Optomechanical oscillator state squeezing by variation in its heat bath temperature

M. Kolář [1], **A. Ryabov** [2], **and R. Filip** [1]
[1] Department of Optics, Palacký University, 771 46 Olomouc, Czech Republic
[2] Department of Macromolecular Physics, Charles University in Prague, 180 00 Praha 8, Czech Republic

E-mail: kolar@optics.upol.cz

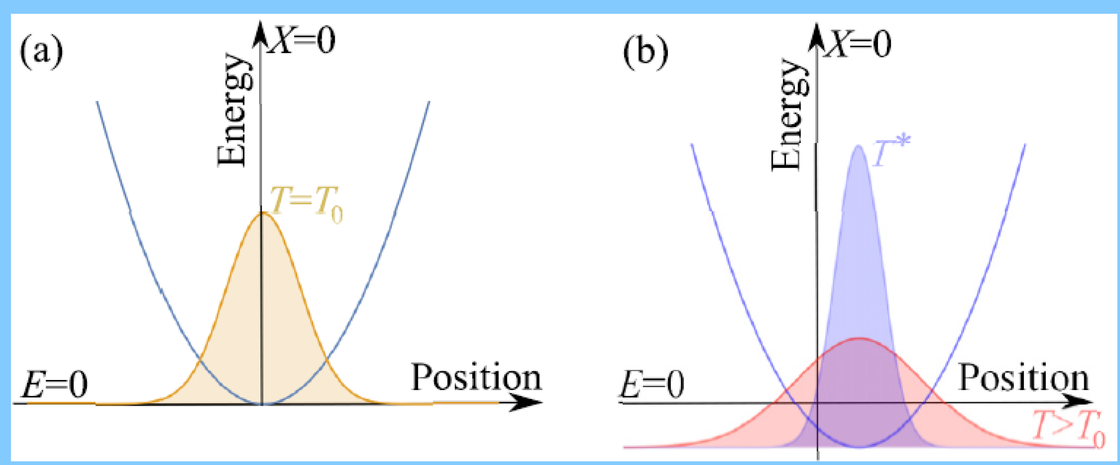
We analyze the situation in which a quantum optomechanical oscillator (the membrane) is driven by an external force resulting from the interaction with an external system (the piston). Both systems are embedded in a common heat bath at certain temperature *T*. The driving force exerted on the oscillator is bath temperature-dependent. Initially the piston is quadratically coupled to the membrane. The bath temperature *T* is then reversibly changed. The change of temperature changes the frequency of the membrane, but simultaneously also affects its fluctuations. The resulting equilibrium state of the membrane is analyzed from the point of view of mechanical, as well as of thermodynamic characteristics.

Effects of linear coupling were analyzed in [1]

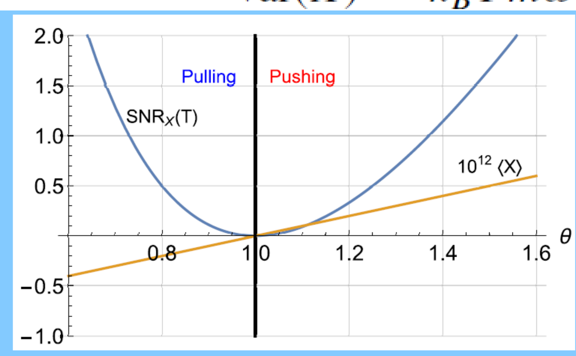
$$\hat H_0 = \frac{\hat P^2}{2m} + \frac{m\omega^2}{2}\hat X^2$$

$$\hat H_I(T) = -f(T)\hat X$$

$$\hat H(T) = \frac{\hat P^2}{2m} + \frac{m\omega^2}{2}\left[\hat X - \frac{f(T)}{m\omega^2}\right]^2 - \frac{f(T)^2}{2m\omega^2}$$



$$\mathrm{SNR}_X(T) = \frac{\langle X\rangle^2}{\mathrm{Var}(X)} = \frac{f(T)^2}{k_B T m\omega^2}$$

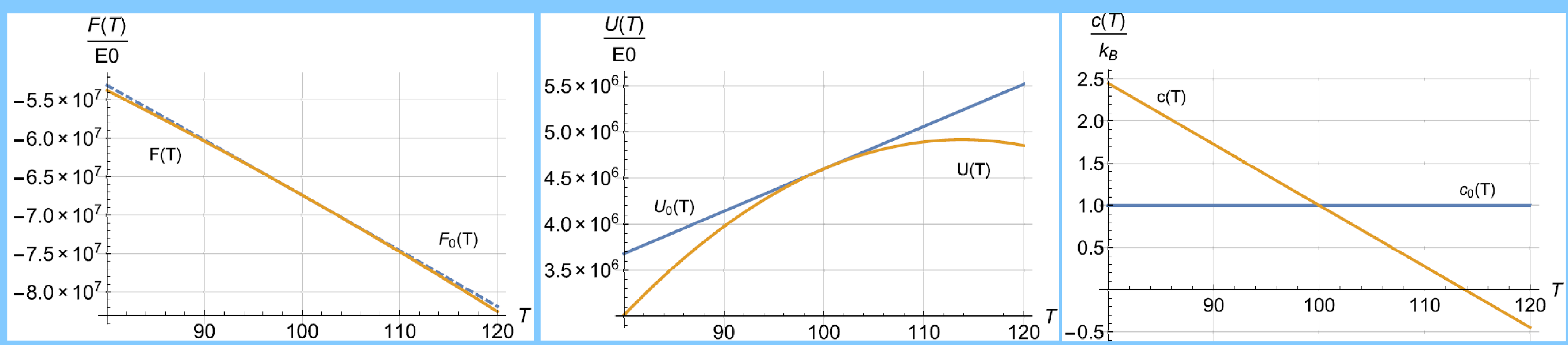


$$\theta = \frac{T}{T_0}$$

$$U = U_0(T) - \frac{f(T)^2}{2m\omega^2} \qquad U_0(T) = \langle \hat H_0\rangle_{f=0}$$

$$F = F_0(T) - \frac{f(T)^2}{2m\omega^2}$$

$$S = -\frac{\partial F_0}{\partial T}$$



$$c_0(T) = \frac{dU_0}{dT} = T\frac{\partial S}{\partial T}$$

OR

$$c(T) = \frac{dU}{dT} = T\frac{\partial S}{\partial T} - \frac{1}{2m\omega^2}\frac{df^2}{dT}$$

The analysis of the quadratic coupling

$$\hat H_0 = \frac{\hat P^2}{2m} + \frac{m\omega_0^2}{2}\hat X^2 \qquad \hat H_I(T) = \frac{mf(T)}{2}\hat X^2 \qquad \hat H(T) \equiv \frac{\hat P^2}{2m} + \frac{m\omega(T)^2}{2}\hat X^2$$

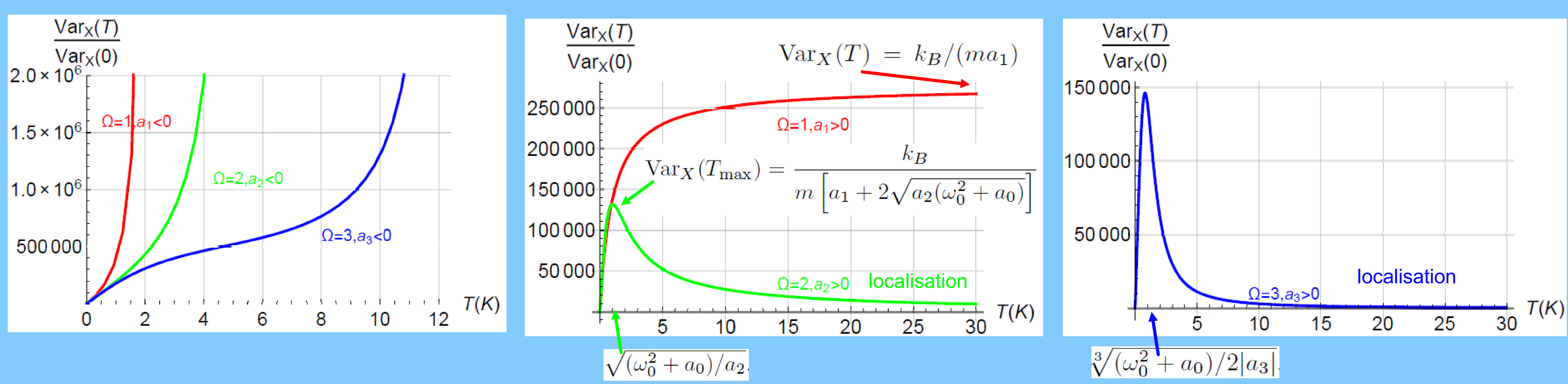
Assumption: $\omega(T) = \sqrt{\omega_0^2 + f(T)} = \sqrt{\omega_0^2 + \sum_{k=0}^{\Omega} a_k T^k}$ $\quad \hat\rho = \frac{\exp[-\hat H/k_BT]}{Z} = \exp\left[\frac{F-\hat H}{k_BT}\right]$

The mechanical characteristics:

$$\mathrm{Var}_X(T) \approx \frac{k_BT}{m\omega(T)^2}$$

$$\mathrm{Var}_P(T) \approx mk_BT$$

$$\frac{\partial\, \mathrm{Var}_X(T)}{\partial T} = \frac{k_B(\omega_0^2 + a_0 - a_2T^2 - 2a_3T^3)}{m\omega(T)^4}$$

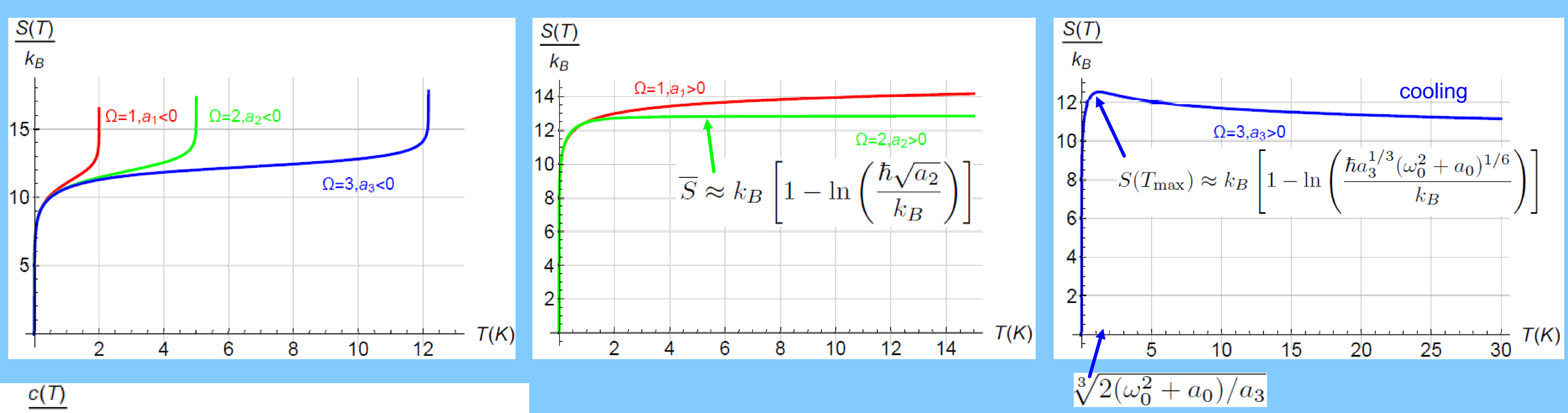


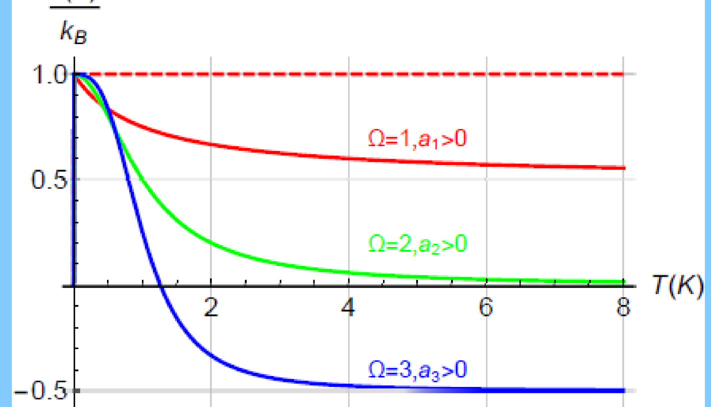
The thermodynamic characteristics:

$$S \approx k_B\left[1 - \ln\left(\frac{\hbar\omega(T)}{k_BT}\right)\right]$$

$$S = -\frac{\partial F}{\partial T} + U\frac{\partial}{\partial T}[\ln\omega(T)]$$

$$\frac{\partial S(T)}{\partial T} = k_B\frac{2(\omega_0^2 + a_0) + a_1T - a_3T^3}{2T\omega(T)^2}$$





$$c(T) = \frac{\mathrm{d}U}{\mathrm{d}T} \approx k_B$$

OR

$$c_0(T) = T\frac{\partial S}{\partial T}$$

Conclusions:

- we have studied an oscillator quadratically coupled to its surroundings by a bath temperature *T*-dependent force
- its position variance can diverge, saturate, or decrease (localisation) with increasing bath temperature
- its entropy can as well diverge, saturate, or decrease (cooling) with increasing bath temperature
- these effects are moreless independent of each other

localisation \ cooling	YES	NO
YES	$\Omega = 3,\ a_3 > 0$	$\Omega = 2,\ a_2 > 0$
NO	$\emptyset$	$\Omega \le 3,\ a_\Omega < 0$; $\Omega = 1,\ a_1 > 0$

References: [1] M. Kolář, A. Ryabov, and R. Filip, Phys. Rev. A 95, 042105 (2017).
[2] H.-P. Breuer, F. Petruccione: The Theory of Open Quantum Systems (Oxford, 2002).
[3] W. Greiner, *et. al.*, Thermodynamics and Statistical Mechanics, Springer (1997).

The support of the Czech Science Foundation (GACR) projects GA 14-36681G and No. 17-06716S is acknowledged.