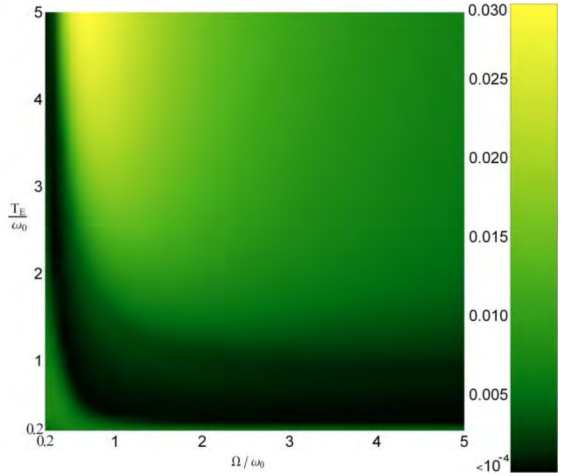
$$\frac{\partial}{\partial \omega} J_{eff}(\omega, \Omega, T_E)_{|\omega=\omega_0} = 0$$

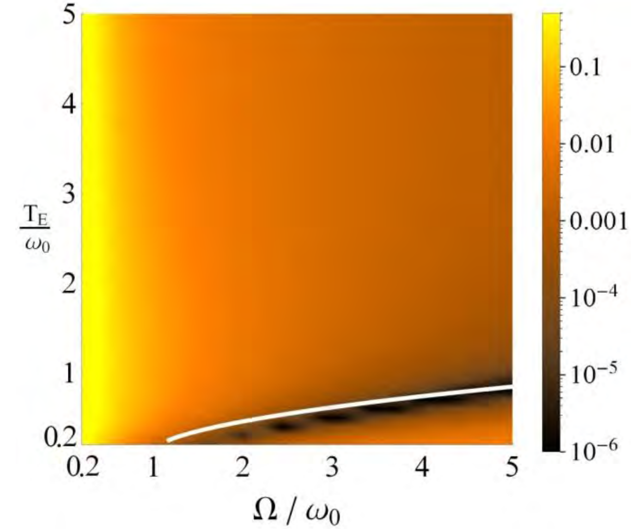
$$J_{eff}(\omega, \Omega, T_E) \equiv J(\omega) \coth\left(\frac{\omega}{2T_E}\right)$$

- Locally white-noise spectrum around the system's transition frequency

G. Clos, H. – P. Breuer, PRA **86**, 012115 (2012)

Relationship with the non-Markovanity





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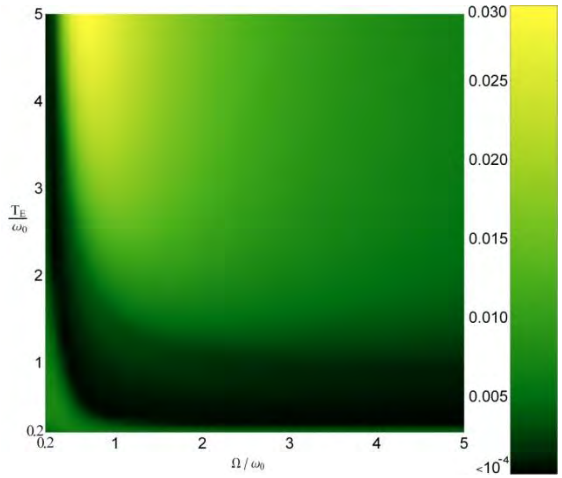
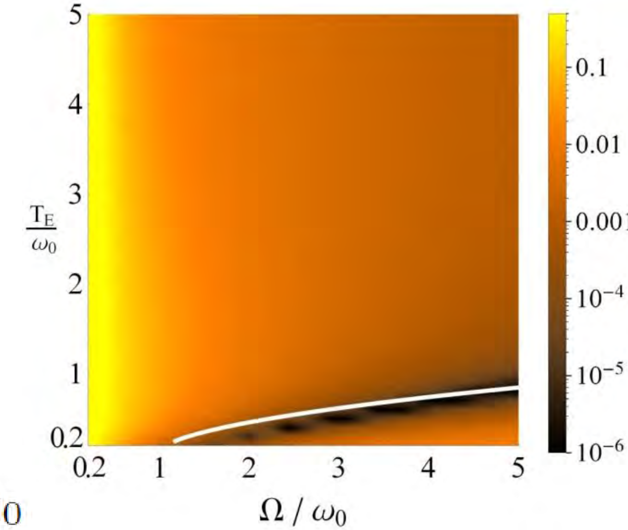
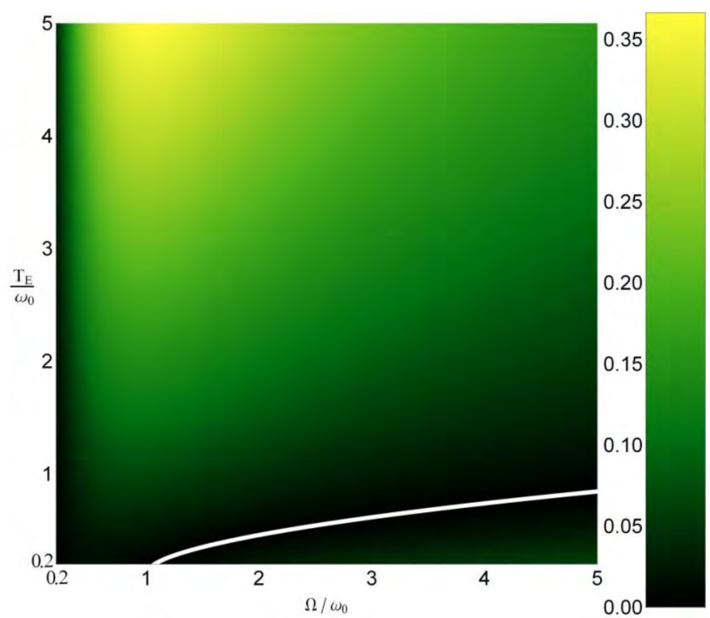




$$\partial_\omega J_{eff}(\omega, \Omega, T_E)_{|\omega=\omega_0}$$



The heat backflow measure is suppressed whenever the resonance condition approximately holds (i.e. on the black region)

The non-Markovianity measure is suppressed whenever the resonance condition strictly holds (i.e. on the white line)

G. Guarnieri, C. Uchiyama, B. Vacchini, PRA **93**, 012118 (2016)

Application to the QBM

- An harmonic oscillator is coupled to an environment consisting of an infinite number of bosonic modes.

- $$\mathcal{H} = \frac{\omega_0}{2}\left(a^\dagger a + 1/2\right) + \sum_k \omega_k b_k^\dagger b_k + X \otimes B_E$$

$$X = 2^{-1/2}(a + a^\dagger)$$

$$(P = 2^{-1/2} i (a^\dagger - a))$$

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Weak coupling regime

- Analytic Fokker-Planck master equation

$$\frac{d}{dt}\chi^{(\eta)}(q,p,t) = \Big\{\omega_0\left(q\partial_p - p\partial_q\right) - V_1(\eta,t)\left(\partial^2_{qq} + \partial^2_{pp}\right) - \left(2\Delta(t) + V_1(\eta,t)\right)\frac{q^2+p^2}{4} + \left(V_2(\eta,t) - \gamma(t)\right)\left(q\partial_q + p\partial_p\right) + V_2(\eta,t)\Big\}\chi^{(\eta)}(q,p,t)$$

- Analytic solution for the heat flow rate

$$\theta(t) = 2\sigma(0,t)\left(\frac{1}{2}D_2(t)\cos(\omega_0 t) + \omega_0\gamma(t)\right) + \frac{1}{2}D_1(t)\sin(\omega_0 t) - \omega_0\Delta(t)$$

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Strong coupling regime

- Fully numerical simulation with finite-number of environmental modes

$$\mathcal{H} = \frac{\mathbf{P}^T\mathbf{P}}{2} + \mathbf{X}^T\mathbf{M}\mathbf{X}$$

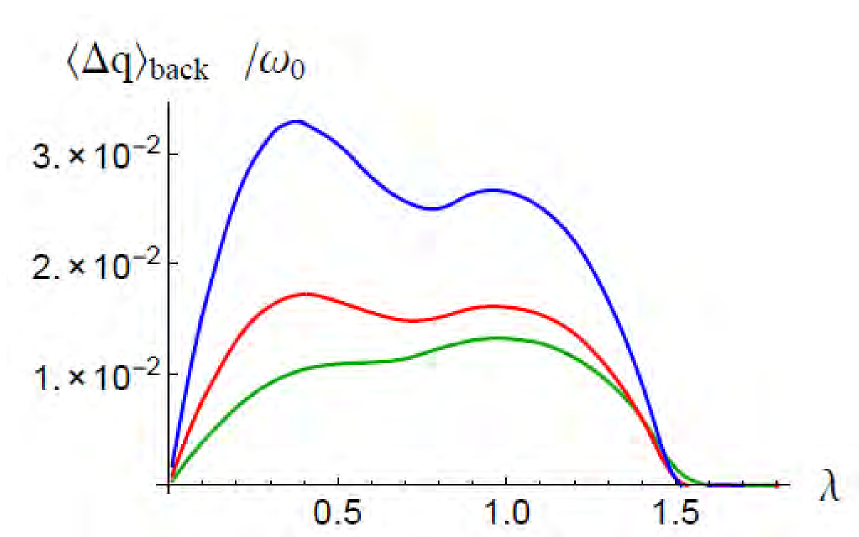
$$X_i(t) = \sum_{j=1}^{N+1}\left[\mathbf{M}_{ij}^{XX}(t)X_j(0) + \mathbf{M}_{ij}^{XP}(t)P_j(0)\right]$$

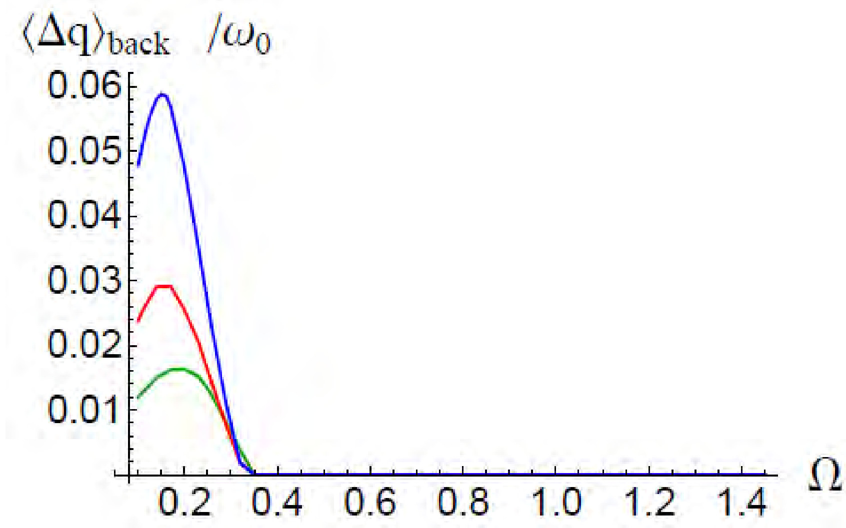
$$P_i(t) = \sum_{j=1}^{N+1}\left[\mathbf{M}_{ij}^{PX}(t)X_j(0) + \mathbf{M}_{ij}^{PP}(t)P_j(0)\right]$$

Application to the QBM

$$\langle \Delta q \rangle_{back} = \max_{\rho_S(0)} \frac{1}{2} \int_0^{+\infty} dt \, \left(|\theta(t)| - \theta(t) \right)$$

$T_S = T_E$



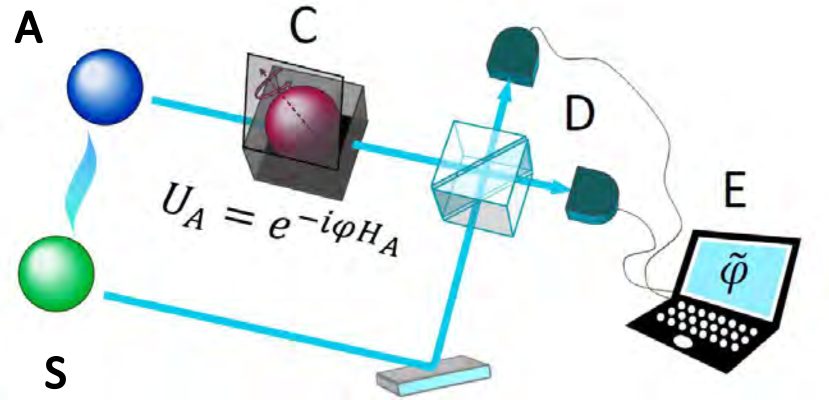


- Cut-off values in λ and Ω for the heat backflow

G. Guarnieri, J. Nokkala, R. Schmidt, S. Maniscalco, B. Vacchini, PRA 94 (2016)

Relationship with the non-Markovanity

Gaussian Interferometric Power



$$\mathcal{Q}\left(\rho_{SA}\right) = \frac{1}{4} \inf_{\mathcal{H}_A} \mathcal{J}\left(\rho_{SA}^{\mathcal{H}_A}\right)$$

$\mathcal{J}\left(\rho_{SA}^{\mathcal{H}_A}\right)$ Quantum Fisher Information

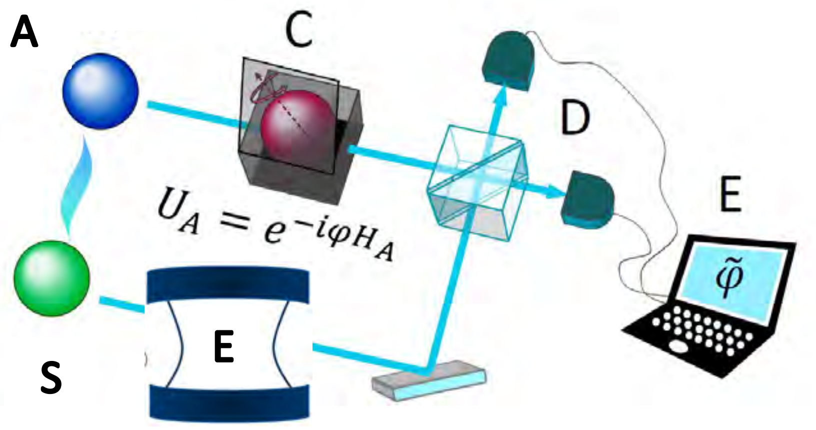
- Likewise the trace distance, this figure of merit is a contraction under the action of CPT maps

⇨ Non-Markovian dynamics if $\frac{d}{dt}\mathcal{Q}\left(\rho_{SA}\right) > 0$

$$\mathcal{N}_{QIP} = \max_{\rho_{SA}(0)} \frac{1}{2} \int_{\mathbb{R}+} dt \left(\left| \frac{d}{dt}\mathcal{Q}\left(\rho_{SA}\right) \right| + \frac{d}{dt}\mathcal{Q}\left(\rho_{SA}\right) \right)$$

H. S. Dhar, M. N. Bera, G. Adesso, PRA **91**, 032115 (2015)

Gaussian Interferometric Power



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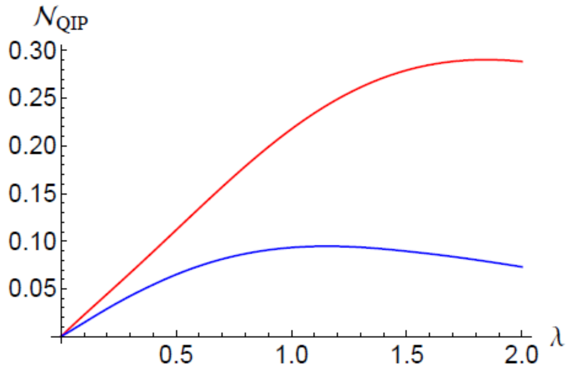
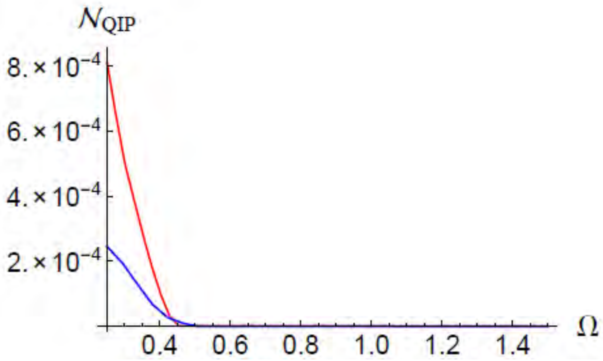
H. S. Dhar, M. N. Bera, G. Adesso, PRA **91**, 032115 (2015)

Relationship with the non-Markovanity

Gaussian Interferometric Power

For the QBM model





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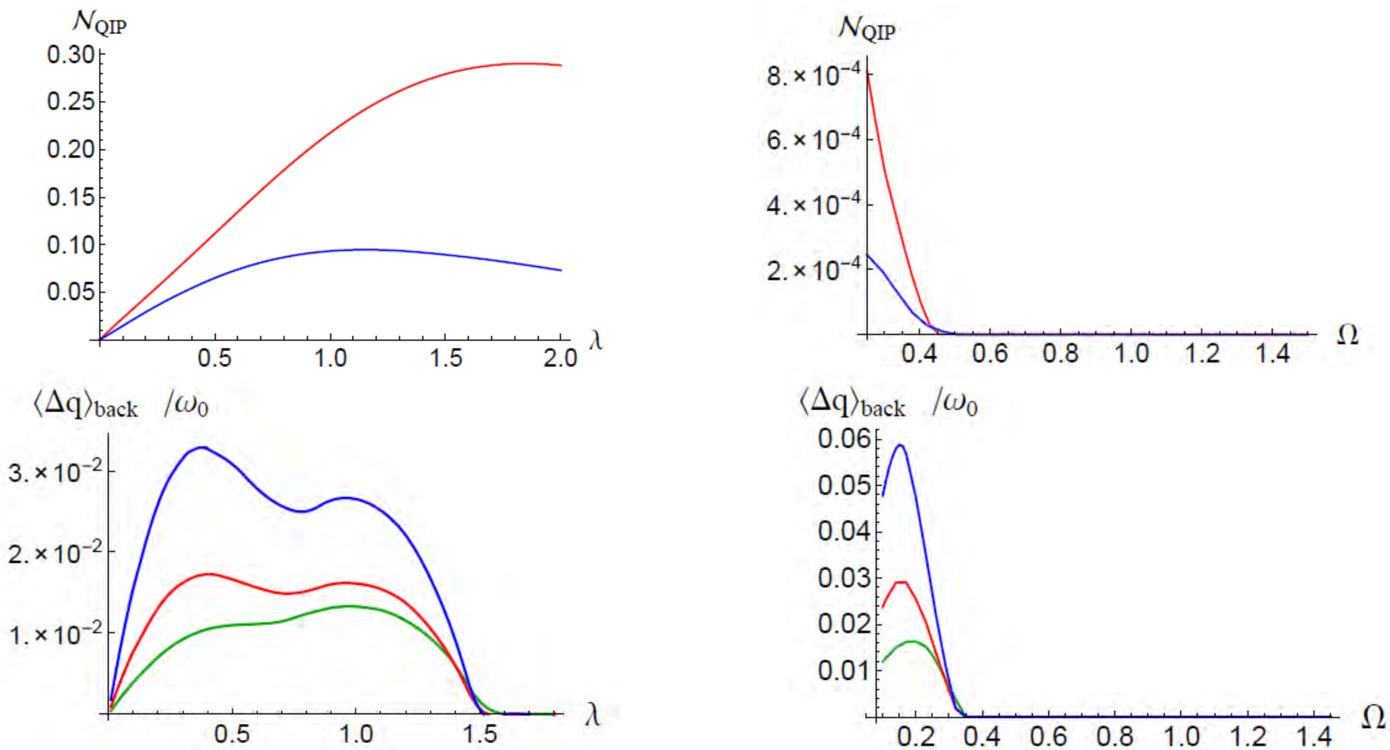

- λ dependence: cut-off for the heat backflow, absent for the nM measure
- Ω dependence: very similar behavior, cut-off for frequency both measures

Conclusions

For both the spin – boson model and for the quantum Brownian motion,

we had that:

- A generally non-Markovian description of the reduced dynamics causes the heat flow to oscillate and even come back from the environment to the system.

- The heat backflow measure is maximized whenever the initial temperatures of system and environment are equal to each other

- The occurrence of heat backflow represents a stricter condition than non-Markovianity, in the sense that the latter is required in order to witness the former and that, on the contrary, a Markovian dynamics prevents its observation.

Contents

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- QUANTUM NON - MARKOVIANITY
- HEAT IN OPEN QUANTUM SYSTEMS
- FULL - COUNTING STATISTICS
- HEAT BACKFLOW OCCURRENCE AND MEASURE
  - SPIN - BOSON
  - QUANTUM BROWNIAN MOTION
- **LOWER BOUND TO THE MEAN DISSIPATED HEAT**
  - LANDAUER'S PRINCIPLE
  - NON - EQUILIBRIUM LOWER BOUND
  - XX - COUPLED AND DRIVEN V-SYSTEM
- CONCLUSIONS

Landauer's principle

$$\beta Q_E \geq \Delta S$$

Lower bound to the mean dissipated heat

ERASURE PROTOCOL SCENARIO

1. S and E described in terms of Hilbert spaces;

2. the environment is initially described in terms of a thermal state $\rho_\beta = e^{-\beta \mathcal{H}_E}/Z_E$, with $\mathcal{H}_E$ being a self-adjoint Hamiltonian operator on $\mathscr{H}_E$, $\beta \in [-\infty, +\infty]$ its inverse temperature and $Z_E = \mathrm{Tr}_E\left[e^{-\beta \mathcal{H}_E}\right]$;

3. the total initial state is uncorrelated $\rho_{SE}(0) = \rho_S(0) \otimes \rho_\beta$;

4. the evolution of the overall system $S + E$ is given in terms of an unitary operator, i.e. $\rho_{SE}(t) = U(t,0)\rho_{SE}(0)\, U^\dagger(t,0)$.

D. Reeb and M. M. Wolf, New J. Phys. **16**, 103011 (2014)

Non-equilibrium lower bound

A thermodynamic-based lower bound

- $$\rho'_E = \mathrm{Tr}_S \left[\mathcal{U} \left(\rho_S(0) \otimes \rho_\beta \right) \mathcal{U}^\dagger \right] = \sum_k A_k \rho_\beta A_k^\dagger$$

Transformation of the environmental state

- $$p(\Delta E) = \sum_{l,n,m} \langle r_n | \hat{A}_l | r_m \rangle (\rho_E)_{mm} \langle r_m | \hat{A}_l^\dagger | r_n \rangle \delta \left(\Delta E - (E_n - E_m) \right)$$

constructed by means of the A_k's contains far more information than just

$$Q_E \equiv \langle \Delta E \rangle = \int p(\Delta E)\, \Delta E\, d(\Delta E)$$

- $$\langle e^{-\beta \Delta E} \rangle = \int e^{-\beta \Delta E} p(\Delta E) d(\Delta E) = \mathrm{Tr}_E \left[\rho_\beta \mathbf{A} \right] \qquad \mathbf{A} = \sum_k A_k A_k^\dagger$$

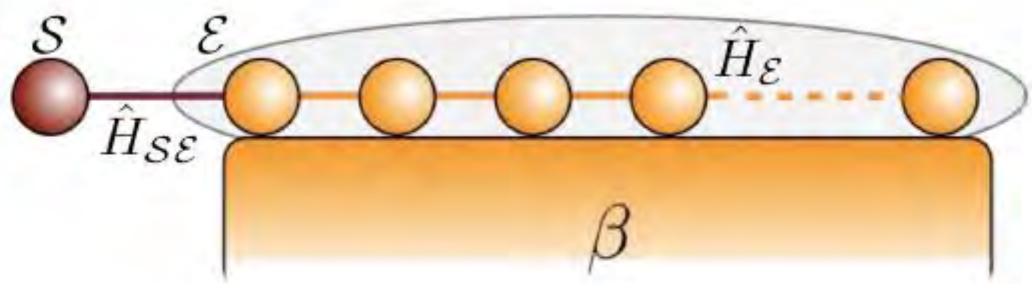
Non – unitality degree of the environmental map

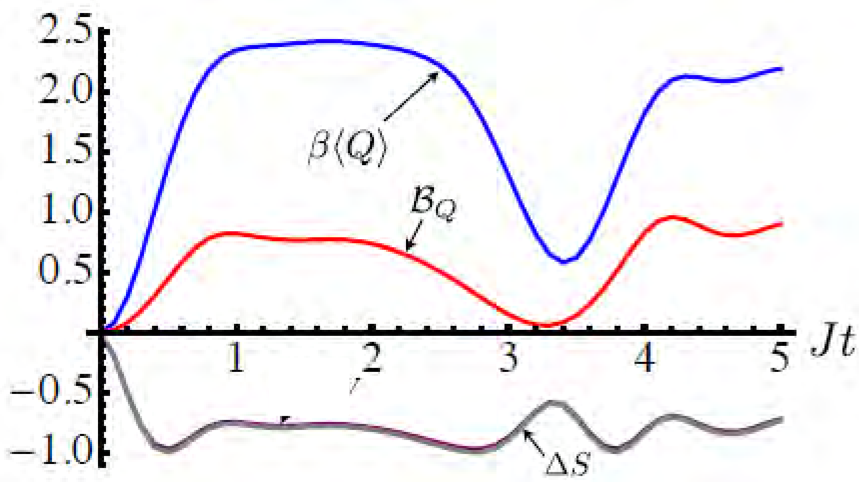
Jensen's inequality $\langle f(x) \rangle \geq f(\langle x \rangle)$ $\Rightarrow$ $\beta Q_E \geq - \ln \mathrm{Tr}_E \left[\rho_\beta \mathbf{A} \right]$

J. Goold and M. Paternostro and K. Modi, Phys. Rev. Lett. **114**, 060602 (2015)

Non-equilibrium lower bound

Spin system coupled to an isotropic XX model





The thermodynamical bound here outperforms Landauer's (altough not in general)

J. Goold and M. Paternostro and K. Modi, Phys. Rev. Lett. **114**, 060602 (2015)

Non-equilibrium lower bound

A full – counting statistics – based approach

$\beta Q_E \geq \Delta S$ Obtained exploiting FCS

Laplace transform of the probability distribution

$$\tilde{\Theta}(\eta,\beta,t) \equiv \ln\langle e^{-\eta\Delta E}\rangle_t = \ln\int d(\Delta E)p_t(\Delta E)e^{-\eta\Delta E}$$

- **Hölder's inequality**

$$\| fg \|_1 \leq \| f \|_p \| g \|_q, \qquad 0 \leq q,p, \leq 1,\ \frac{1}{p}+\frac{1}{q}=1$$

Convexity property of the cumulant generating function

$$\begin{aligned}\tilde{\Theta}([\alpha\eta_1+(1-\alpha)\eta_2],\beta,t) &= \ln\langle e^{-[\alpha\eta_1+(1-\alpha)\eta_2]\Delta E}\rangle_t \\ &\leq \alpha\ln\langle e^{-\eta_1\Delta E}\rangle_t + (1-\alpha)\ln\langle e^{-\eta_2\Delta E}\rangle_t = \alpha\tilde{\Theta}(\eta_1,\beta,t)+(1-\alpha)\tilde{\Theta}(\eta_2,\beta,t)\end{aligned}$$

Non-equilibrium lower bound

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$$\leq \alpha \ln\langle e^{-\eta_1 \Delta E}\rangle_t + (1-\alpha)\ln\langle e^{-\eta_2\Delta E}\rangle_t = \alpha\tilde{\Theta}(\eta_1, \beta, t) + (1-\alpha)\tilde{\Theta}(\eta_2, \beta, t)$$

$$\tilde{\Theta}(\eta, \beta, t) \geq \eta \frac{\partial}{\partial\eta}\tilde{\Theta}(\eta, \beta, t)|_{\eta=0}$$

Non-equilibrium lower bound

$$\tilde{\Theta}([\alpha\eta_1 + (1-\alpha)\eta_2]\,, \beta, t) = \ln\langle e^{-[\alpha\eta_1+(1-\alpha)\eta_2]\Delta E}\rangle_t$$
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$$\boxed{\beta Q_E(t) \geq -\frac{\beta}{\eta}\tilde{\Theta}(\eta,\beta,t) \equiv \mathcal{B}_Q^{\eta}(t), \qquad \eta > 0}$$

- New one-parameter family of lower bounds to the mean dissipated heat
- It applies to a non-equilibrium scenario

Non-equilibrium lower bound

- Consider the family of lower bounds $\mathcal{B}_Q^{\eta}(t) = -\frac{\beta}{\eta}\tilde{\Theta}(\eta, \beta, t)$

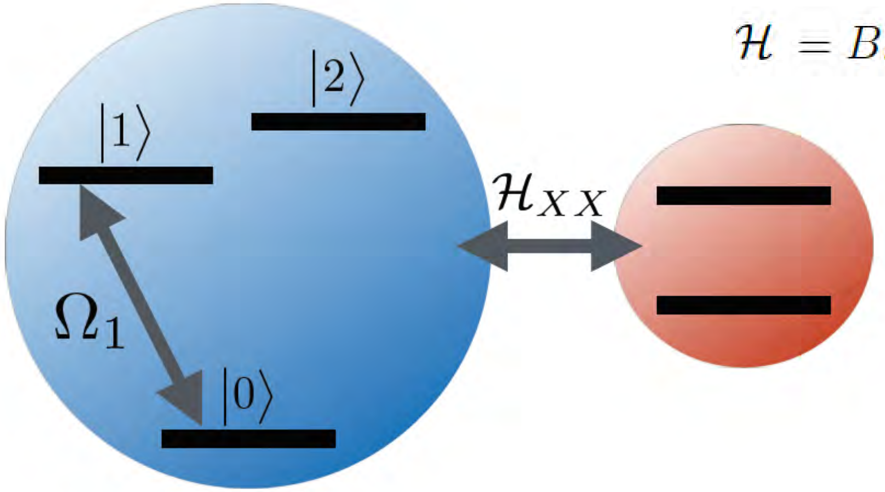
- $\eta = \beta$ case

$$\mathcal{B}_Q^{\beta}(t) = -\tilde{\Theta}(\beta, \beta, t) = \ln \mathrm{Tr}_E\left[\rho_\beta \mathbf{A}\right]$$

For this choice of the counting field parameter we retrieve the lower bound btained by Goold, Modi and Paternostro.

- For $0 < \eta \leq \beta$ we therefore find lower bounds which outperform it

XX – coupled and driven V - system



$$\mathcal{H} = BS_z^{20} + B\sigma_z + J\left(S_x^{20}\otimes\sigma_x + S_y^{20}\otimes\sigma_y\right) + \underline{D}^{20}\cdot\underline{E}$$

$$\underline{D}^{20} = \underline{d}S_-^{20} + \underline{d}^* S_+^{20}$$

$$S_\pm^{20} = \frac{1}{2}\left(S_x^{20} \pm i S_y^{20}\right)$$

$$\underline{E} = \varepsilon + \varepsilon^*$$

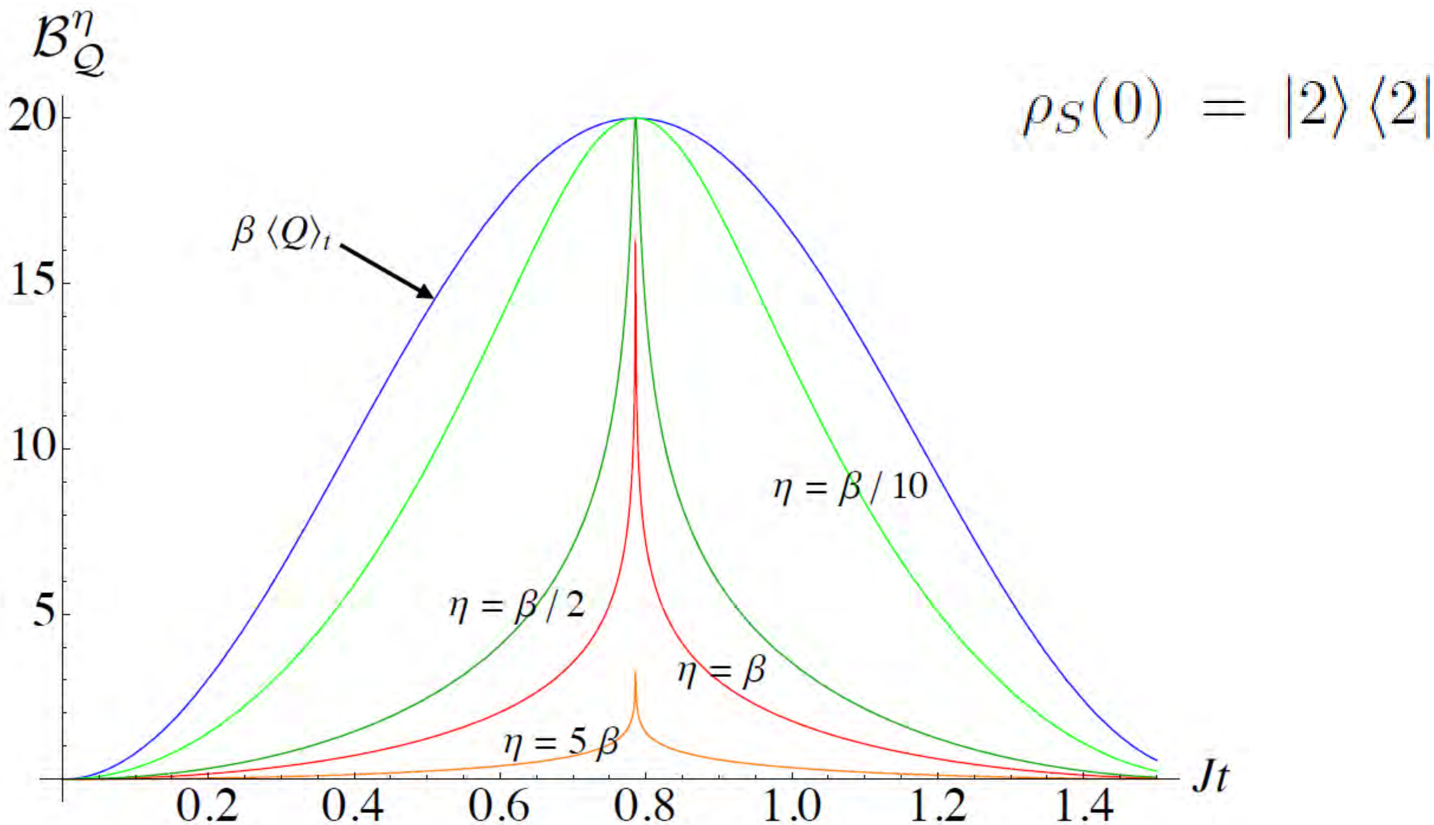
$$\varepsilon = i\sum_{\mathbf{k}}\sum_{\lambda=1,2}\left(\sqrt{2\pi\omega_{\mathbf{k}}/V}\right)\mathbf{e}_\lambda(\mathbf{k})b_\lambda(\mathbf{k})$$

- Interaction picture $\mathcal{H} = J\left(S_x^{20}\otimes\sigma_x + S_y^{20}\otimes\sigma_y\right) + \Omega_1 S_x^{20}\otimes\mathbb{1}_2$

- $$\Theta(\eta,\beta,t) = \log\left([1+\tanh(\beta B)]\frac{16J^2\Omega_1^2 e^{-2B\eta}\sin^4\left(\frac{\omega_1}{2}t\right) + 4J^2 e^{-2B\eta}\sin^2\left(\omega_1 t\right)}{2\omega_1^4} + [1+\tanh(\beta B)]\frac{\left(4J^2\cos\left(\omega_1 t\right)+\Omega_1^2\right)^2}{2\omega_1^4}\frac{1-\tanh(\beta B)}{2}\right),$$

- $$Q_E(t) = [1+\tanh(\beta B)]\frac{16BJ^2\sin^2\left(\frac{\omega_1}{2}t\right)\left[-4J^2\sin^2\left(\frac{\omega_1}{2}t\right)+\omega_1^2\right]}{\omega_1^4}$$

XX – coupled and driven V - system



- Asymptotically tight family of lower bounds to the mean dissipated heat
- Even the $\eta = \beta$ case outperforms Landauer's result in this case, since the change in the system's entropy is here a non-positive quantity at any time

Conclusions (1)

- Heat, in an open quantum systems' scenario, is a delicate concept. One useful way to characterize it is by using the so-called full counting statistics. By means of it, we have studied the time-behavior of the mean value of heat in a generally non-Markovian regime, introducing a condition/quantifer for the occurrence/amount of heat backflow.

- Explicit calculations in a spin-boson model and in a quantum Brownian motion model have shown that heat backflow is maximized when the system and environment initially start at the same temperature.

Conclusions (2)

- A comparative analysis with suitable quantifier of non-Markovianity in both models have moreover shown that occurrence of heat backflow represents a stricter condition than non-Markovianity, in the sense that the latter is required in order to witness the former and, viceversa, a Markovian dynamics prevents the observation of heat backflow.

- Finally, exploiting again full counting statistics, we have derived a family of lower bounds to the mean dissipated heat in an environmental-assisted erasure protocol scenario. The latter has been characterized in a specific model consisting of an externally driven ad XX-coupled V-system, where the new lower bound can be proven to outperform original Landauer's bound.

THANK YOU
FOR YOUR ATTENTION