Calculation of switching rates of phase bistability in strongly coupled dissipative Jaynes-Cummings model

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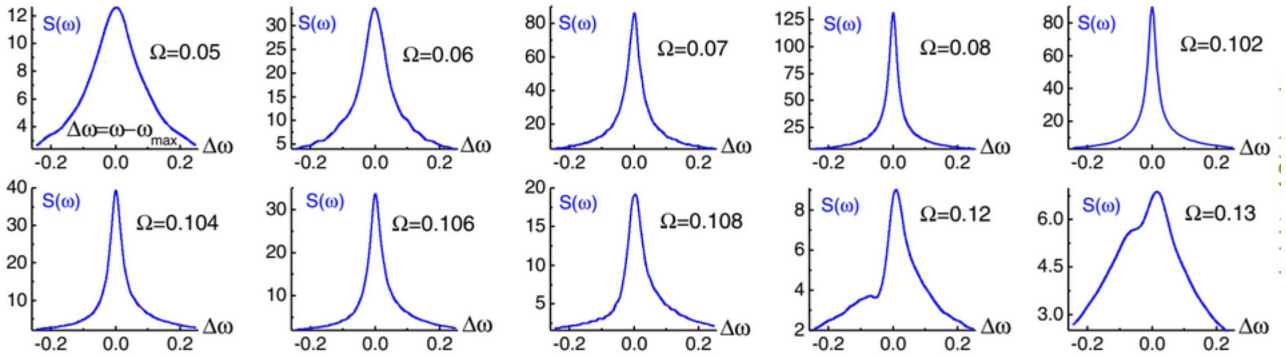
T. Chlouba et al., Opt. Lett. 41, 5821 (2016)

Outline

- Introduction
  - Model definition, numerical methods and results.
- Semiclassical approach
  - Neoclassical equations of motion. Comparison with numerical results. Limitations of semiclassics.
- Full quantum approach
  - Structure of metastable states. Analytical rate formula.

Introduction - Motivation

- E. Andrianov, N. Chtchelkatchev et al.: *Noisy metamolecule: strong narrowing of fluorescence line*, Optics Letters 40, 3536 (2015).



- „no man`s land... advanced computational methods required....”

Introduction - Model

- Resonantly driven Jaynes-Cummings system with Markovian dissipation

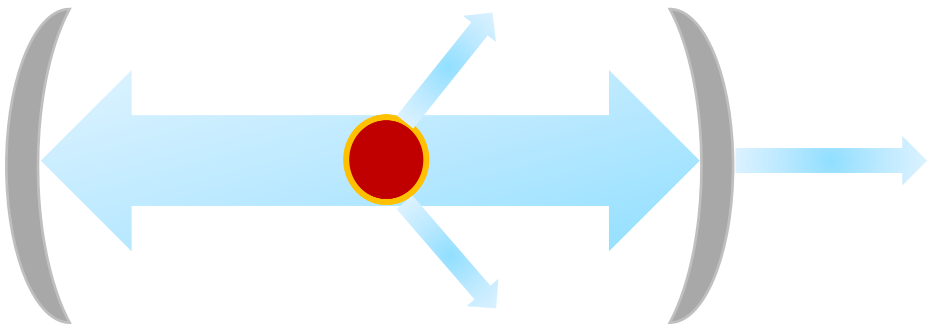
$$H_{\text{spin}} = \omega\sigma^{\dagger}\sigma + \Omega_a \cos(\omega t)(\sigma + \sigma^{\dagger})$$

$$H_{\text{boson}} = \omega a^{\dagger} a + \Omega_b \cos(\omega t)(\mathrm{a} + \mathrm{a}^{\dagger})$$

$$H_{\text{int}} = g(\sigma + \sigma^{\dagger})(a + a^{\dagger})$$

$$\sigma^{\dagger} = \frac{1}{2}(\sigma_x + \sigma_y)$$

$$\sigma = \frac{1}{2}(\sigma_x - \sigma_y)$$



$$\frac{d\rho}{dt} = -i[H, \rho] + \frac{\gamma_a}{2}(2\sigma\rho\sigma^{\dagger} - \rho\sigma^{\dagger}\sigma - \sigma^{\dagger}\sigma\rho) + \frac{\gamma_b}{2}(2a\rho a^{\dagger} - a^{\dagger}a\rho - \rho a^{\dagger}a)$$

Introduction – Approximation

- RWA approximation + (canonical transformation)

$$g = 0.35$$
$$\gamma_b = 0.25$$
$$\gamma_a = 0.002$$
$$\Omega_a = \Omega_b/9$$

$$H_{\text{spin}} = \cancel{\omega\sigma^\dagger\sigma} + \boxed{\Omega_a(\sigma + \sigma^\dagger)}$$

$$H_{\text{boson}} = \cancel{\omega a^\dagger a} + \Omega_b(a + a^\dagger)$$

$$H_{\text{int}} = g(\sigma^\dagger a + \sigma a^\dagger) + \cancel{g(\sigma^\dagger a^\dagger + \sigma a)}$$

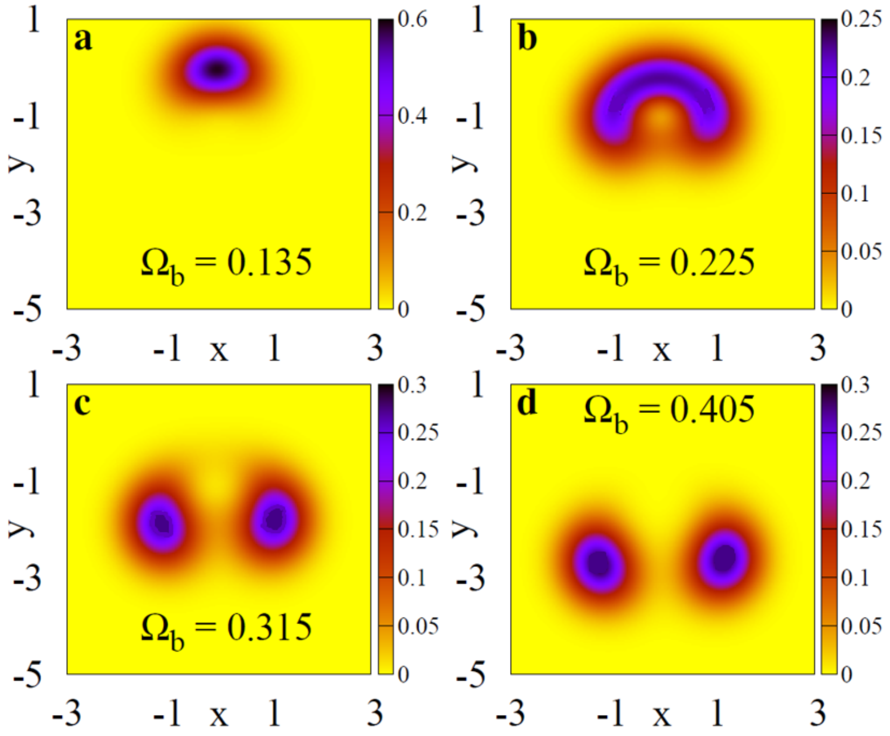
$$\boxed{H_{\text{RWA}} = g(\sigma^\dagger a + \sigma a^\dagger) + \Omega_b(a + a^\dagger) + \Omega_a(\sigma + \sigma^\dagger)}$$

$$\frac{d\rho}{dt} = -i[H_{\text{RWA}}, \rho] + \frac{\gamma_a}{2}(2\sigma\rho\sigma^\dagger - \rho\sigma^\dagger\sigma - \sigma^\dagger\sigma\rho) + \frac{\gamma_b}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$$

E. Andrianov et al., Opt. Lett. 40, 3536 (2015)

Introduction - Numerical methods, Results

- System **easily** solvable by sparse numerical methods or quantum trajectories in QuTip
- Wigner functions: Two-peak structure above certain drive threshold



Semiclassical approach – Neoclassical equations of motion

$$\frac{d\langle a\rangle}{dt} = -i(\Omega_b + g\langle\sigma\rangle) - \frac{\gamma_a}{2}\langle a\rangle$$

$$\alpha \equiv \langle a\rangle_{\text{stat.}} = -\frac{2i}{\gamma_b}(\Omega_b + g\langle\sigma\rangle)$$

$$H_{\text{eff}} = \Omega^*(\alpha)\sigma + \Omega(\alpha)\sigma^\dagger$$

$$\Omega(\alpha) = (\Omega_a + g\alpha) = \Delta e^{i\phi}$$

$$|+\rangle = \frac{1}{\sqrt{2}}(1, -e^{i\phi_+})^T \qquad -\Delta \qquad \langle\sigma\rangle_+$$

$$|-\rangle = \frac{1}{\sqrt{2}}(1, e^{i\phi_-})^T \qquad \Delta \qquad \langle\sigma\rangle_-$$

$$\boxed{\alpha_\pm = -2i\frac{\Omega_b}{\gamma_b} \pm i\frac{g}{\gamma_b}e^{i\phi_\pm}}$$

$$\Delta = \frac{g^2}{\gamma_b}\sqrt{\left(\frac{\Omega_b}{\Omega_{b,c}}\right)^2 - 1}$$

$$e^{i\phi_\pm} = \frac{\Omega_a - i\frac{2\Omega_b g}{\gamma_b}}{\Delta \mp i\frac{g^2}{\gamma_b}}$$

Semiclassical approach – Neoclassical equations of motion

$$\frac{d\langle a\rangle}{dt} = -i(\Omega_b + g\langle\sigma\rangle) - \frac{\gamma_a}{2}\langle a\rangle \quad \Rightarrow \quad \alpha \equiv \langle a\rangle_{\text{stat.}} = -\frac{2i}{\gamma_b}(\Omega_b + g\langle\sigma\rangle)$$

$$\Rightarrow \quad H_{\text{eff}} = \Omega^*(\alpha)\sigma + \Omega(\alpha)\sigma^\dagger \quad \begin{cases} |+\rangle \\ |-\rangle \end{cases} \Rightarrow \quad \begin{matrix} \langle\sigma\rangle_+ \\ \langle\sigma\rangle_- \end{matrix} \quad \Rightarrow \quad \boxed{\alpha_\pm = -2i\frac{\Omega_b}{\gamma_b} \pm i\frac{g}{\gamma_b}e^{i\phi_\pm}}$$

$$\Omega(\alpha) = \Delta e^{i\phi}$$

$$e^{i\phi_\pm} = \frac{\Omega_a - i\frac{2\Omega_b g}{\gamma_b}}{\Delta \mp i\frac{g^2}{\gamma_b}} \qquad \Delta = \frac{g^2}{\gamma_b}\sqrt{\left(\frac{\Omega_b}{\Omega_{b,c}}\right)^2 - 1} \quad \Rightarrow \quad \boxed{\Omega_{b,c} = \frac{g^2}{\sqrt{4g^2 + \gamma_b^2\Omega_b^2/\Omega_a^2}}}$$

Onset of luminescence threshold

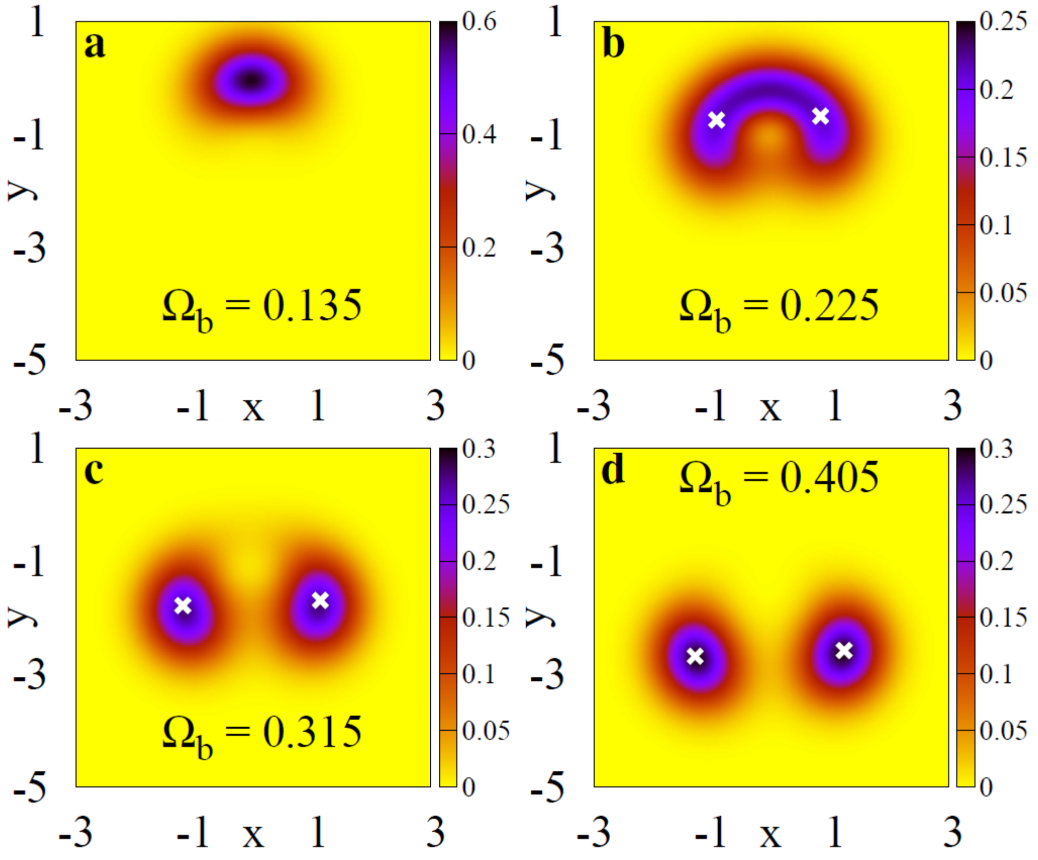
Semiclassical approach - Results

$$\alpha_{\pm} = -2i\frac{\Omega_b}{\gamma_b} \pm i\frac{g}{\gamma_b}e^{i\phi_{\pm}}$$

$$\Omega_{b,c} = g^2/\sqrt{4g^2 + \gamma_b^2\Omega_b^2/\Omega_a^2} \approx 0.175$$

Onset of luminescence threshold

For large drives coexistence of 2 metastable states

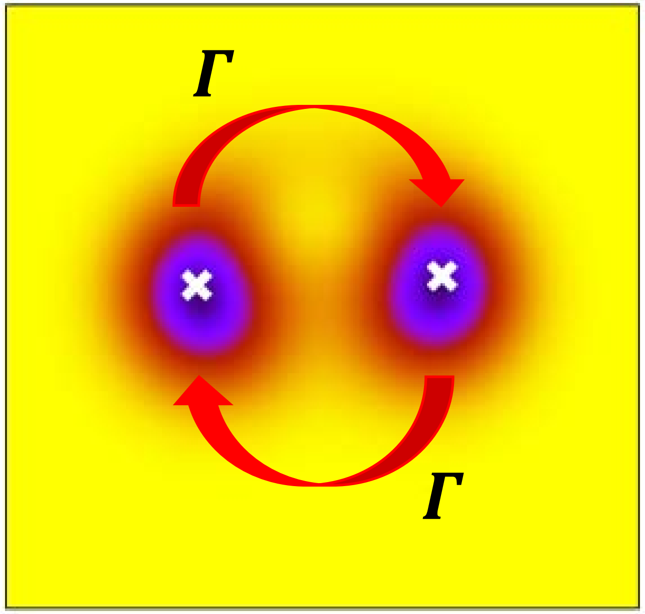


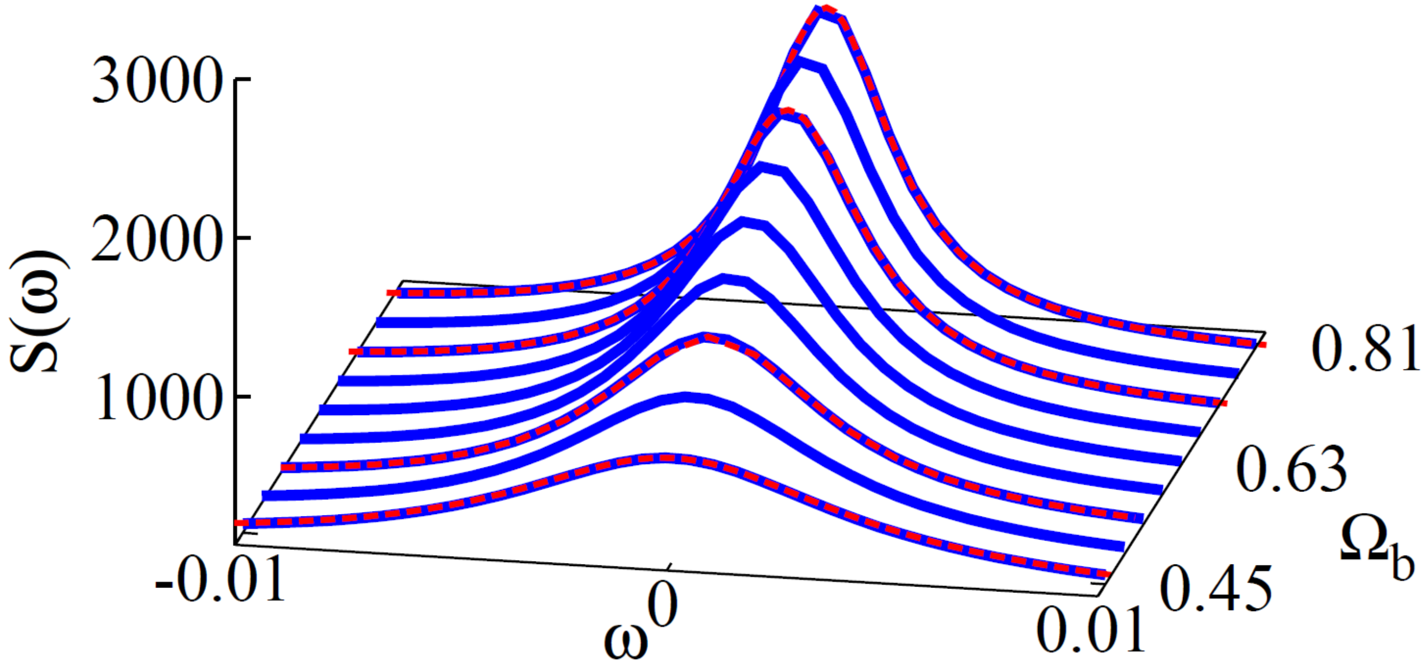
Semiclassical approach - Luminescence spectrum

- Numerical calculation by Quantum regression theorem
- Dichotomous noise formula, fits.

C. W. Gardiner, P. Zoller, Quantum Noise (Springer 2000).

$$S(\omega) = \frac{\Gamma|\alpha_+ - \alpha_-|^2}{\omega^2 + 4\Gamma^2}$$



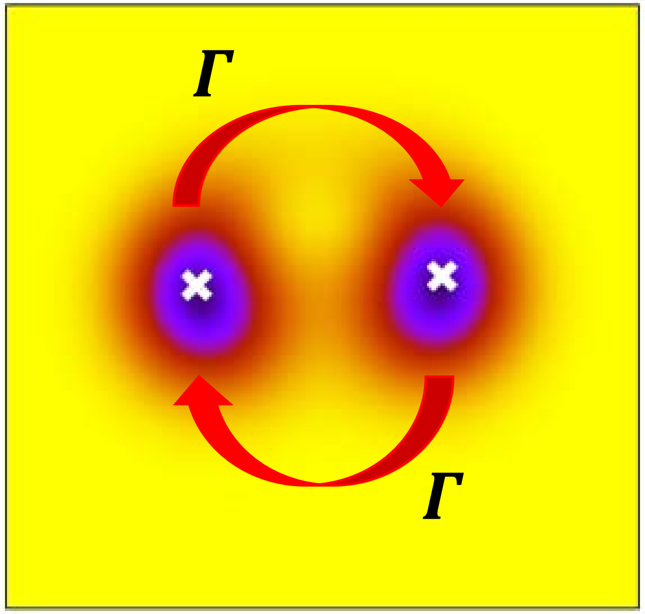


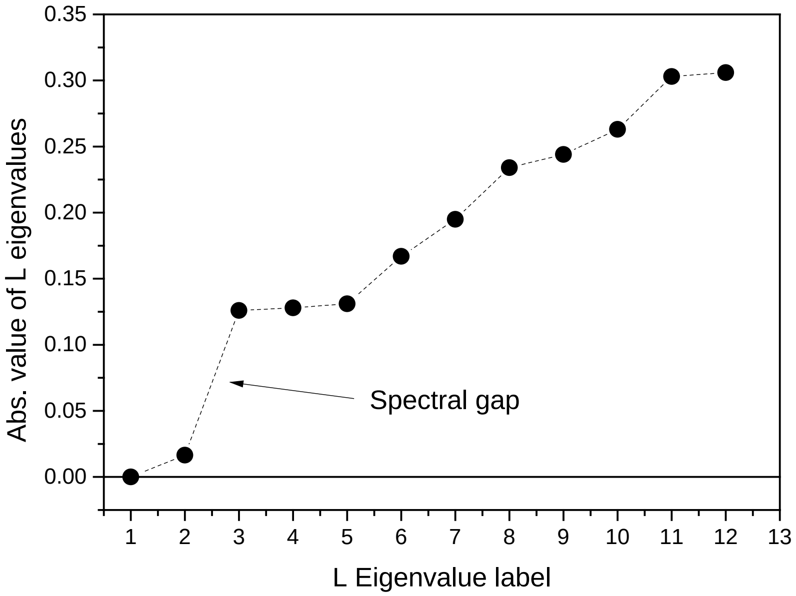
Towards a theory of metastability in open quantum dynamics: K. Macieszczak et al., PRL 116, 1 (2016).

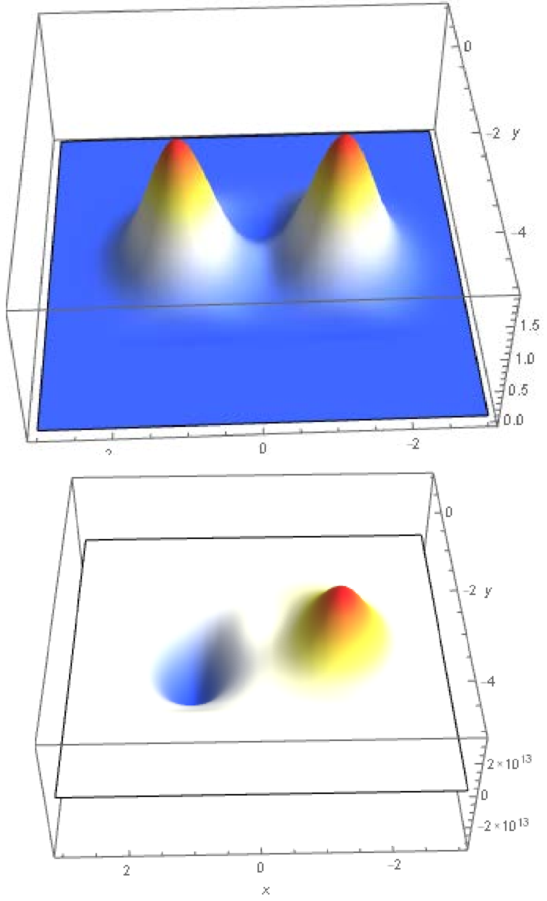
Semiclassical approach – Bistability identification

Towards a theory of metastability in open quantum dynamics: K. Macieszczak et al., PRL 116, 1 (2016).

- Spectral decomposition of Liouvillean
- Wigners of stat. state and “excited density matrix”







- Attempt to calculate switching rates semiclassicaly.

Semiclassical approach – Switching rates

Towards a theory of metastability in open quantum dynamics: K. Macieszczak et al., PRL 116, 1 (2016).

- Restriction to subspace given by two (nonorthogonal) metastable states

$$\frac{d}{dt}\begin{pmatrix} 1 & S \\ S & 1 \end{pmatrix}\begin{pmatrix} p_+ \\ p_- \end{pmatrix} = \begin{pmatrix} -\Gamma & \Gamma \\ \Gamma & -\Gamma \end{pmatrix}\begin{pmatrix} p_+ \\ p_- \end{pmatrix}$$

$$\rho = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} \qquad p_+ = Tr\rho_{++}$$

- Gives a set of coupled equations. Can be solved in P representation.

Semiclassical approach – Switching rates

Towards a theory of metastability in open quantum dynamics: K. Macieszczak et al., PRL 116, 1 (2016).

- Restriction to subspace given by two (nonorthogonal) metastable states

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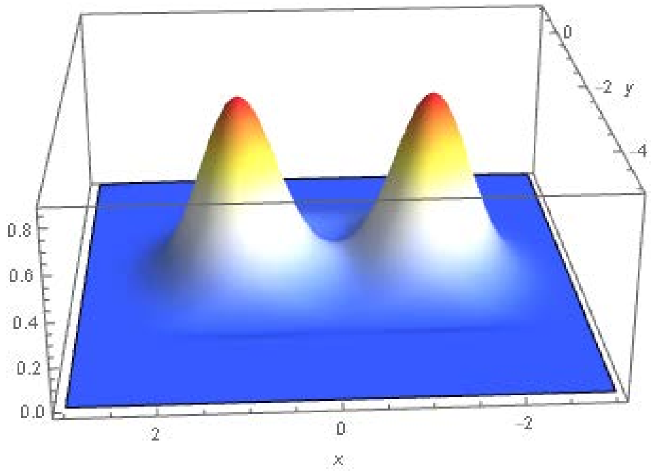
- Gives a set of coupled equations. Can be solved in P representation

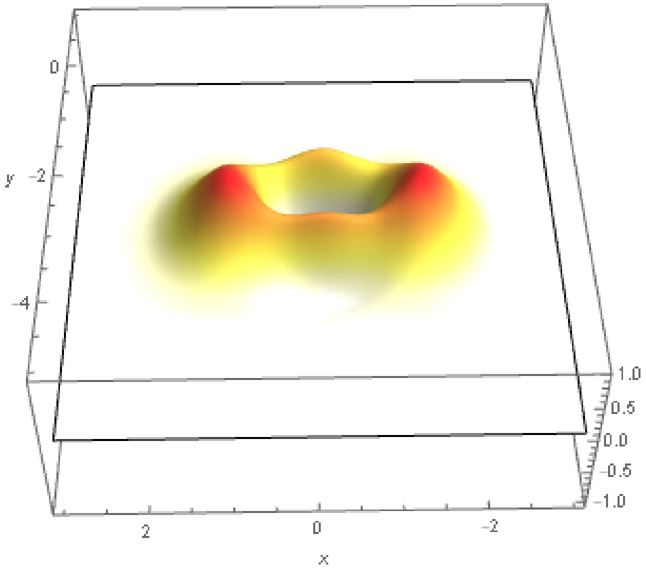
- Does not give correct solution. Why?

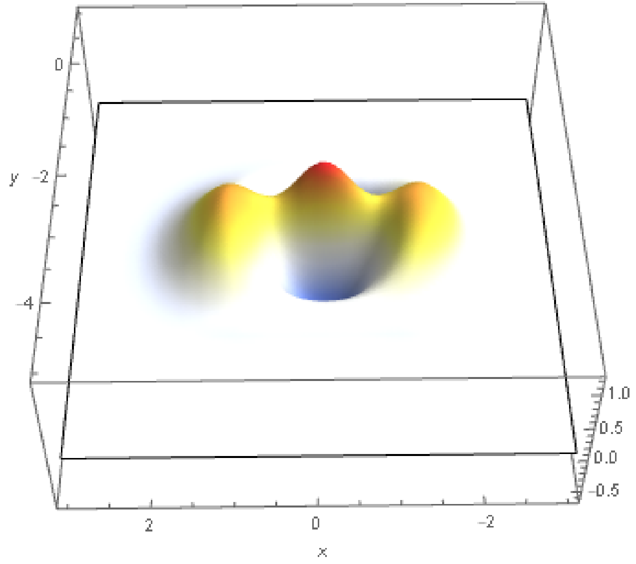
Semiclassical approach – Switching rates

Towards a theory of metastability in open quantum dynamics: K. Macieszczak et al., PRL 116, 1 (2016).

- Spectral decomposition of stationary solution:
  - Eigenvalues {0.469, 0.457, 0.03, 0.02, 0.01…}
  - Wigners of two dominant contributions
  - Some linear combination of coherent peaks



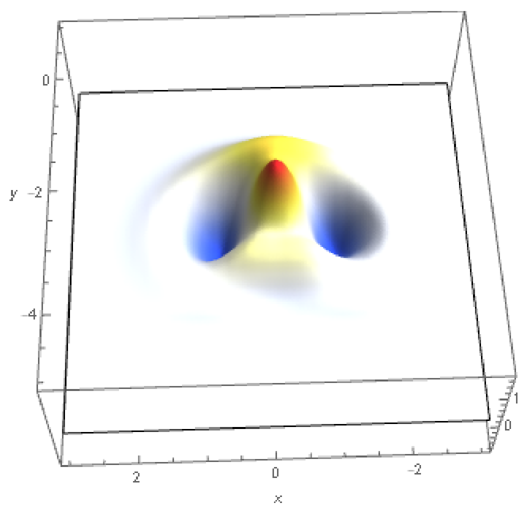


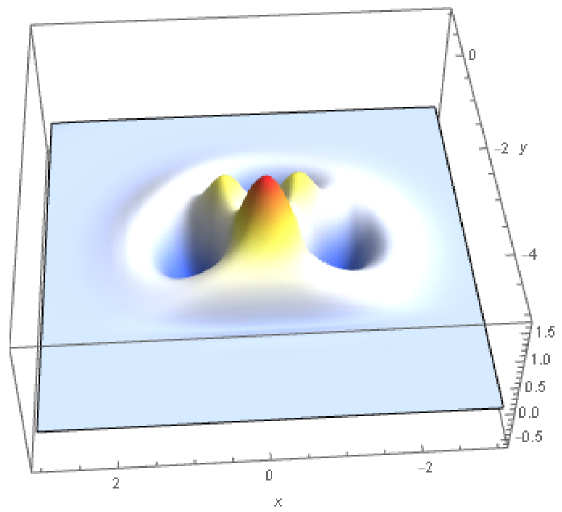


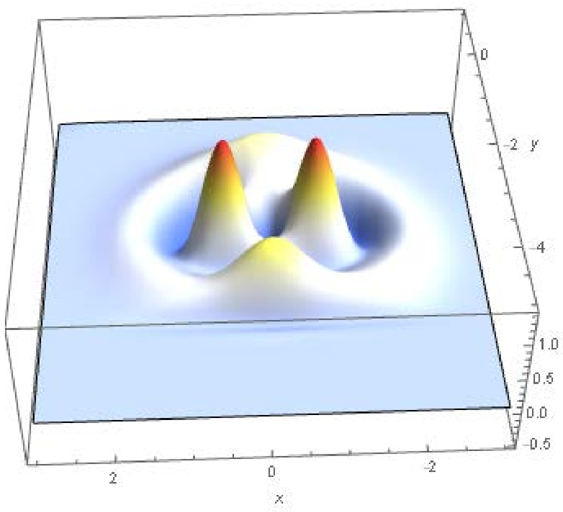
Semiclassical approach – Switching rates

Towards a theory of metastability in open quantum dynamics: K. Macieszczak et al., PRL 116, 1 (2016).

- Spectral decomposition of stationary solution:
  - Eigenvalues {0.469, 0.457, 0.03, 0.02, 0.01…}
  - Wigners of following "eigenvectors"…
  - Their eigenvalue is not small enough for calculation of small switching rates







H. J. Carmichael, Phys. Rev. X 5, **031028** (2015).

Full quantum approach

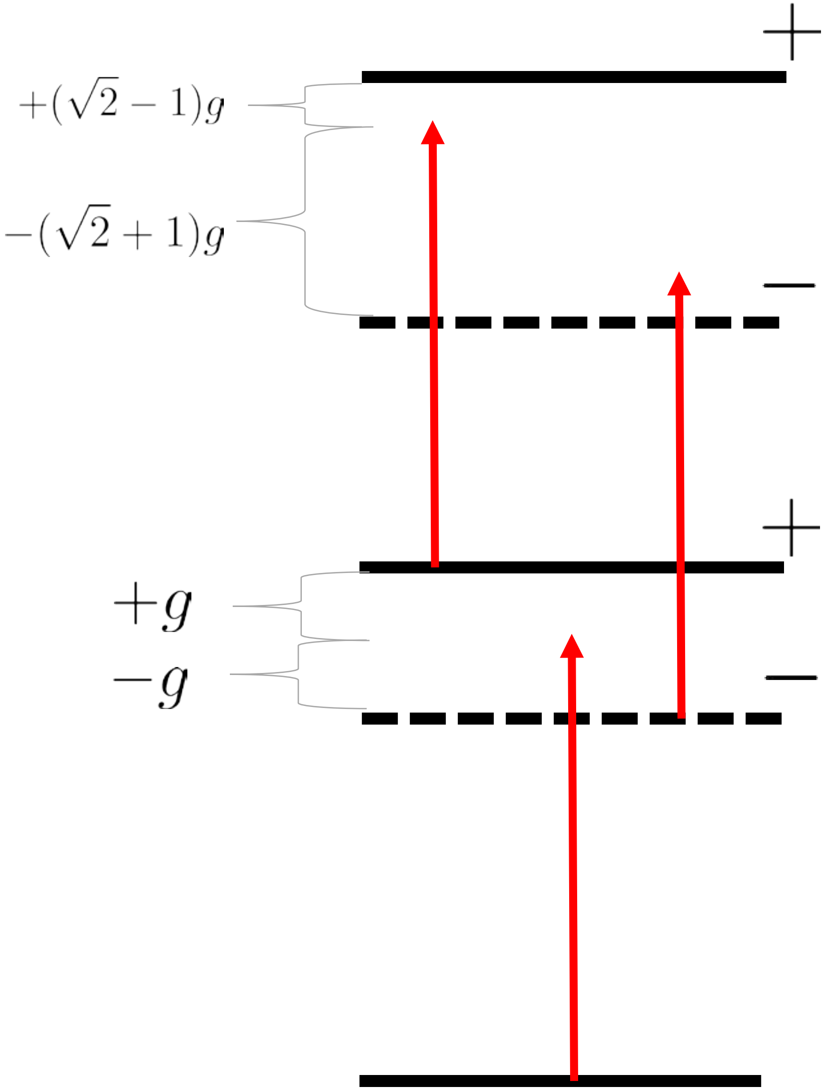
- True nature of bistable states and **photon blockade**

$$|n,+\rangle \propto |n\rangle|\downarrow\rangle + |n-1\rangle|\uparrow\rangle \qquad E_{n,+} = (n-1/2)\omega + g\sqrt{n}$$

$$|n,-\rangle \propto |n\rangle|\downarrow\rangle - |n-1\rangle|\uparrow\rangle \qquad E_{n,-} = (n-1/2)\omega - g\sqrt{n}$$

Ansatz:

$$\boxed{|\psi_+\rangle \propto \sum_{n=1}^{\infty} \frac{\alpha_+^n}{\sqrt{n!}}|n,+\rangle + \frac{1}{\sqrt{2}}|\mathrm{vac}\rangle}$$



Full quantum approach - Solution

- Fermi Golden Rule-like approach

$$\rho(0) = |\psi_+\rangle\langle\psi_+|$$

1. Calculate population of + state

$$\rho_{++}(t) = \langle\psi_+|\rho(t)|\psi_+\rangle = \mathrm{Tr}^+\rho(\mathrm{t})$$

2. Plug into Liouville-Lindblad equation

$$\dot{\rho}_{++}(t) = \mathrm{Tr}^+\dot{\rho}(\mathrm{t}) = \langle\psi_+|\mathcal{L}\rho(\mathrm{t})|\psi_+\rangle$$

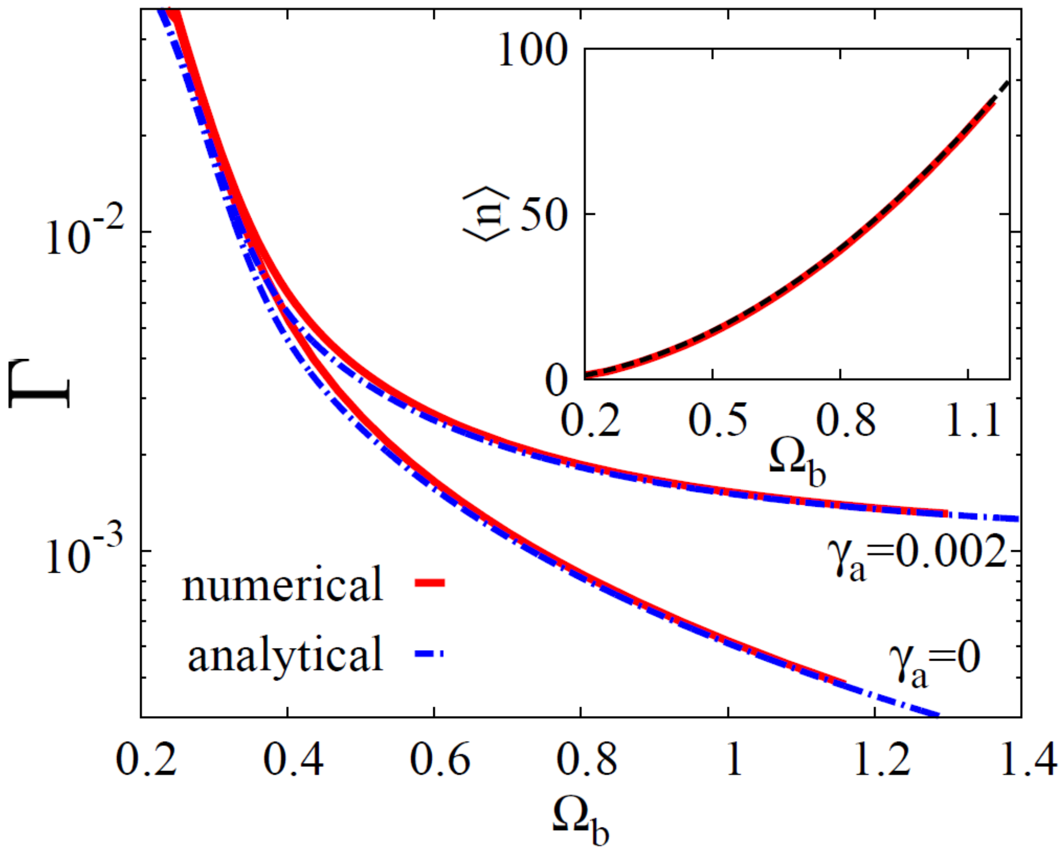
3. Assume exponential decay and do the algebra

$$\boxed{\Gamma = -\frac{\dot{\rho}_{++}(0)}{\rho_{++}(0)}} = -\frac{\gamma_b}{4}[e^{|\alpha|^2}-1/2]^{-1}[(e^{|\alpha|^2}-2|\alpha|^2e^{|\alpha|^2}-1)+2|\alpha|^2\sum_{k=0}^{\infty}\frac{|\alpha|^{2k}}{k!}\sqrt{\frac{k}{k+1}}]+\frac{\gamma_a}{2}\frac{1}{1-\frac{1}{2e^{|\alpha|^2}}} \boxed{\approx \frac{\gamma_b}{16\langle n\rangle}+\frac{\gamma_a}{2}}$$

Full quantum approach

- Fine structure of states $|n, +(-)\rangle$ (correlations between spin and photon) is key for calculations!

- Semiclassical approach is not sufficient for rates!



Thank you for your attention!

References

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6. C. W. Gardiner and P. Zoller, *Quantum Noise*, Springer (2000).