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Quasicontinuous variable quantum computation with collective spins in multi-path interferometers

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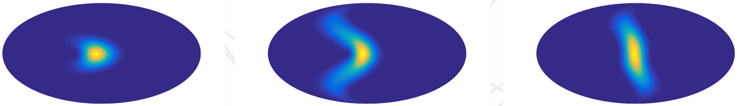
Outline

- Basics of continuous variable quantum computation
- Physical model: trapped atoms in optical resonator
- Coupling between different atomic samples
- Higher power Hamiltonians
- Summary



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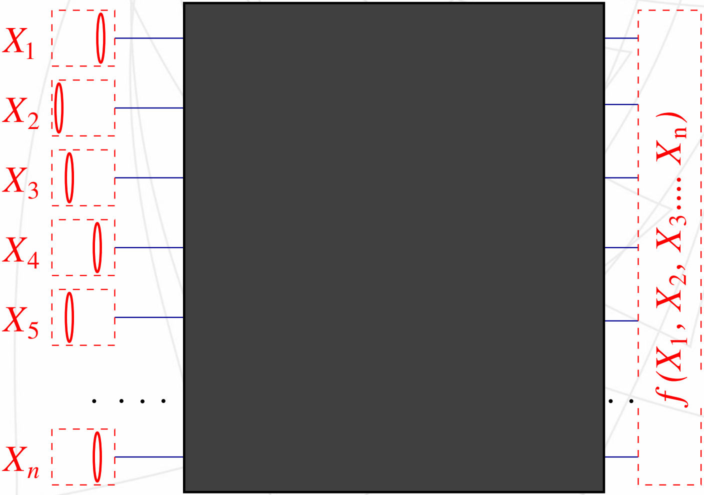
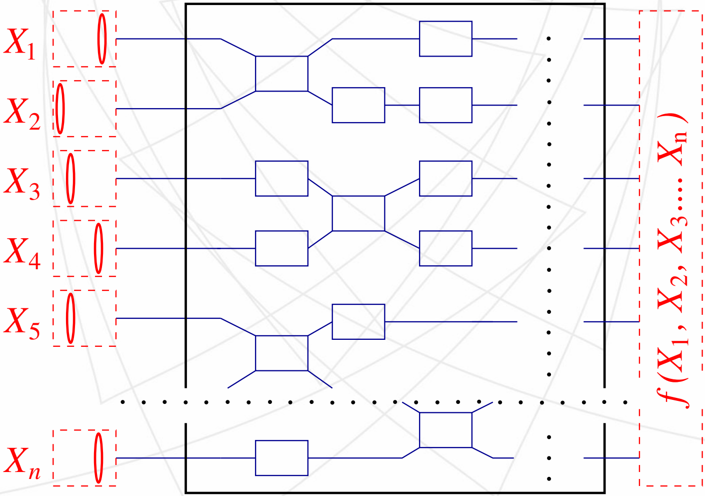
Basics of continuous variable quantum computation

Quantum computation over continuous variables: Lloyd & Braunstein, PRL **82,** 1784 (1999).

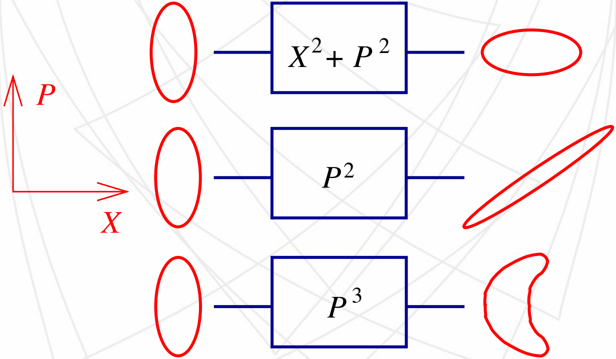


Basics of continuous variable quantum computation

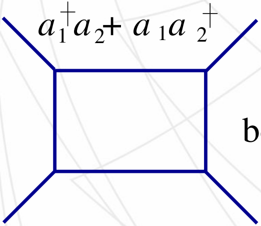
$$[X_k, P_k] = i$$



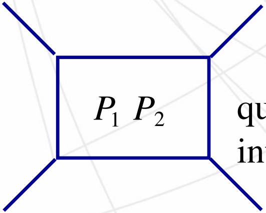
Single-mode transformations



Two-mode transformations



beam splitter



quantum non–demolition
interaction

Collective spin operators:

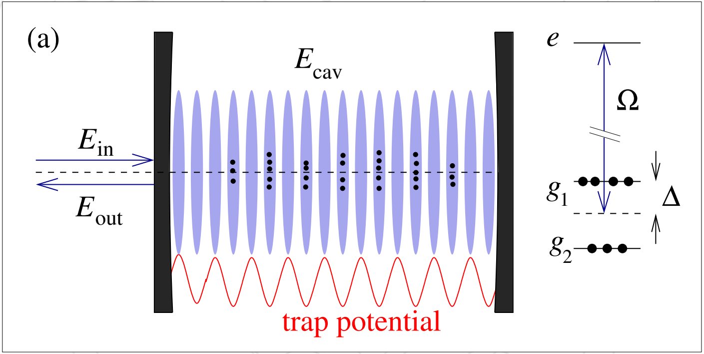
$$\begin{aligned}\hat{X} &= \frac{1}{2}(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1 \hat{a}_2^\dagger),\\ \hat{Y} &= \frac{1}{2i}(\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_1 \hat{a}_2^\dagger),\\ \hat{Z} &= \frac{1}{2}(\hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2)\end{aligned}$$

commutation relations $[\hat{X}, \hat{Y}] = i\hat{Z}$, $[\hat{Y}, \hat{Z}] = i\hat{X}$, and $[\hat{Z}, \hat{X}] = i\hat{Y}$.

In a confined region near $Y \approx -N/2$, $\hat{X}$ and $\hat{Z}$ of spins have similar properties as $\hat{X}$ and $\hat{P}$ of a harmonic oscillator.

Physical model: trapped atoms in optical resonators

Atoms in a single resonator:



Hamiltonian:

$$H = \hbar\left(\omega\hat{Z} + \chi\hat{Z}^2\right)$$

[Schleier-Smith et al, PRL **104,** 073602 (2010); PRA **81,** 021804(R) (2010)]

Physical model: trapped atoms in optical resonators

Atoms in a single resonator:

$$\hat{H} = \hbar\chi\hat{Z}^2$$

One-axis twisting.

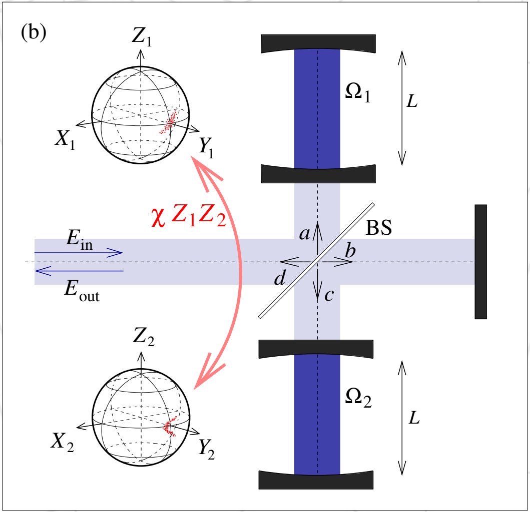
By rotating the spins and switching the sign of the nonlinearity:

$$\begin{aligned}\hat{H} &= \hbar\chi\left(\hat{X}^2 - \hat{Y}^2\right)\\ \hat{H} &= \hbar\chi\left(\hat{X}\hat{Z} + \hat{Z}\hat{X}\right)\end{aligned}$$

Two-axis countertwisting.

Coupling between different atomic samples

Two resonators in an interferometer:



[T.O., PRL **119,** 010502 (2017)]

Coupling between different atomic samples

Atomic interactions:

$$\hat{H} = \hbar\left[\omega(\hat{Z}_1 + T_B\hat{Z}_2) + \chi(\hat{Z}_1 - \hat{Z}_2)^2\right],$$

where

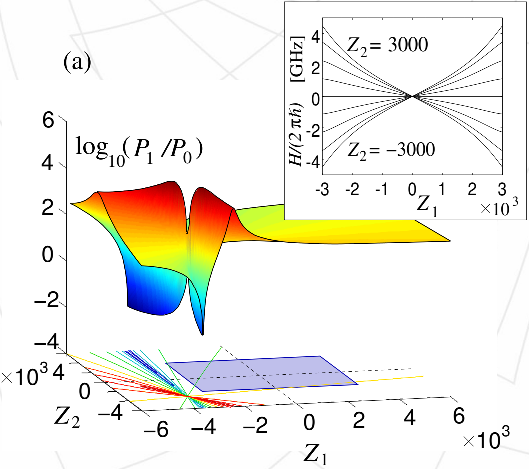
$$\omega = \frac{2^6\cdot 3}{\pi^2}\frac{R_B}{(1+T_B)^2}\frac{1}{T}\left(\frac{\lambda}{w}\right)^2\frac{\Gamma}{\Delta}\mathcal{R},$$
$$\chi = -\frac{2^8\cdot 3^2}{\pi^4}\frac{R_B T_B}{(1+T_B)^3}\frac{1}{TL\Delta k}\left(\frac{\lambda}{w}\right)^4\left(\frac{\Gamma}{\Delta}\right)^2\mathcal{R}.$$

Leading to the QND interaction

$$\hat{H}_{\mathrm{QND}} = -\hbar 2\chi\hat{Z}_1\hat{Z}_2.$$

Coupling between different atomic samples

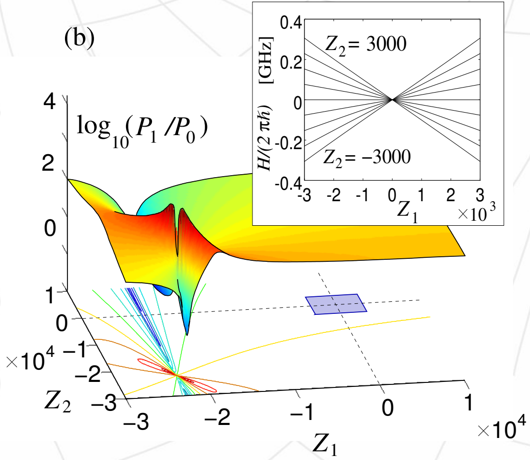
Power in a cavity:



Detuning $L\Delta k = 0.08\,T$

Coupling between different atomic samples

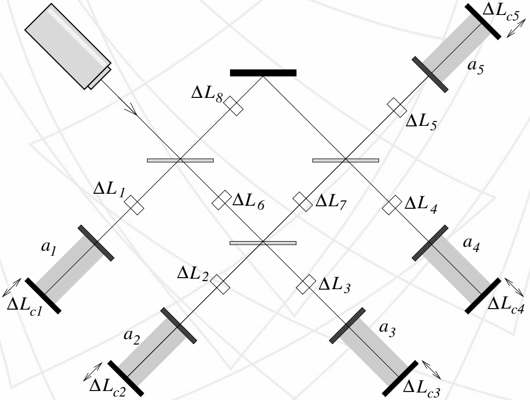
Power in a cavity:



Detuning $L\Delta k = 0.5\,T$

Coupling between different atomic samples

Generalization to multiple resonators:



Higher power Hamiltonians

Taking advantage of spin commutation rules

Expansion of commutators

$$e^{-i\hat{A}\Delta t}e^{-i\hat{B}\Delta t}e^{i\hat{A}\Delta t}e^{i\hat{B}\Delta t} = e^{[\hat{A},\hat{B}]\Delta t^2} + \mathcal{O}(\Delta t^3)$$

Single-mode cubic Hamiltonian out of quadratic ones:

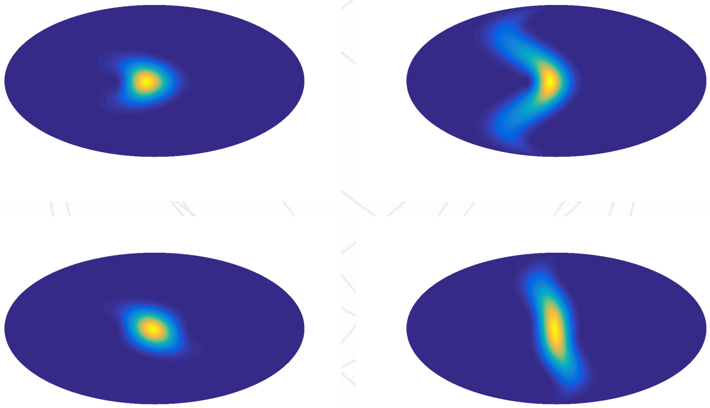
$$\begin{aligned}\hat{X}^3 &= \frac{i}{4}\left[(\hat{Z}^2 - \hat{Y}^2),(\hat{Y}\hat{Z} + \hat{Z}\hat{Y})\right] \\ &\quad + \frac{i}{4}\left[(\hat{X}\hat{Z} + \hat{Z}\hat{X}),(\hat{X}\hat{Y} + \hat{Y}\hat{X})\right] + \frac{1}{4}\hat{X}\end{aligned}$$

Two-mode Hamiltonian:

$$\begin{aligned}\hat{X}_1^3\hat{Z}_2 &= \frac{1}{4}\hat{X}_1\hat{Z}_2 + \frac{1}{4}\left[(\hat{Z}_1^2 - \hat{Y}_1^2),\left[\hat{Z}_1^2,\hat{X}_1\hat{Z}_2\right]\right] \\ &\quad - \frac{1}{4}\left[\hat{X}_1\hat{Z}_1 + \hat{Z}_1\hat{X}_1,\left[\hat{X}_1^2,\hat{Z}_1\hat{Z}_2\right]\right]\end{aligned}$$

Higher power Hamiltonians

Resulting transformations of spin coherent and spin squeezed states:



Calculations by Šimon Bräuer

Conclusion and Summary

Challenges:

- Precise combination of cavities into interferometers
- Losses — connected with the dispersive interaction as $\epsilon \sim N(\lambda/w)^2(\Gamma/\Delta)^2$
- Decoherence: phase of the atomic spins influenced by the fluctuating light intensity.

Main advantages:

- Higher order Hamiltonians come naturally from the spin commutators.
- For large atomic numbers: collective spin close to continuous variables.
- Possibility to simulate dynamics of CV quantum systems.

Conclusion and Summary

Summary:

- Basics of continuous variable quantum computation
- Physical model: trapped atoms in optical resonator
- Coupling between different atomic samples
- Higher power Hamiltonians

Thank you for your attention!

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