Experimental demonstration of photonic quantum Fredkin gate and its applications

R. Stárek, M. Mičuda, M. Miková, I. Straka,
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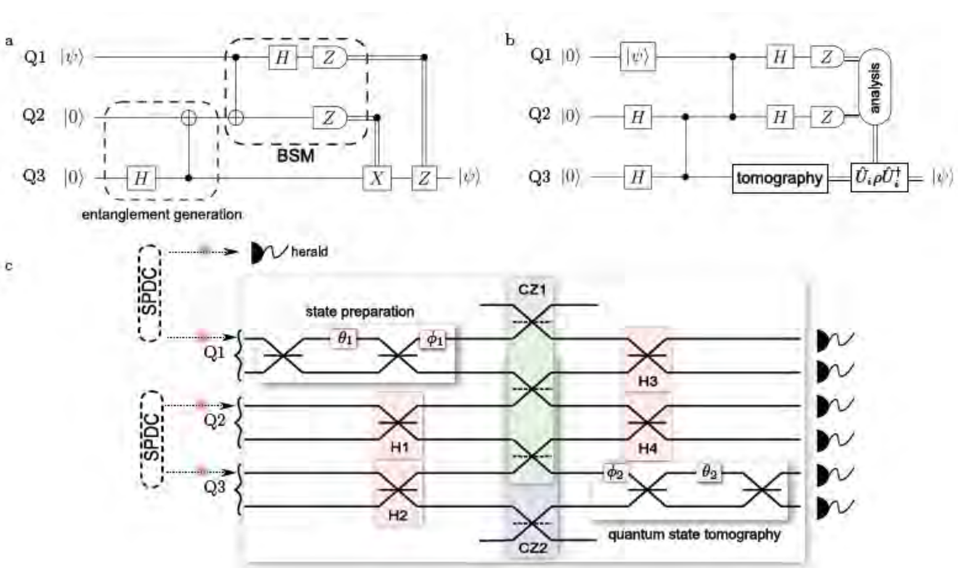
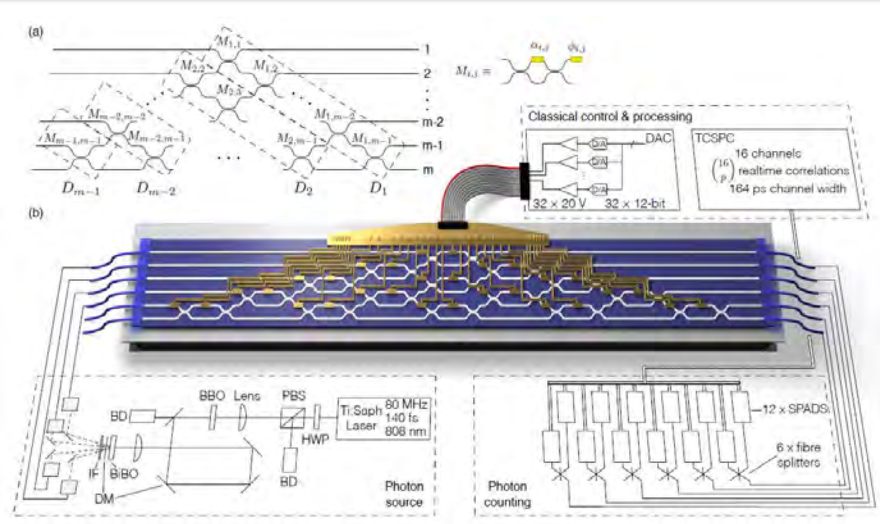
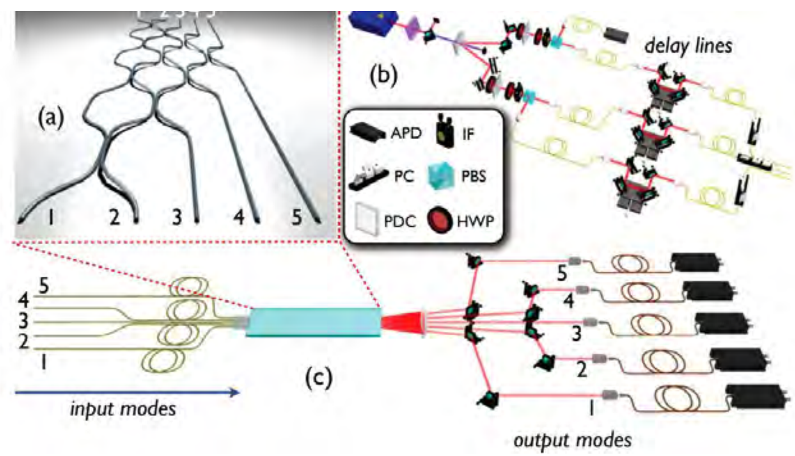
Outline of the talk

- Paths to increase the complexity of optical quantum logic circuits
- Implementations of linear optical quantum Fredkin gate
- Hybrid scheme based on encoding qubits into path and polarization of single photons
- Symmetrization and anti-symmetrization with quantum Fredkin gate and applications
- Conclusions

Increasing the complexity of linear optics quantum logic circuits







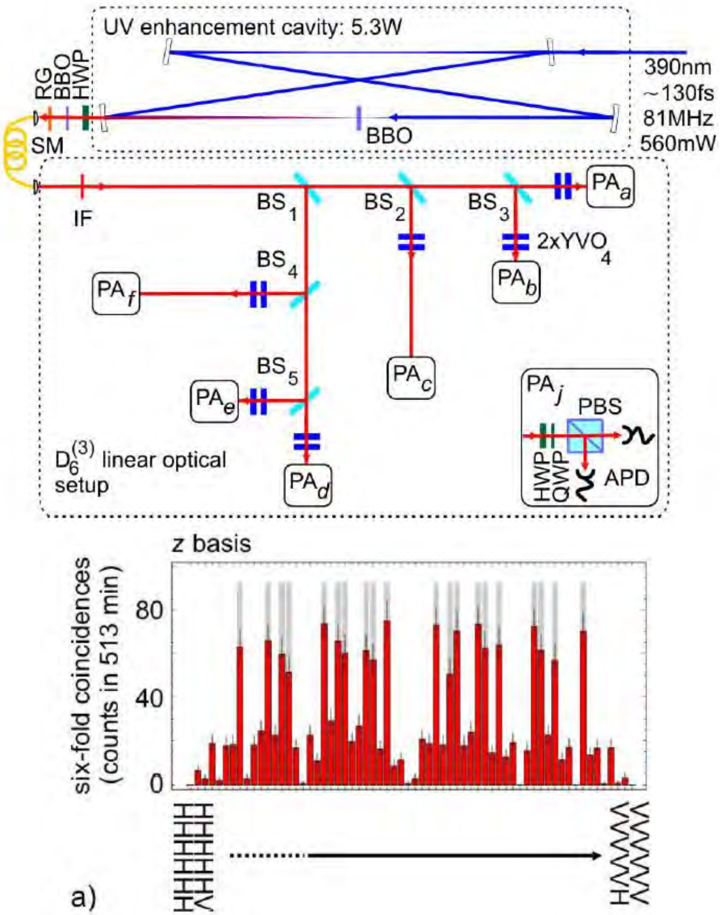
Integrated optics on a photonic chip

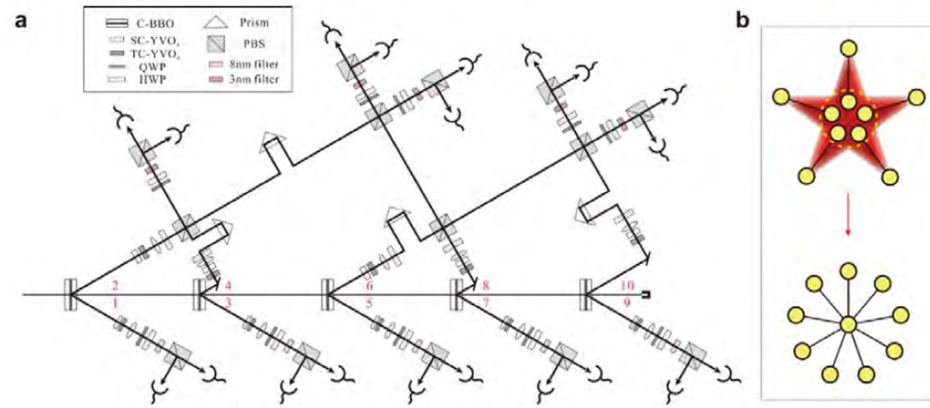
A. Crespi, R. Osellame, R. Ramponi, D. J. Brod, E. F. Galvao, N. Spagnolo, C. Vitelli, E. Maiorino, P. Mataloni, F. Sciarrino, Nature Photonics **7**, 545 (2013).

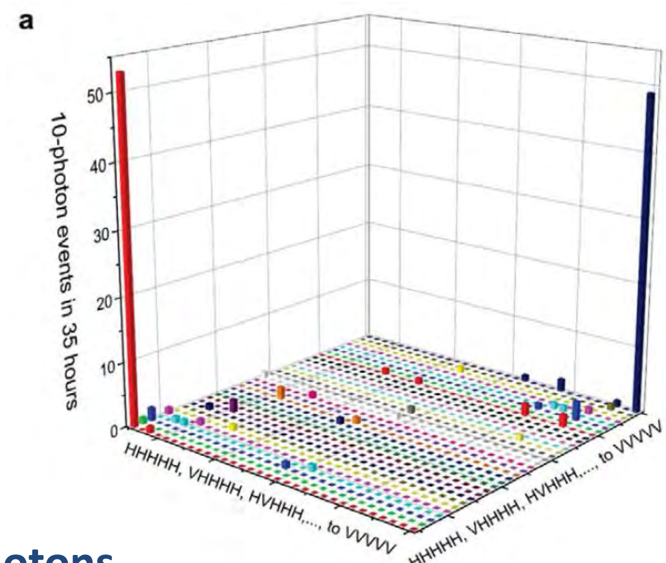
B.J. Metcalf, J.B. Spring, P.C. Humphreys, N. Thomas-Peter, M. Barbieri, W. S. Kolthammer, X.-M. Jin, N.K. Langford, D. Kundys, J.C. Gates, B.J. Smith, P.G.R. Smith, and I.A. Walmsley, Nature Photonics **8**, 770-774 (2014).

J. Carolan, C. Harrold, C. Sparrow, E. Martin-Lopez, N.J. Russell, J.W. Silverstone, P.J. Shadbolt, N. Matsuda, M. Oguma, M. Itoh, G.D. Marshall, M.G. Thompson, J.C.F. Matthews, T. Hashimoto, J.L. O'Brien, and A. Laing, Science **349**, 711 (2015).

Increasing the complexity of linear optics quantum logic circuits





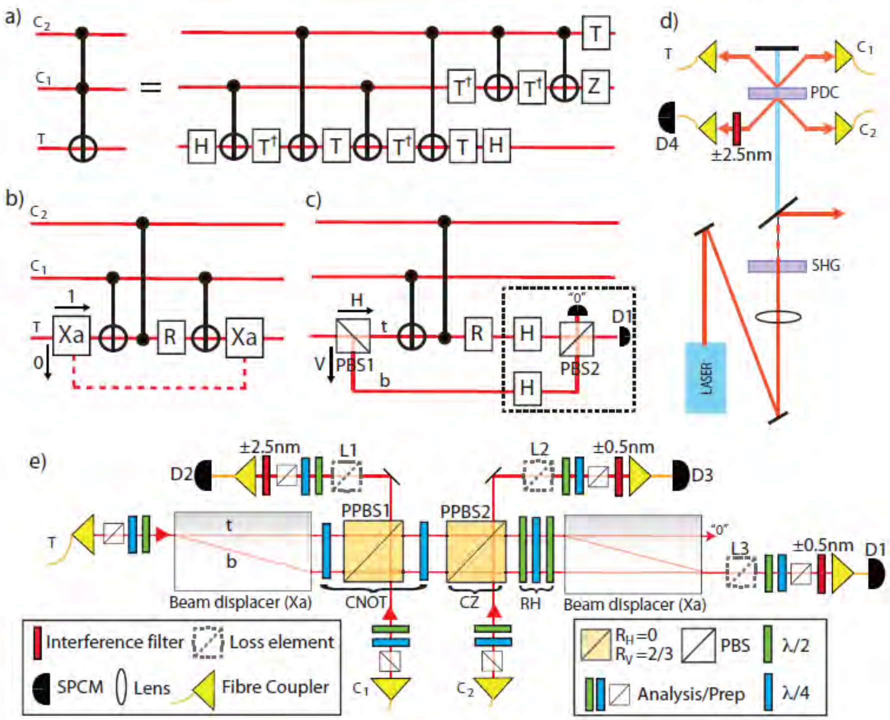


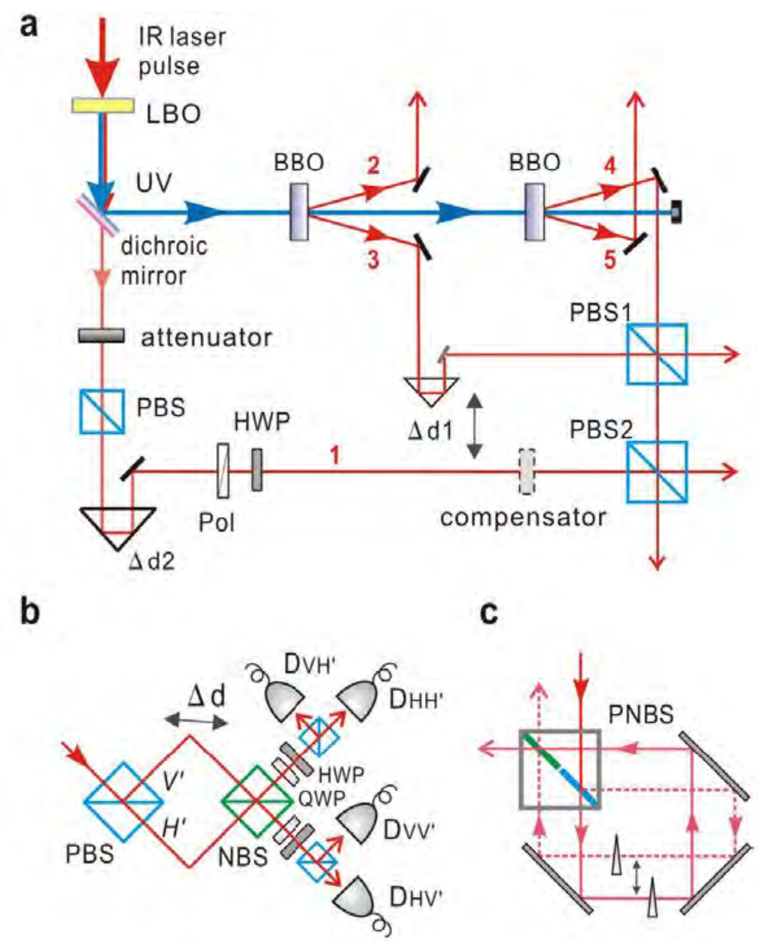
Increasing the number of photons

W. Wieczorek, R. Krischek, Nikolai Kiesel, P. Michelberger, G. Toth, H. Weinfurter Phys. Rev. Lett. **103**, 020504 (2009).

X.-L. Wang, L.-K. Chen, W. Li, H.-L. Huang, C. Liu, C. Chen, Y.-H. Luo, Z.-E. Su, D. Wu, Z.-D. Li, H. Lu, Y. Hu, X. Jiang, C.-Z. Peng, L. Li, N.-L. Liu, Y.-A. Chen, C.-Y. Lu, and J,-W. Pan, arXiv:1605.08547.

Increasing the complexity of linear optics quantum logic circuits





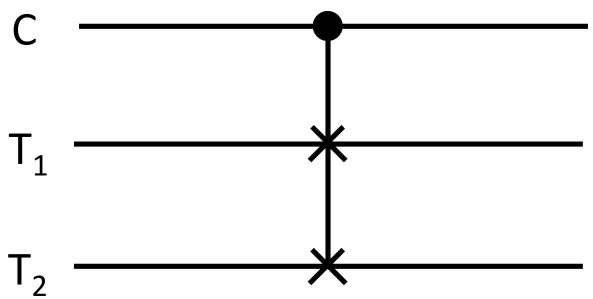
Utilization of inherently stable interferometers
and exploitation of several degrees of freedom of single photons

B. P. Lanyon, M. Barbieri, M. P. Almeida, T. Jennewein, T. C. Ralph, K. J. Resch, G. J. Pryde, J. L. O'Brien, A. Gilchrist, A. G. White, Nature Physics **5**, 134 (2009).

W.-B. Gao, C.-Y. Lu, X.-C. Yao, P. Xu, O. Gühne, A. Goebel, Y.-A. Chen, C.-Z. Peng, Z.-B. Chen, and J.-W. Pan, Nature Physics, **6**, 331 (2010).

Quantum Fredkin gate

Controlled-SWAP gate



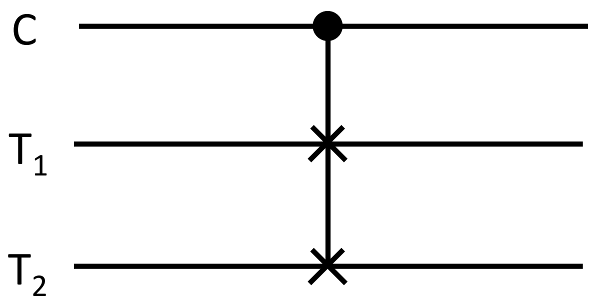
$$\hat{U}_{FRED} = |0\rangle\langle 0|\otimes\hat{I}_{T_1T_2} + |1\rangle\langle 1|\otimes\widehat{W}_{T_1T_2}$$

The SWAP operator

$$\widehat{W}|i\rangle|j\rangle = |j\rangle|i\rangle$$

Quantum Fredkin gate

Controlled-SWAP gate



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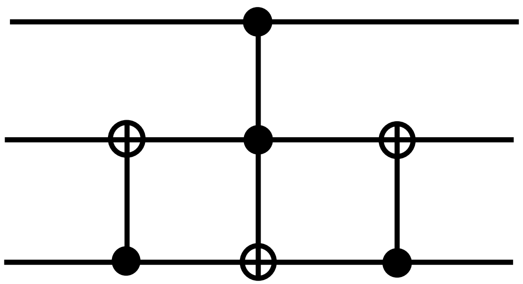
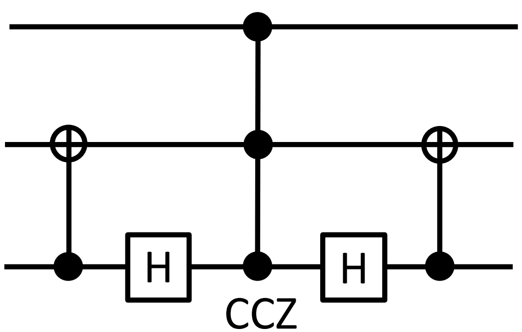
The SWAP operator

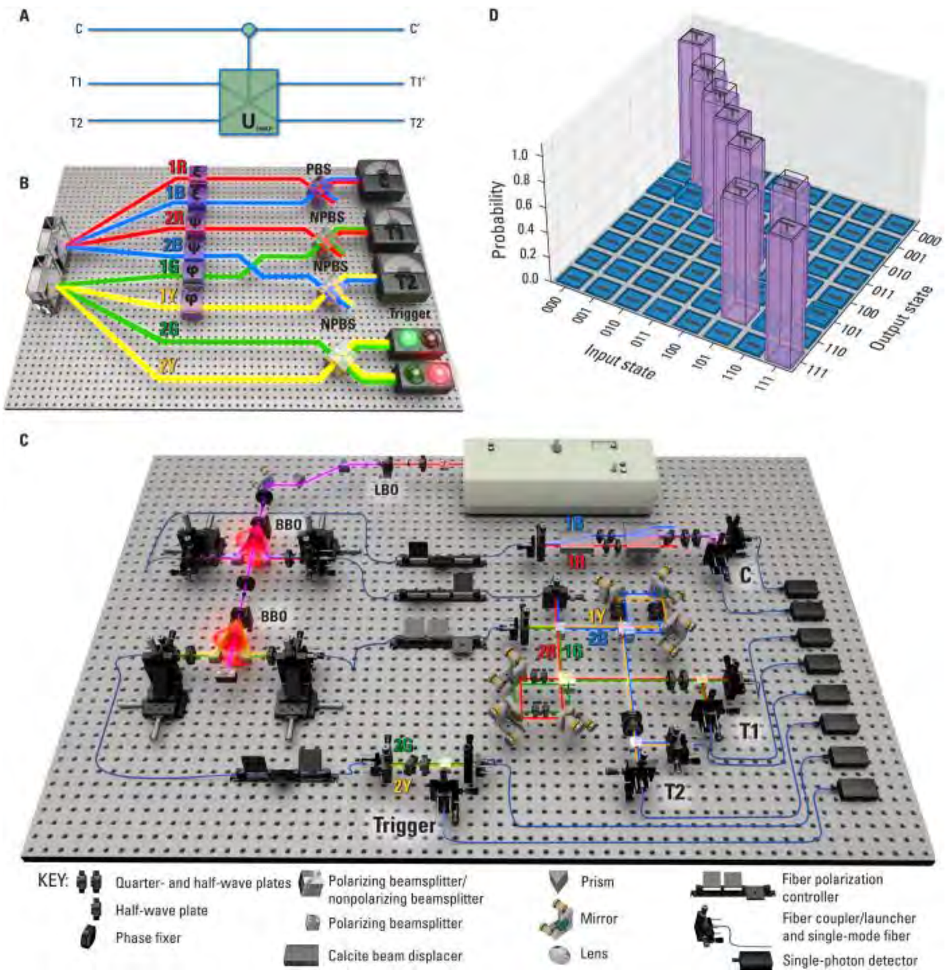
$$\widehat{W}|i\rangle|j\rangle = |j\rangle|i\rangle$$

Decomposition into Toffoli and CNOT gates





Previous implementations of liner optical quantum Fredkin gate

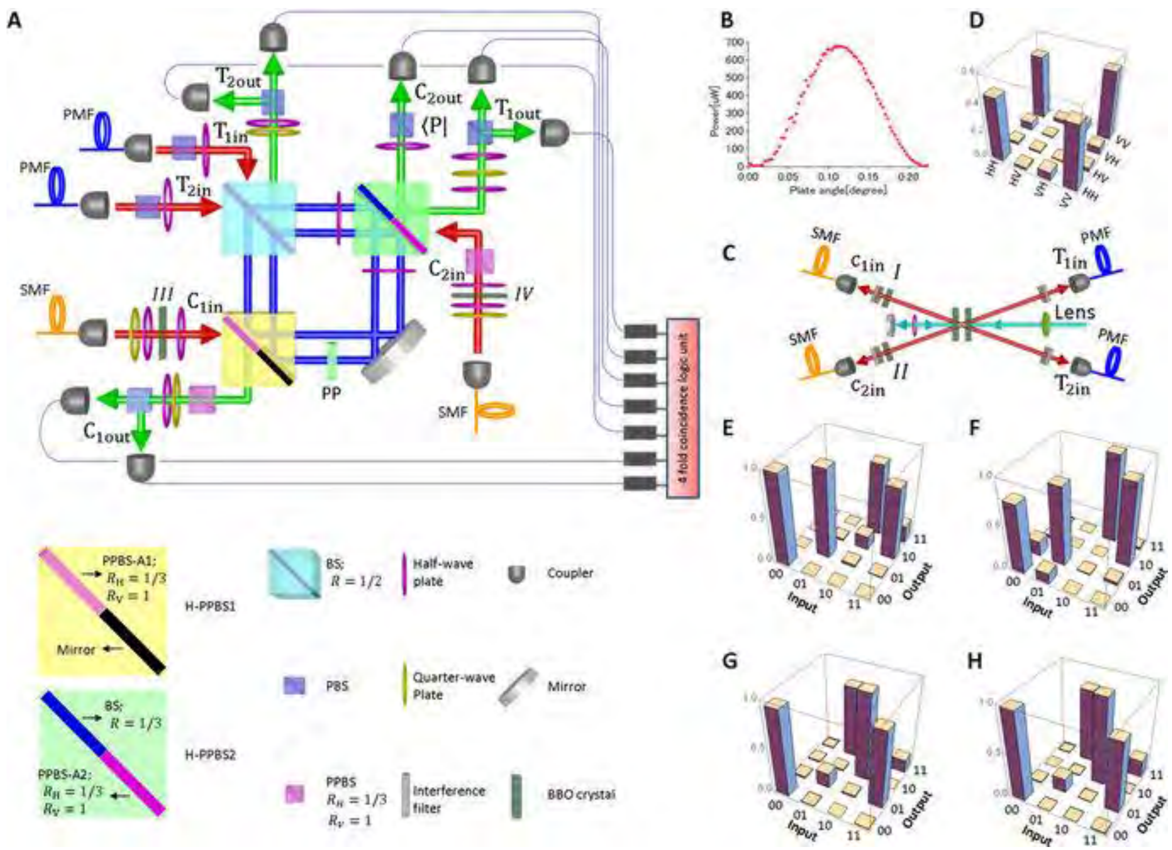


Requires entanglement in the auxiliary spatial degrees of freedom.

Average fourfold coincidence rate of 2.2 per minute at the output of the gate.

R.B. Patel, J. Ho, F. Ferreyrol, T.C. Ralph and G.J. Pryde, Science Advances **2**, e1501531 (2016).

Previous implementations of liner optical quantum Fredkin gate



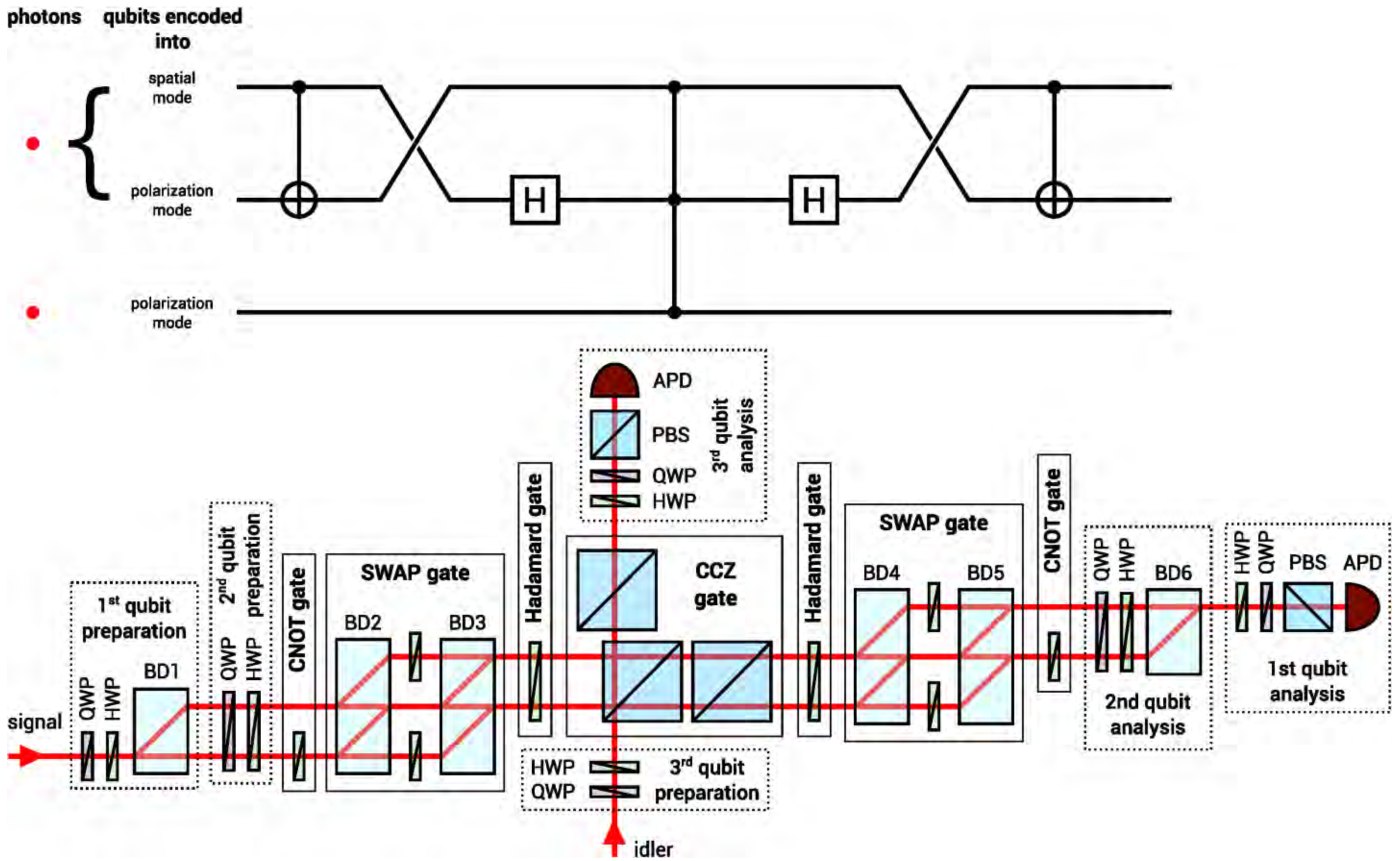
Requires encoding the control qubit into entangled two-photon state.

Low success probability of 1/162 implied low count rate of 10^{-4} Hz.

T. Ono, R. Okamoto, M. Tanida, H. F. Hofmann, and S. Takeuchi, Sci. Rep. **7**, 45353 (2017).

J. Fiurášek, Phys. Rev. A **78**, 032317 (2008).

Our approach to linear-optical quantum Fredkin gate



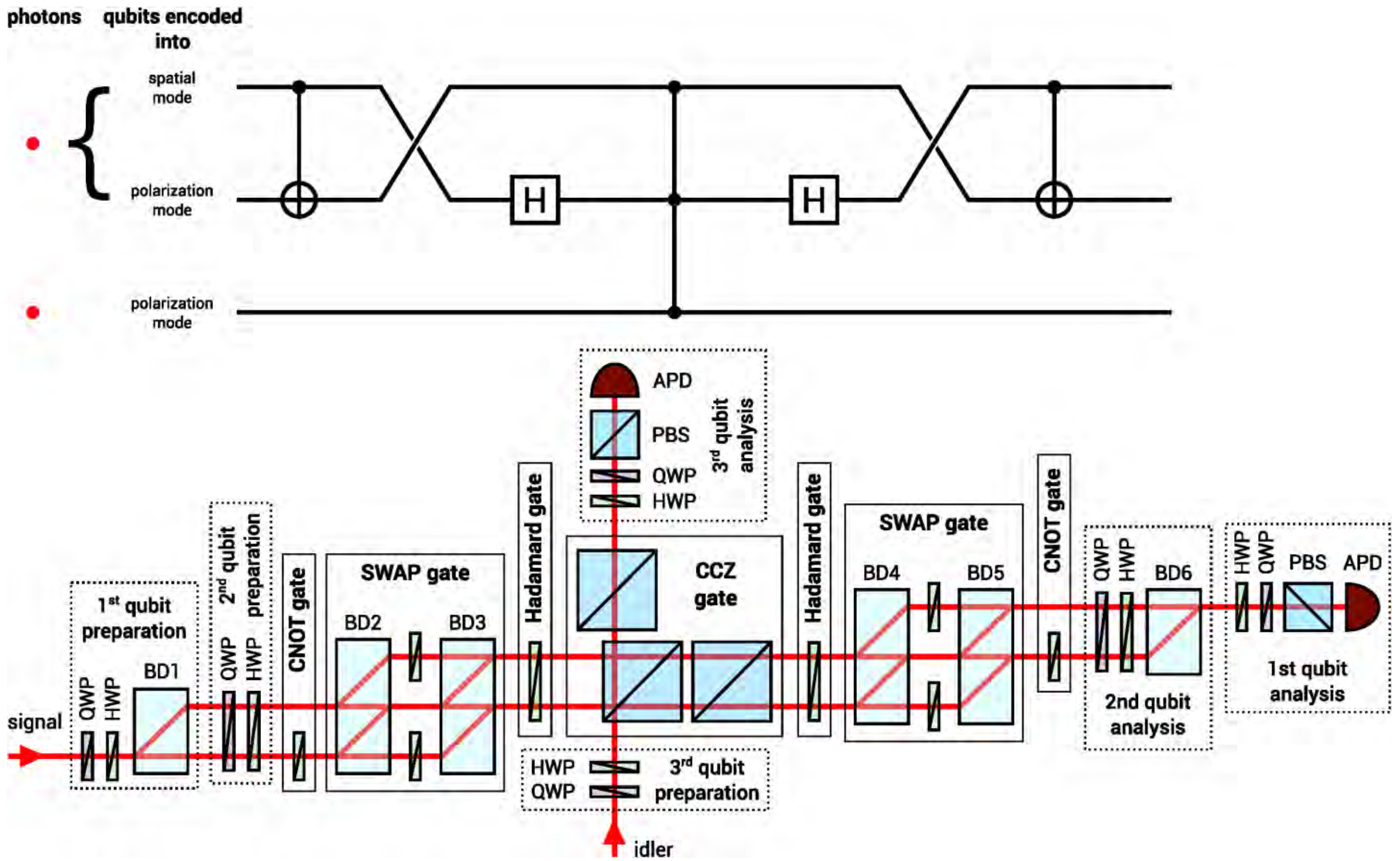
Hybrid encoding of three qubits into quantum states of two photons

Control qubit is encoded into polarization of signal photon

Target qubits are encoded into polarization and path of the idler photon

Two-photon coincidences are measured at the device output

Our approach to linear-optical quantum Fredkin gate



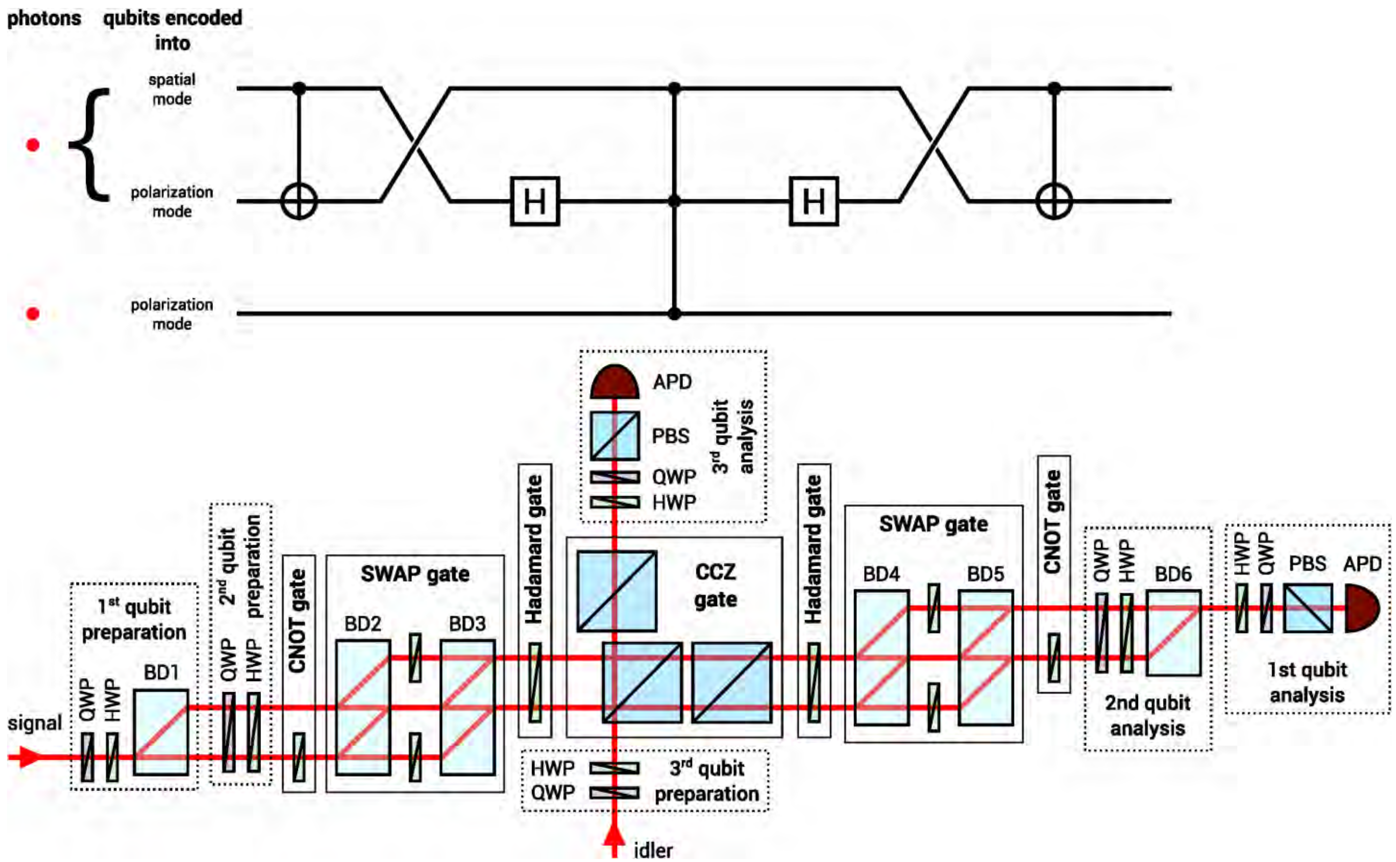
Quantum Toffoli gate is implemented by two-photon interference at a partially polarizing BS

Two-qubit CNOT gates are implemented deterministically using half-wave plates

Polarization and path qubits are twice swapped in our scheme

The interferometer formed by calcite beam displacers is inherently stable

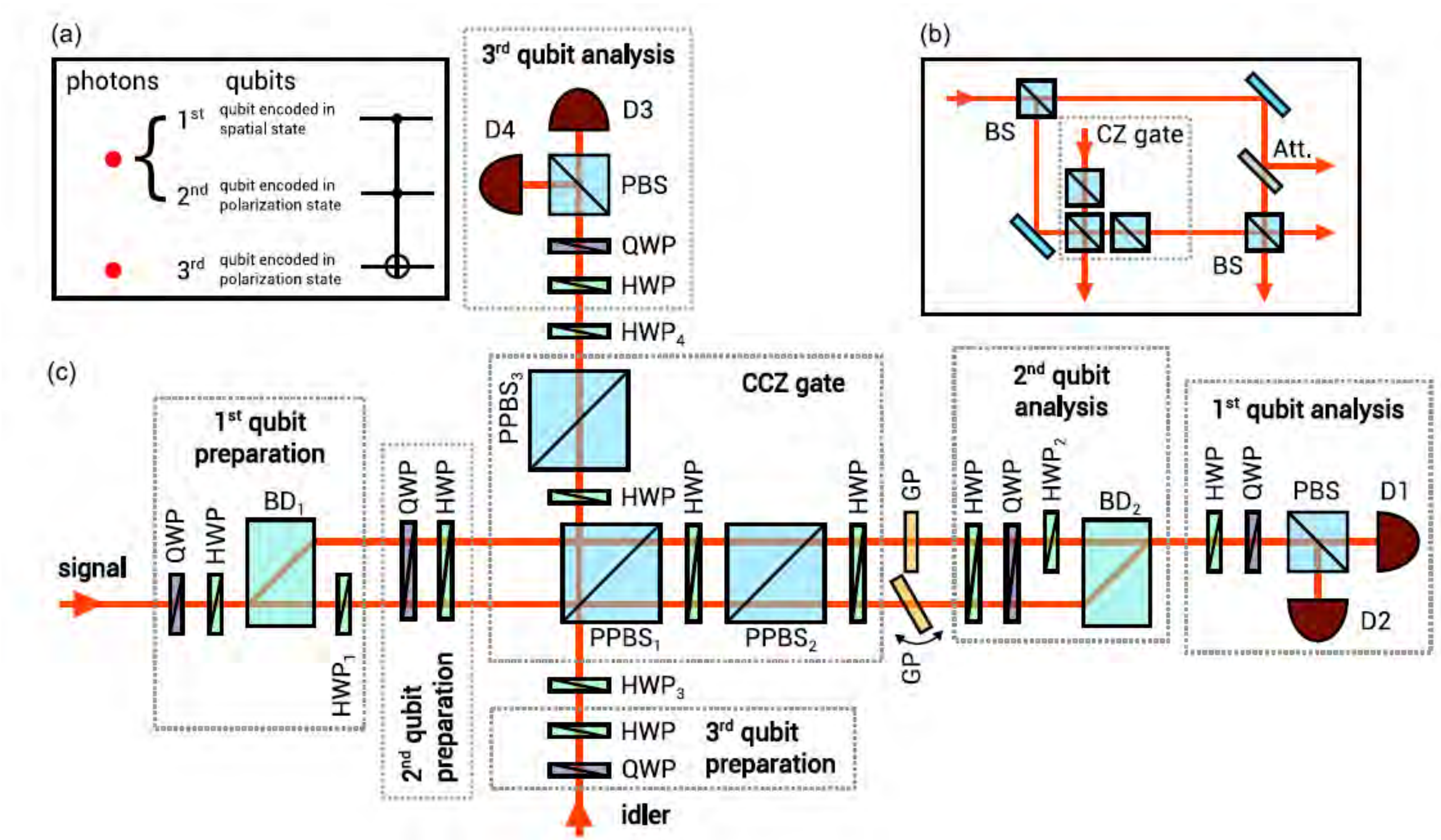
Our approach to linear-optical quantum Fredkin gate



Features of our implementation:

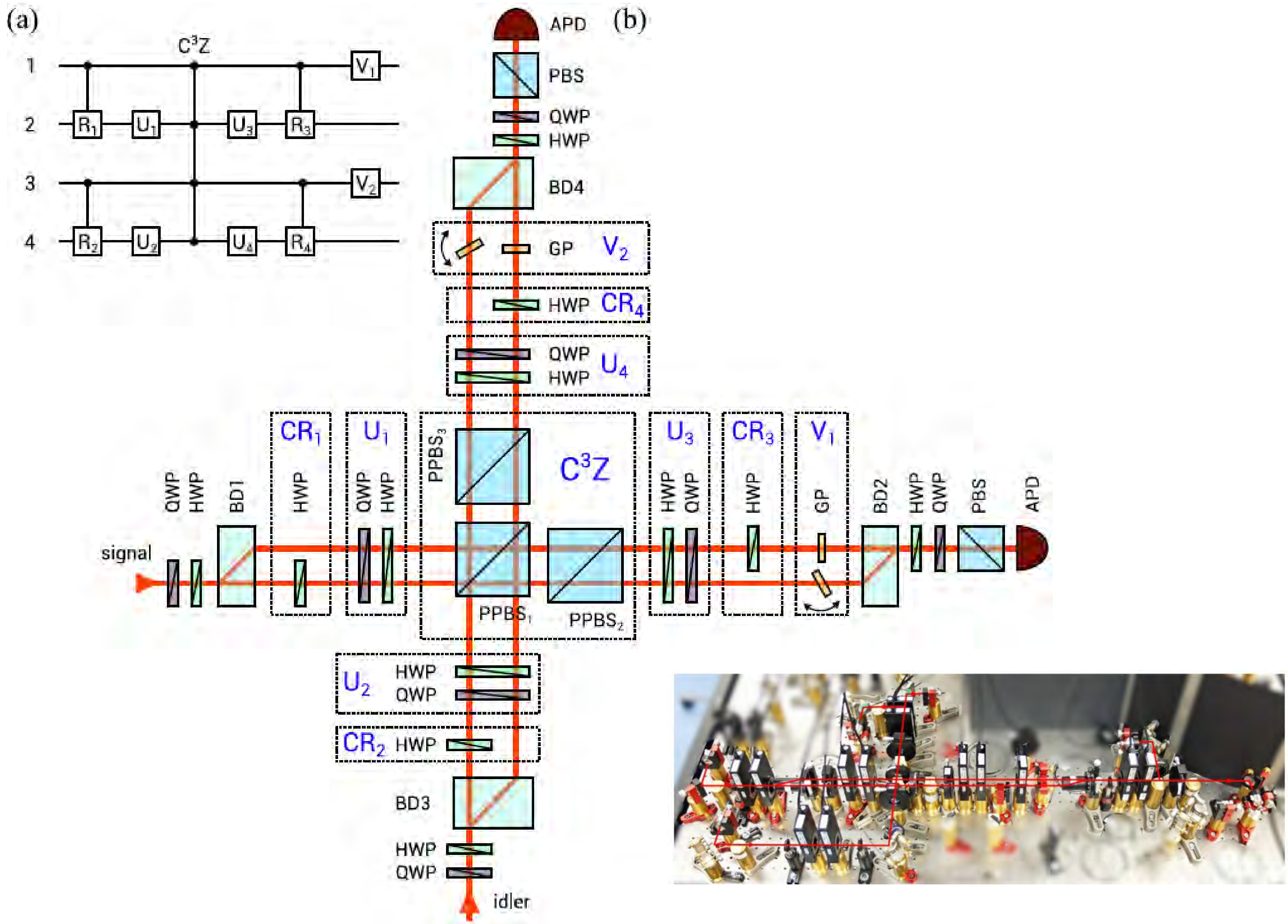
- High coincidence rate – 300 two-photon coincidences per second at the gate output
- Inherently stable setup
- Peculiar qubit encoding
- Not fully scalable (number of components scales exponentially when several qubits are encoded into a single photon)

Three-qubit linear optical quantum Toffoli gate



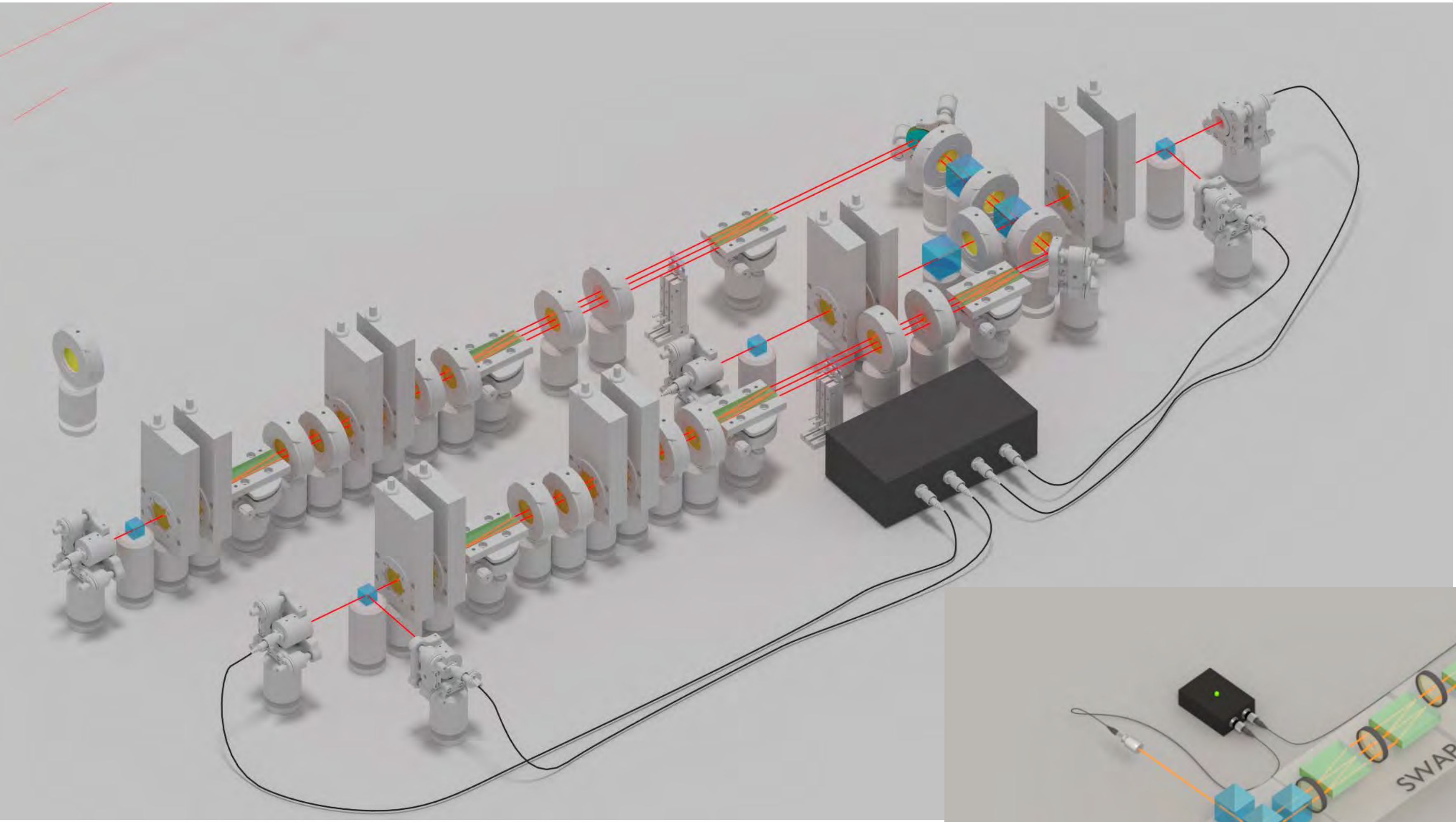
M. Mičuda, M. Sedlák, I. Straka, M. Miková, M. Dušek, M. Ježek, and J. Fiurášek, Phys. Rev. Lett. **111**, 160407 (2013).

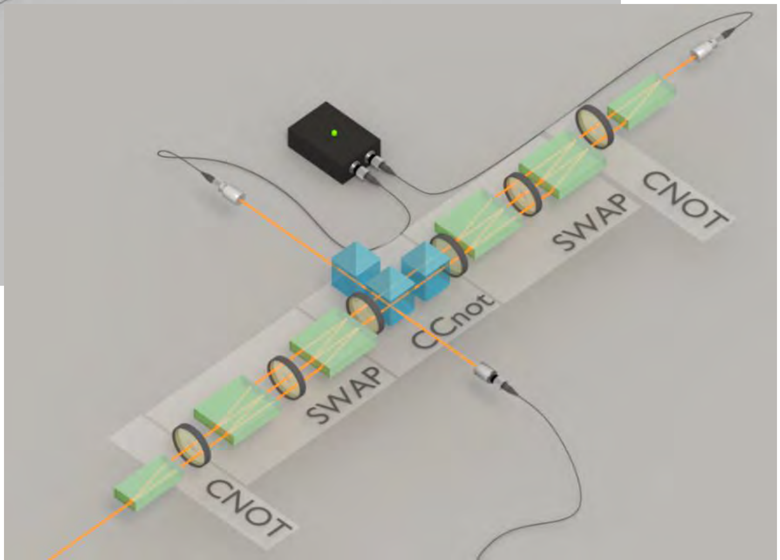
Four-qubit linear optical quantum CCCZ gate



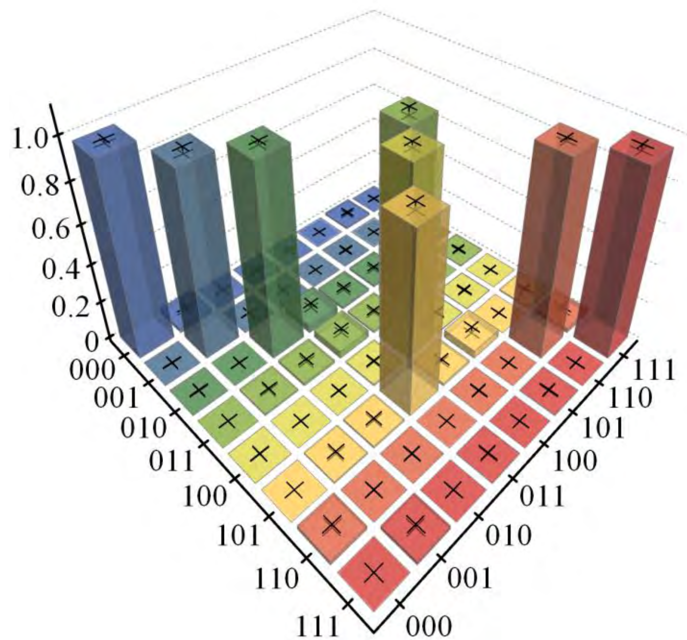
R. Stárek, M. Mičuda, M. Miková, I. Straka, M. Dušek, M. Ježek, and J. Fiurášek, Sci. Rep. **6**, 33475 (2016).

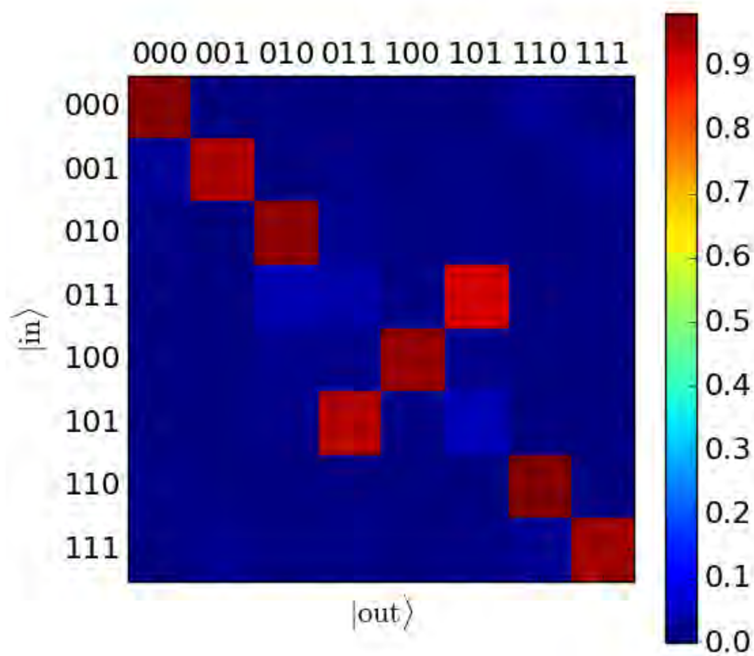
Linear optical quantum Fredkin gate





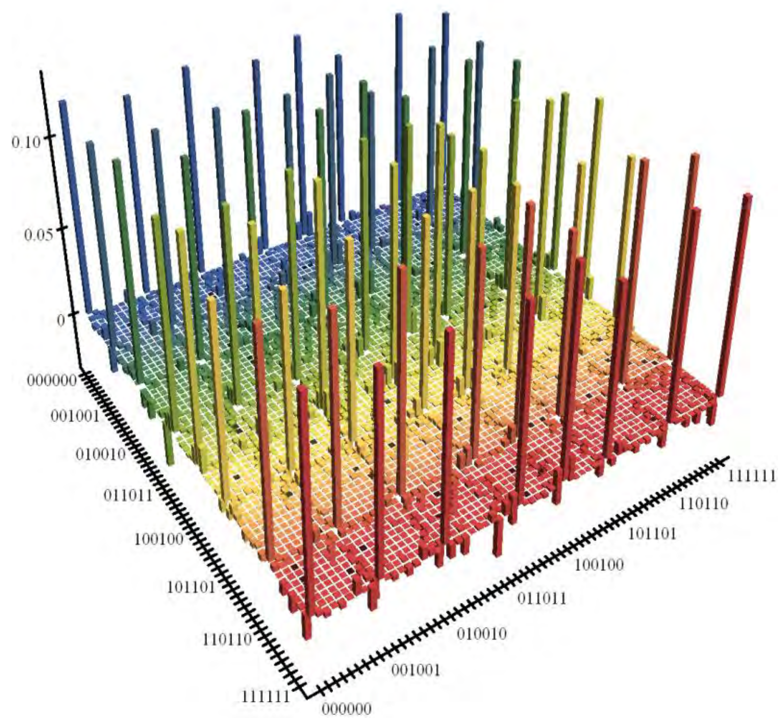
Measured truth table

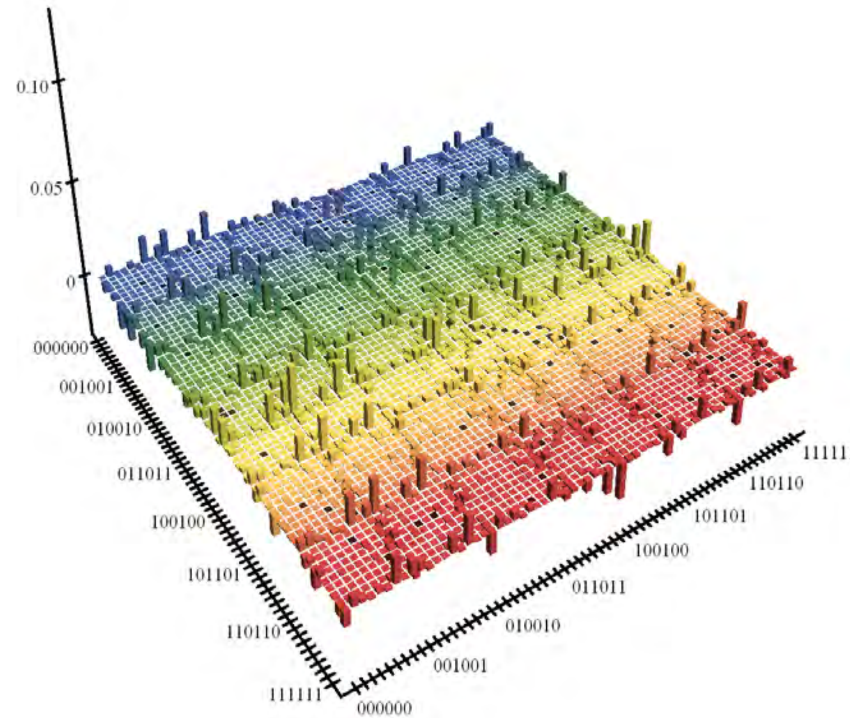




The truth table was measured in the computational basis.

Full quantum process tomography of quantum Fredkin gate

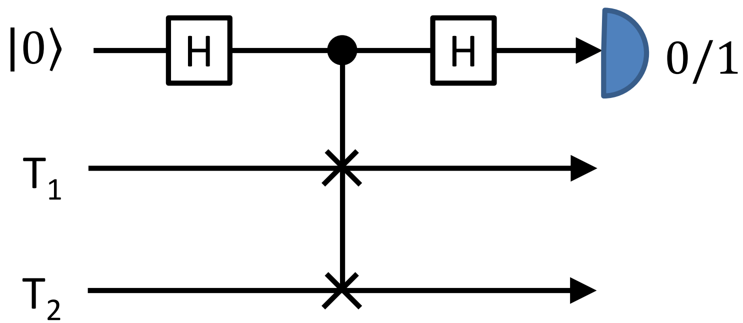




$$F = 0.901 \pm 0.001$$

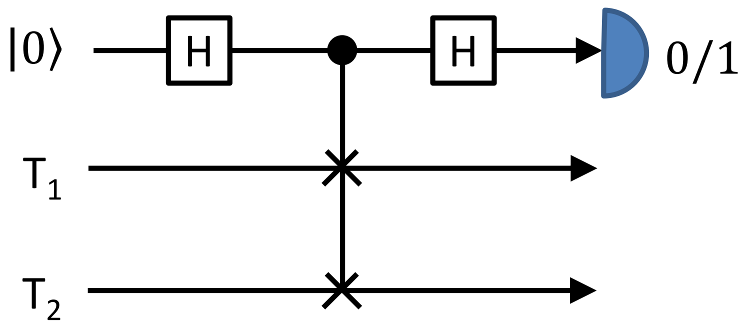
- The Fredkin gate was probed by all 6^3=218 combinations of product states H,V,D,A,R,L.
- Each output state was characterized by measurements in 3^3=27 product bases formed by combinations of single-qubit bases H/V, D/A, R/L.

Quantum symmetrization and anti-symmetrization



C. Miquel, J. P. Paz, M. Saraceno, E. Knill, R. Laflamme, and C. Negrevergne, Nature London **418**, 59 (2002).
R. Filip, Phys. Rev. A **65**, 062320 (2002).
A.K. Ekert, C.M. Alves, D.K.L. Oi, M. Horodecki, P. Horodecki, and L.C. Kwek, Phys. Rev. Lett. **88**, 217901 (2002).

Quantum symmetrization and anti-symmetrization



Projection of the control qubit on state 0 heralds symmetrization of the target qubits:

$$\widehat{W}|i\rangle|j\rangle = |j\rangle|i\rangle \qquad \hat{S} = \frac{1}{2}\left(\hat{I}_{T_1T_2} + \widehat{W}_{T_1T_2}\right) = \hat{I}_{T_1T_2} - |\Psi^-\rangle\langle\Psi^-|$$

Projection of the control qubit on state 1 heralds anti-symmetrization of the target qubits:

$$\hat{A} = \frac{1}{2}\left(\hat{I}_{T_1T_2} - \widehat{W}_{T_1T_2}\right) = |\Psi^-\rangle\langle\Psi^-|$$

C. Miquel, J. P. Paz, M. Saraceno, E. Knill, R. Laflamme, and C. Negrevergne, Nature London **418**, 59 (2002).
R. Filip, Phys. Rev. A **65**, 062320 (2002).
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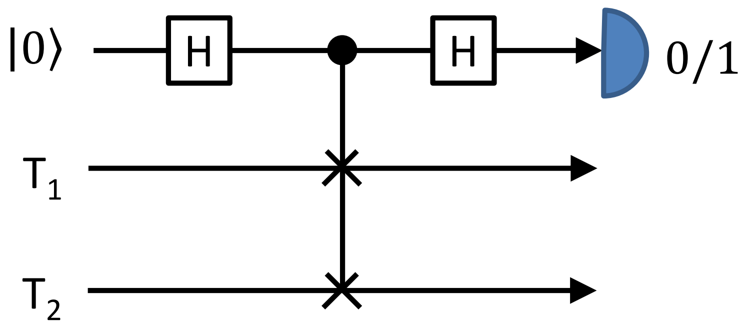
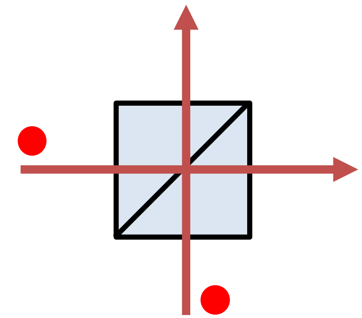
Quantum symmetrization and anti-symmetrization



vs.



(anti)-symmetrization by two-photon interference on balanced BS

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R. Filip, Phys. Rev. A **65**, 062320 (2002).
A.K. Ekert, C.M. Alves, D.K.L. Oi, M. Horodecki, P. Horodecki, and L.C. Kwek, Phys. Rev. Lett. **88**, 217901 (2002).

Optimal quantum cloning by symmetrization

Input state:

$$|\psi\rangle\langle\psi|_{T_1} \otimes \frac{1}{2}\hat{I}_{T_2}$$

Normalized symmetrized output state:

$$\frac{2}{3}|\psi\rangle\langle\psi|_{T_1} \otimes |\psi\rangle\langle\psi|_{T_2} + \frac{1}{3}|\Psi\rangle\langle\Psi|_{T_1T_2}$$

$$|\Psi\rangle_{T_1T_2} = \frac{1}{\sqrt{2}}(|\psi\rangle|\psi_\perp\rangle + |\psi_\perp\rangle|\psi\rangle)$$

Single- and two-copy fidelities:

$$F_{single} = \frac{5}{6} \qquad F_{global} = \frac{2}{3}$$

W.T.M. Irvine, A. Lamas Linares, M.J.A. de Dood, and D. Bouwmeester, Phys. Rev. Lett. **92**, 047902 (2004).
M. Ricci, F. Sciarrino, C. Sias, and F. De Martini, Phys. Rev. Lett. **92**, 047901 (2004).

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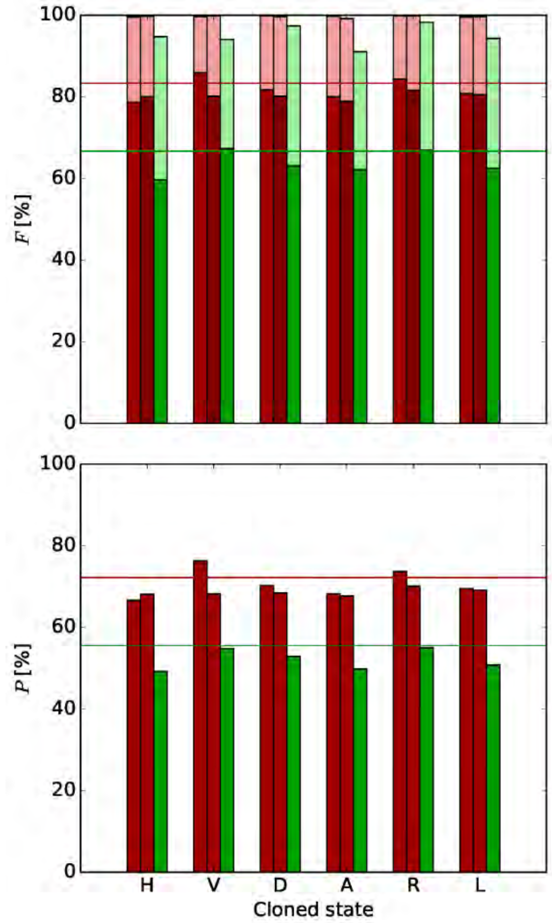

red –single-copy fidelities and purities
green – global fidelities and purities

W.T.M. Irvine, A. Lamas Linares, M.J.A. de Dood, and D. Bouwmeester, Phys. Rev. Lett. **92**, 047902 (2004).
M. Ricci, F. Sciarrino, C. Sias, and F. De Martini, Phys. Rev. Lett. **92**, 047901 (2004).

Purification of single-qubit states

Input mixed state

$$\hat{\rho} = p|\psi\rangle\langle\psi| + \frac{1-p}{2}\hat{I}$$

Purification by symmetrization of two copies of the state

$$\hat{S}_{T_1T_2}(\hat{\rho}\otimes\hat{\rho})\hat{S}_{T_1T_2}$$

Purified single-qubit state

$$\hat{\rho}_{out} = \tilde{p}|\psi\rangle\langle\psi| + \frac{1-\tilde{p}}{2}\hat{I}$$

$$\tilde{p} = \frac{4p}{3+p^2} \geq p$$

J.I. Cirac, A.K. Ekert, and C. Macchiavello, Phys. Rev. Lett. **82**, 4344 (1999).
M. Ricci, F. De Martini, N.J. Cerf, R. Filip, J. Fiurasek, C. Macchiavello, Phys. Rev. Lett. **93**, 170501 (2004) .

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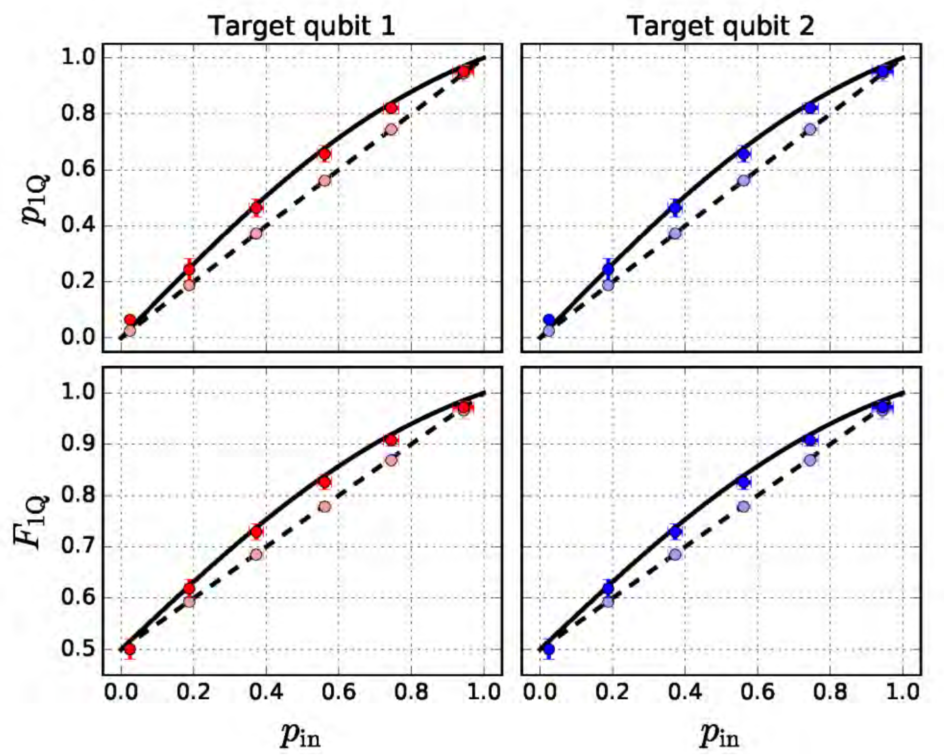

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M. Ricci, F. De Martini, N.J. Cerf, R. Filip, J. Fiurasek, C. Macchiavello, Phys. Rev. Lett. **93**, 170501 (2004) .

Direct estimation of purity and overlap of quantum states

Input state of target qubits:

$$\hat{\rho}_{T_1} \otimes \hat{\sigma}_{T_2}$$

Probability of symmetrization and anti-symmetrization:

$$P_S = \mathrm{Tr}\left[\hat{S}_{T_1 T_2}\left(\hat{\rho}_{T_1} \otimes \hat{\sigma}_{T_2}\right)\hat{S}_{T_1 T_2}\right] \qquad P_A = \mathrm{Tr}\left[\hat{A}_{T_1 T_2}\left(\hat{\rho}_{T_1} \otimes \hat{\sigma}_{T_2}\right)\hat{A}_{T_1 T_2}\right]$$

Direct estimation of overlap of density matrices:

$$P_S - P_A = \mathrm{Tr}[\hat{\rho}\hat{\sigma}]$$

If $\hat{\rho} = \hat{\sigma}$ then $P_S - P_A = \mathrm{Tr}[\hat{\rho}^2]$

R. Filip, Phys. Rev. A **65**, 062320 (2002).
A.K. Ekert, C.M. Alves, D.K.L. Oi, M. Horodecki, P. Horodecki, and L.C. Kwek, Phys. Rev. Lett. **88**, 217901 (2002).
M. Hendrych, M. Dušek, R. Filip, and J. Fiurášek, Phys. Lett. A **310**, 95 (2003).
J. Du, P. Zou, X. Peng, D. Oi, L. Kwek, C. Oh, and A. Ekert, Phys. Rev. A **74**, 042319 (2006).

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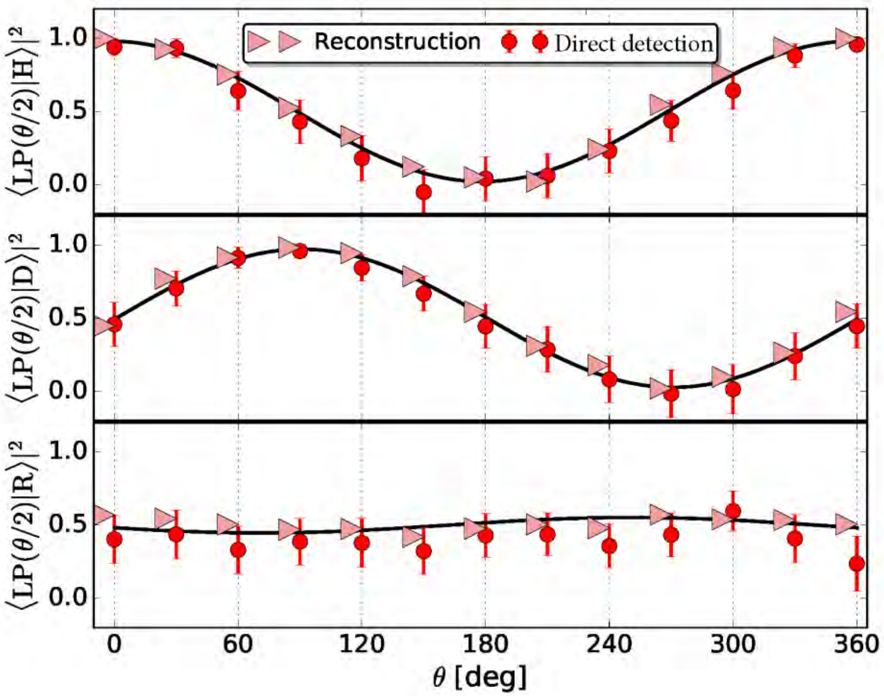

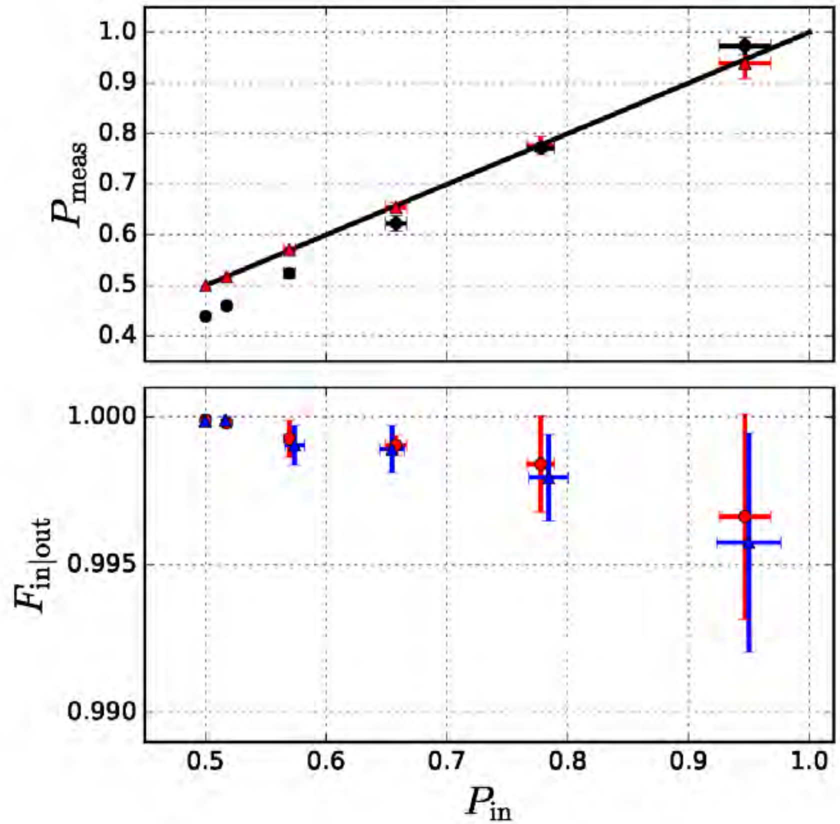

- Overlap and purity are estimated from measurements on the control qubit.
- Output target qubits are accessible and other measurements can be performed on them.
- Non-demolition character of purity estimation: the reduced single-qubit density matrices of the output target states remain equal to the input states.

Single-qubit quantum fingerprinting

Comparison of states of the two target qubits:

$$|\psi\rangle_{T_1} \qquad |\varphi\rangle_{T_2}$$

Probability of projection onto anti-symmetric subspace:

$$P_A = \frac{1}{2}(1 - |\langle\varphi|\psi\rangle|^2)$$

$$P_A > 0 \Leftrightarrow |\psi\rangle \neq |\varphi\rangle$$

S.M. Barnett, A. Chefles, and I. Jex, Phys. Lett. A 307, 189-195 (2003).
J. Niel de Beaudrap, Phys. Rev. A 69, 022307 (2004).
J. Du, P. Zou, X. Peng, D. Oi, L. Kwek, C. Oh, and A. Ekert, Phys. Rev. A **74**, 042319 (2006).
R.T. Horn, S.A. Babichev, K.-P. Marzlin, A.I. Lvovsky, and B.C. Sanders, Phys. Rev. Lett. **95**, 150502 (2005).

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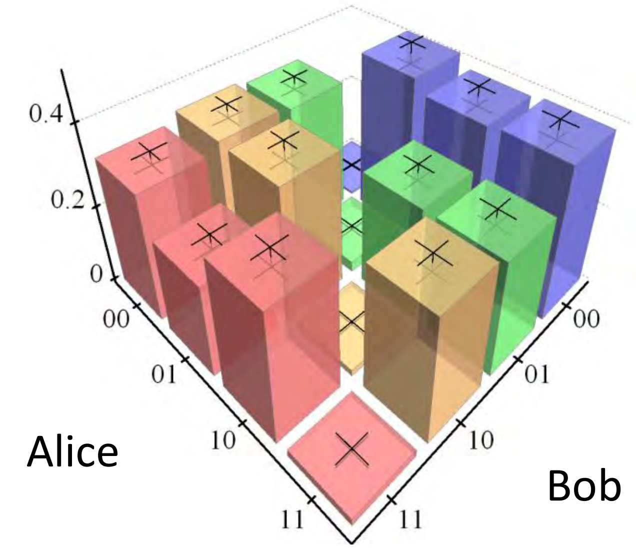


Single-qubit quantum fingerprinting:

- Encode two classical bits into single-qubit states forming vortices of regular tetrahedron on Poincare sphere.
- Compare the two-bit strings by comparing the single-qubit fingerprints.
- Non-zero probability of detection of any difference in the compared pairs of bits.

S.M. Barnett, A. Chefles, and I. Jex, Phys. Lett. A 307, 189-195 (2003).
J. Niel de Beaudrap, Phys. Rev. A 69, 022307 (2004).
J. Du, P. Zou, X. Peng, D. Oi, L. Kwek, C. Oh, and A. Ekert, Phys. Rev. A **74**, 042319 (2006).
R.T. Horn, S.A. Babichev, K.-P. Marzlin, A.I. Lvovsky, and B.C. Sanders, Phys. Rev. Lett. **95**, 150502 (2005).

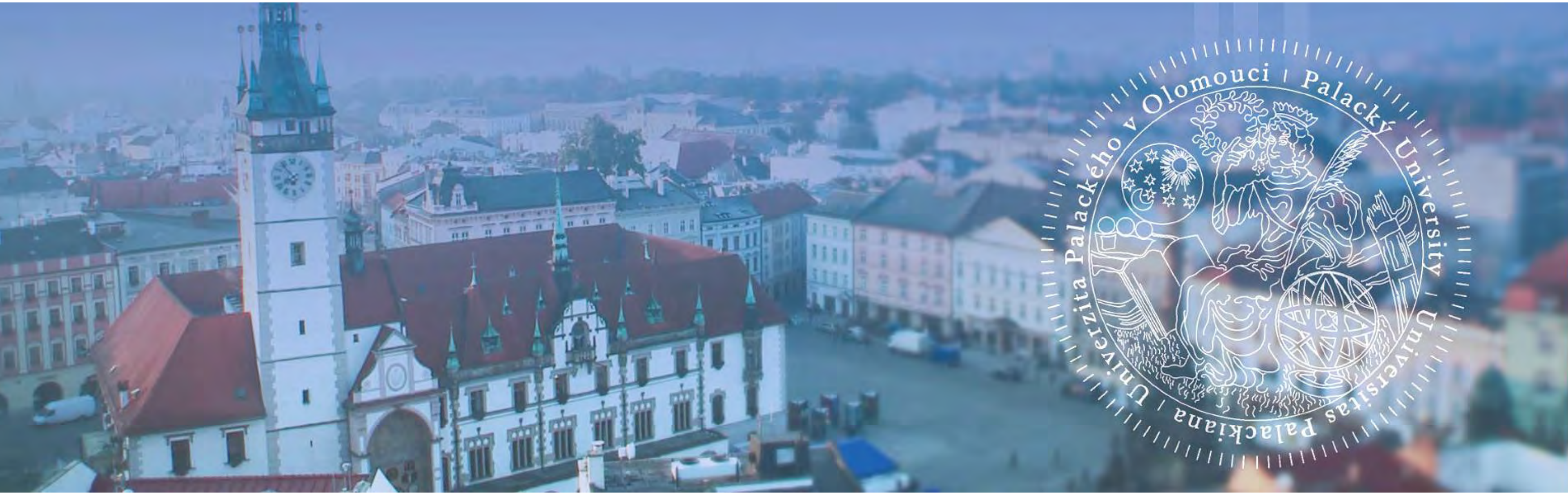
Conclusions

Linear optical quantum Fredkin gate based on hybrid qubit encoding was demonstrated

High coincidence rate enables comprehensive gate characterization

Numerous application of circuit that enables (anti)-symmetrization of the target states

Thank you for your attention!



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