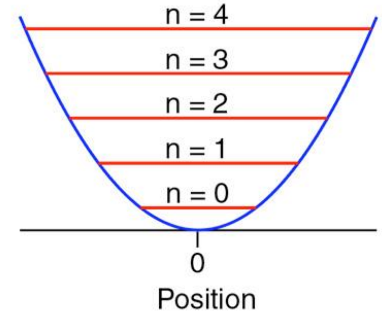
Nonlinear squeezing for quantum information processing

Petr Marek

with many thanks to
R. Filip, A. Furusawa, and others

Palacký University Olomouc

CV Quantum information processing



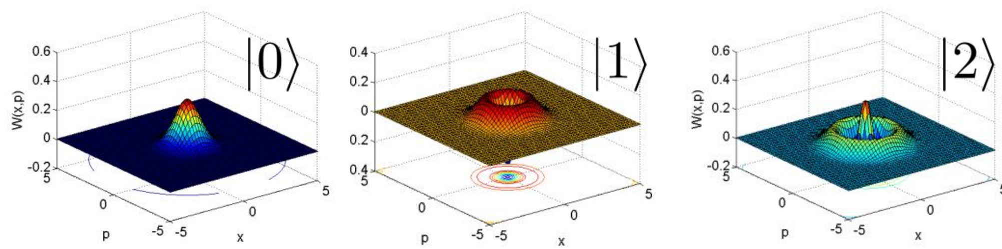
- Preparation of arbitrary quantum states of harmonic oscillators
- Manipulating them in arbitrary fashion
- Measuring them in arbitrary basis

$$\hat{H} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$

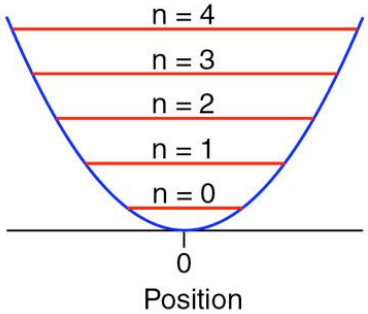
$$\hat{x} = \hat{a} + \hat{a}^\dagger$$

$$\hat{p} = -i(\hat{a} - \hat{a}^\dagger)$$

$W(x,p)$



CV Quantum information processing



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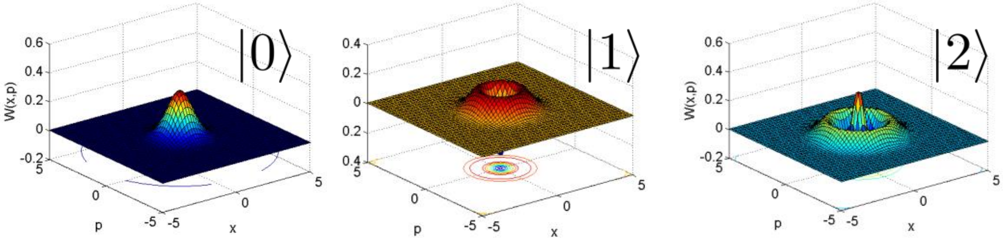


Manipulation: probabilistic VS deterministic

- Probabilistic
  - experimentally feasible
  - allows nonphysical operations
  - not scalable

  - photon subtraction or photon addition

- Deterministic
  - scalable
  - only allows unitary operation or forming mixtures

  - Gaussian operations

Arbitrary unitary transformation of quantum states

$$\hat{U} = e^{-i\hat{H}t} \qquad \hat{H} = \sum_{m,n} c_{m,n}(\hat{x}^m\hat{p}^n + \hat{p}^n\hat{x}^m)$$

- Needs a wide range of nonlinear unitaries
- Alterntively, operations of higher order can be approximated by sequences of lower orders

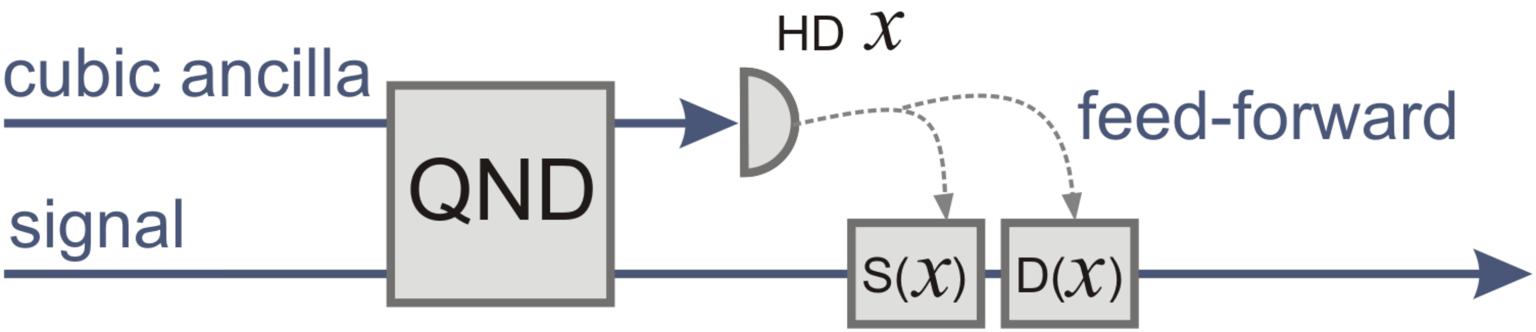
$$e^{-i\hat{A}}e^{-i\hat{B}}e^{i\hat{A}}e^{i\hat{B}} \approx e^{[\hat{A},\hat{B}]}$$

- At least cubic nonlinearity ($\hat{H} = \hat{x}^3$) is required for the synthesis

Seth Lloyd and Samuel L. Braunstein, PRL **82**, 1784 (1999).

How to realize the cubic nonlinearity

$$\hat{x} \to \hat{x}, \quad \hat{p} \to \hat{p} - \chi \hat{x}^2$$

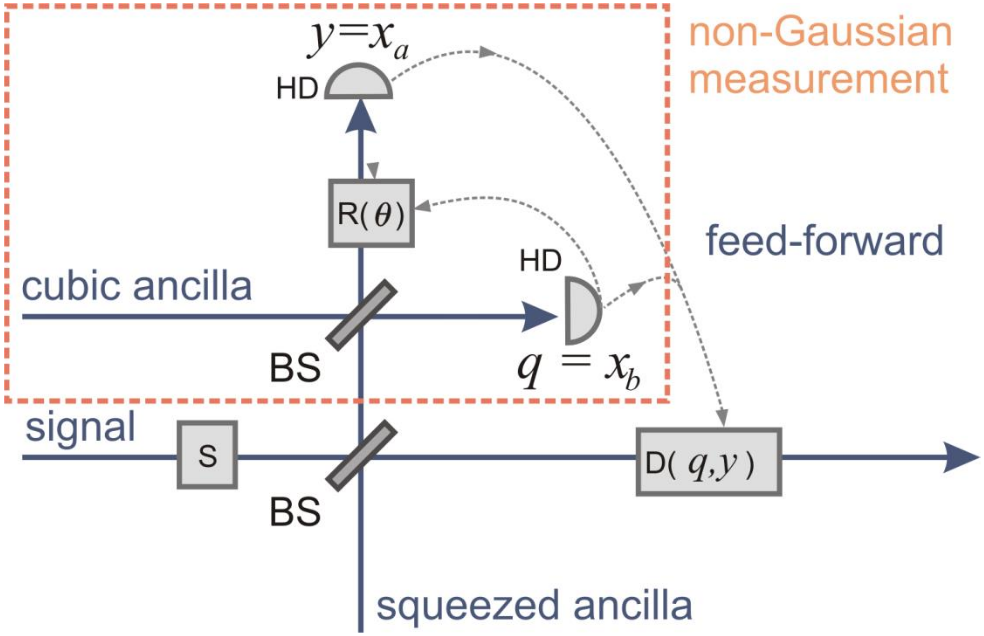


- Apart from the ancilla, everything is Gaussian
- The required ancilla:

$$|A_{x^3}\rangle = \int_{-\infty}^{\infty} e^{-i\chi x^3} |x\rangle dx$$

D. Gottesman, A. Kitaev, and J. Preskill, PRA **64**, 012310 (2001);
P. Marek, R. Filip, and A. Furusawa, PRA **84**, 053802 (2011)

Advanced approach: non-Gaussian measurement



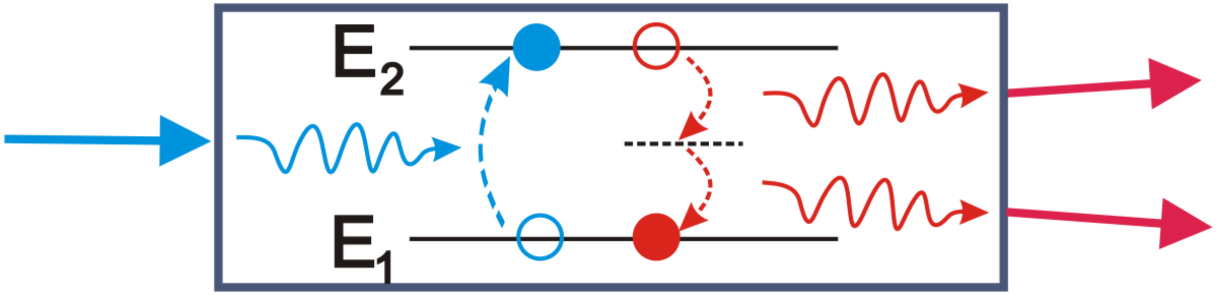
- Measurement projecting on a displaced NG state
- Displacement only feed-forward
- Squeezing compensated in the measurement

K. Miyata, H. Ogawa, P. Marek, R. Filip, H. Yonezawa, J. Yoshikawa, and A. Furusawa, PRA **93**, 022301 (2016);
K. Miyata, H. Ogawa, P. Marek, R. Filip, H. Yonezawa, J. Yoshikawa, and A. Furusawa, Phys. Rev. A **90**, 060302 (2014)

The presentation so far…

- CV quantum information processing
  - realization of the cubic gate
- Nonlinear squeezing

What is squeezing



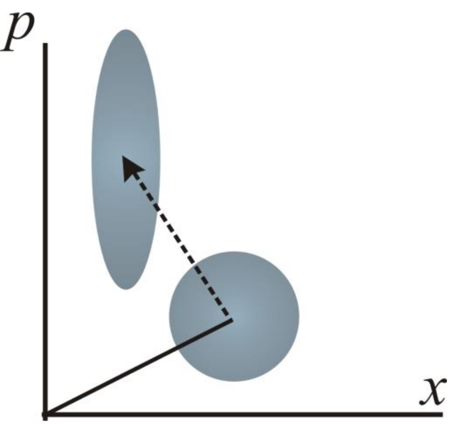
- An elementary technique in quantum optics
- Also known as degenerate SPDC

$$\hat{H} = i\hat{a}^2 - i\hat{a}^{\dagger 2}$$

- Able to generate squeezed states of light

$$e^{-i\hat{H}t}|0\rangle = \sum_{n=0}^{\infty} c_n |2n\rangle$$

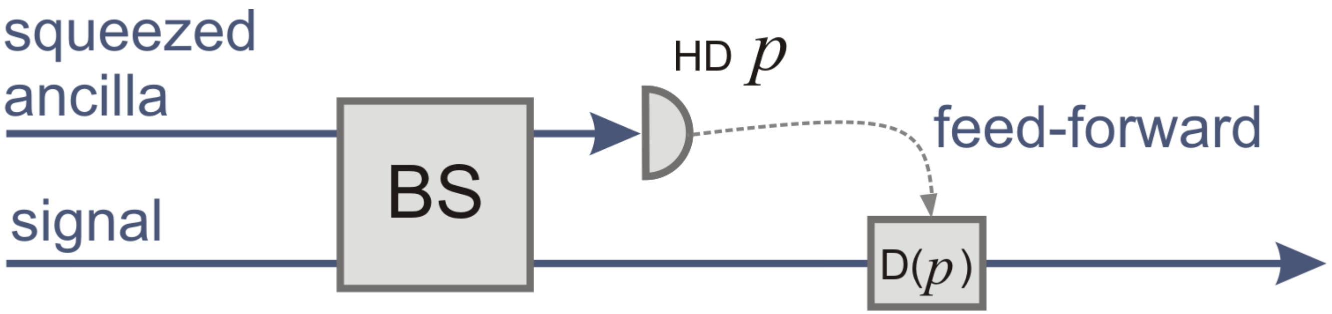
Squeezing in phase space and as active operation



$$\hat{x} \rightarrow g\hat{x}$$
$$\hat{p} \rightarrow \frac{1}{g}\hat{p}$$

- Linear transformation of quadratures
- Can be implemented in existing materials
- Unfortunately only for preparation of the squeezed vacuum state due to:
  - low strength
  - incoupling/outcoupling losses

Measurement induced squeezing



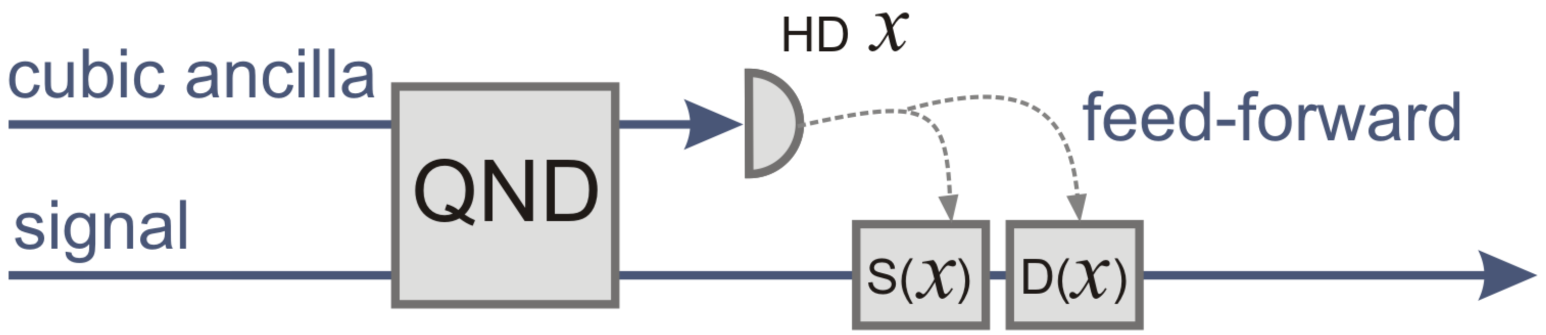
$$\hat{x} \rightarrow g\hat{x} + \sqrt{1-g^2}\,\hat{x}_A$$
$$\hat{p} \rightarrow \frac{1}{g}\hat{p}$$

$$|\mathcal{A}_s\rangle = |x=0\rangle = \int_{-\infty}^{\infty} |p\rangle dp$$

- Squeezing is realized up to a noise term depending on the ancilla
- Ideal ancilla causes no noise
- Combination of measure-and-adjust and quantum erasing

R Filip, P Marek, UL Andersen, Physical Review A 71 (4), 042308 (2005).

Back to the cubic nonlinearity



$$\hat{x} \to \hat{x}, \quad \hat{p} \to \hat{p} - \chi\hat{x}^2 - (\hat{p}_A - \chi\hat{x}_A^2)$$

- Nonlinearity can be realized by the feed-forward
- The ancilla serves to reduce the quantum noise
- Ideal ancilla has reduced fluctuations in a nonlinear quadrature

Linearly and nonlinearly squeezed states

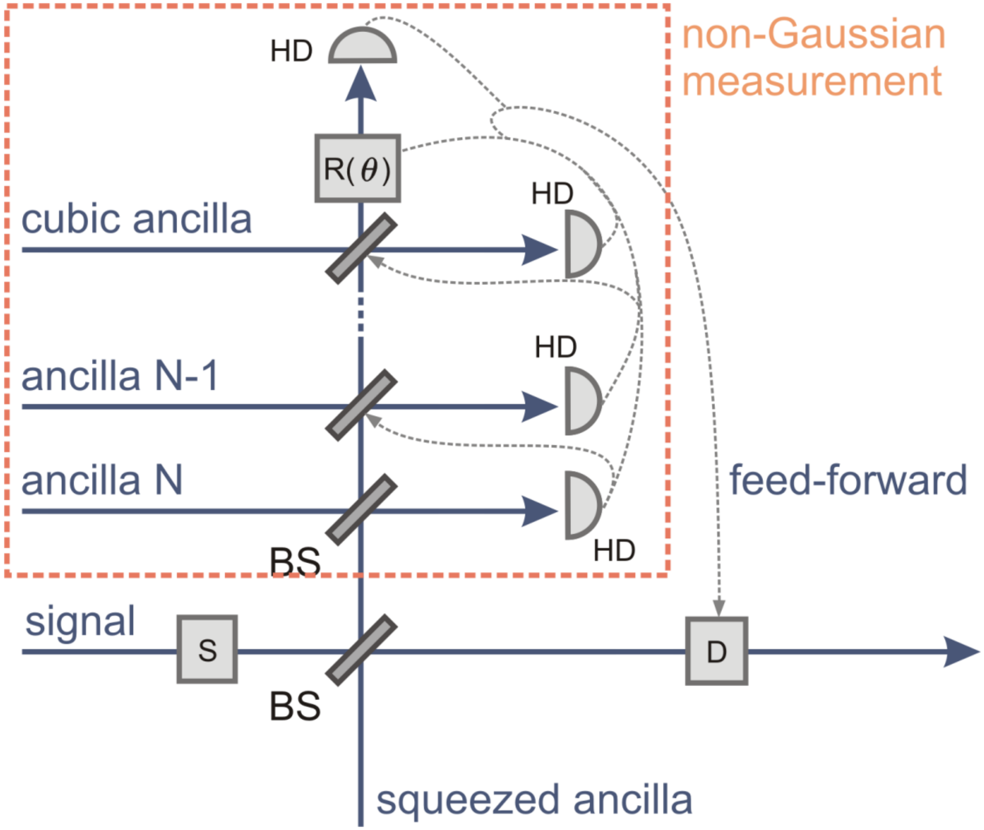
$$\langle[\Delta(\hat{p}-\hat{x})]^2\rangle \to 0$$
$$\langle[\Delta(\hat{p}-\widehat{x^2})]^2\rangle \to 0$$
$$\langle[\Delta(\hat{p}-\widehat{x^3})]^2\rangle \to 0$$
$$\langle[\Delta(\hat{p}-\widehat{x^N})]^2\rangle \to 0$$

- Linear squeezing
- Cubic squeezing
- Quadric squeezing
- Nth order squeezing

Note: parameter χ can be adjusted by linear squeezing

$$\hat{p}-\chi\hat{x}^2 \to \frac{1}{g}\hat{p} - g^2\chi\hat{x}^2 = \frac{1}{g}(\hat{p}-\chi'\hat{x}^2)$$

High order nonlinear operations

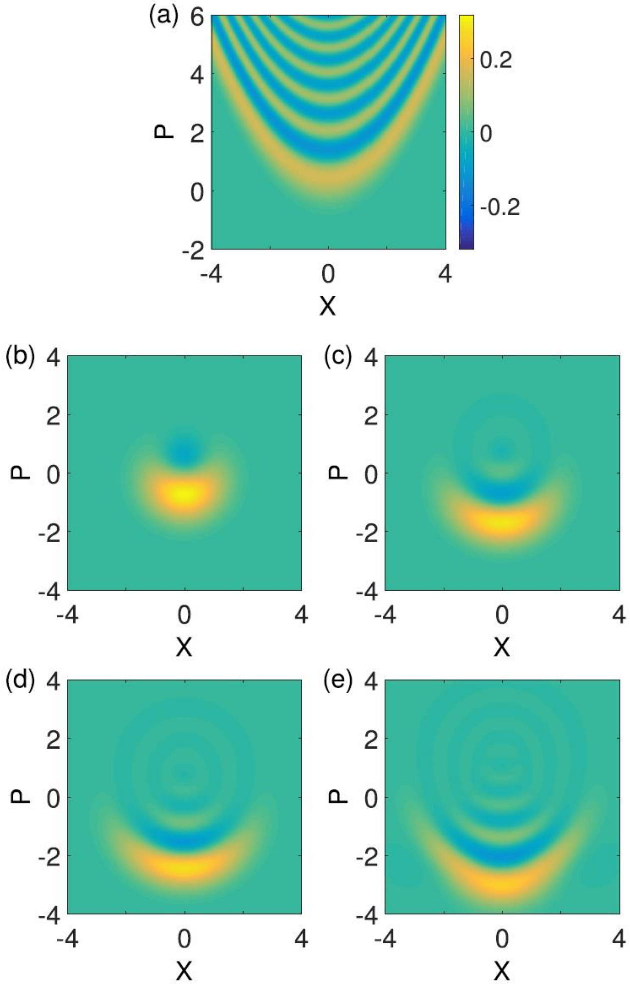


$$\hat{H} = \hat{x}^N$$

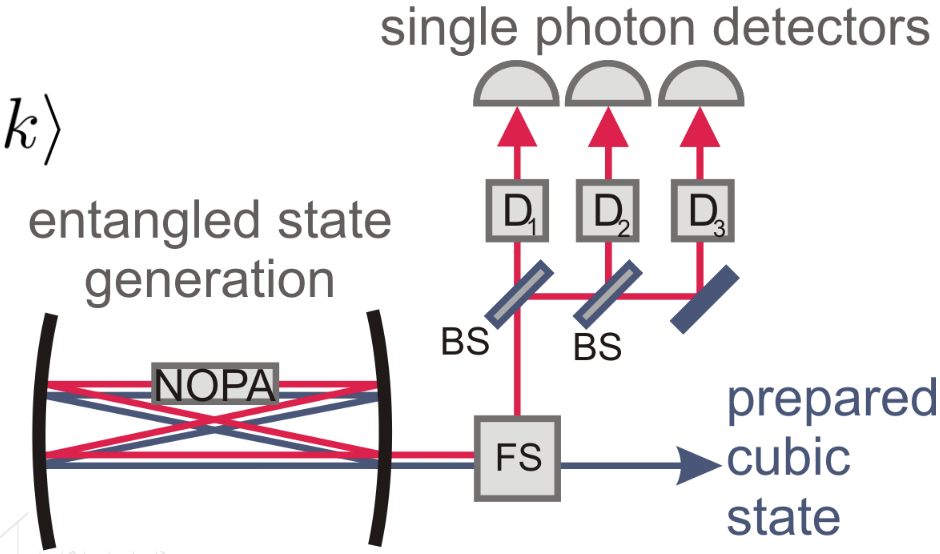
- High order operation can be applied in a single step
- It requires sequence of different nonlinearly squeezed resources, but only single feed-forward operation

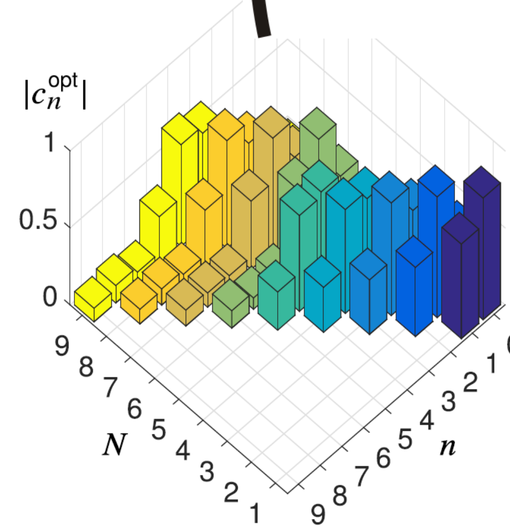
P. Marek, R. Filip, H. Ogawa, A. Sakaguchi, S. Takeda, J. Yoshikawa, and A. Furusawa, Phys. Rev. A **97**, 022329 (2018)

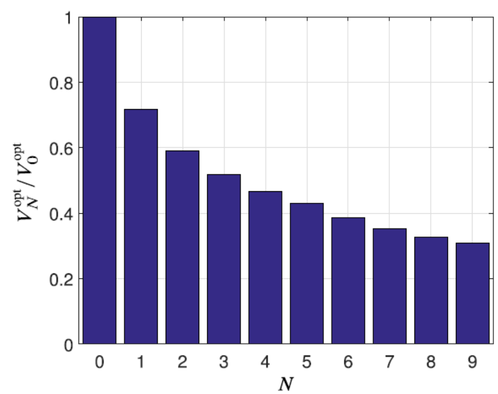
Approximate ancilla in a limited HS



$$\hat{S}\sum_{k=0}^{N_{\max}} c_k|k\rangle$$

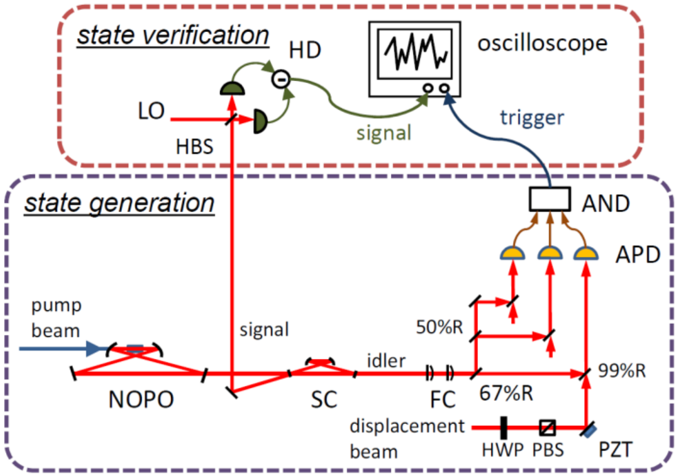






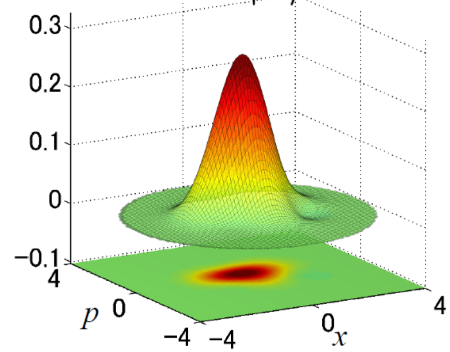
K. Miyata, H. Ogawa, P. Marek, R. Filip, H. Yonezawa, J. Yoshikawa, and A. Furusawa, PRA **93**, 022301 (2016)

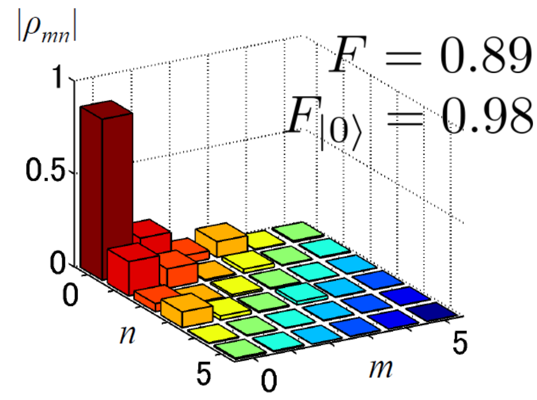
Preparation of the probabilistic ancilla



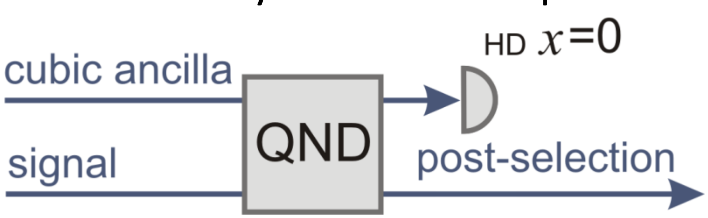
Weak cubic state

$$e^{-i\chi\hat{x}^3}|0\rangle$$





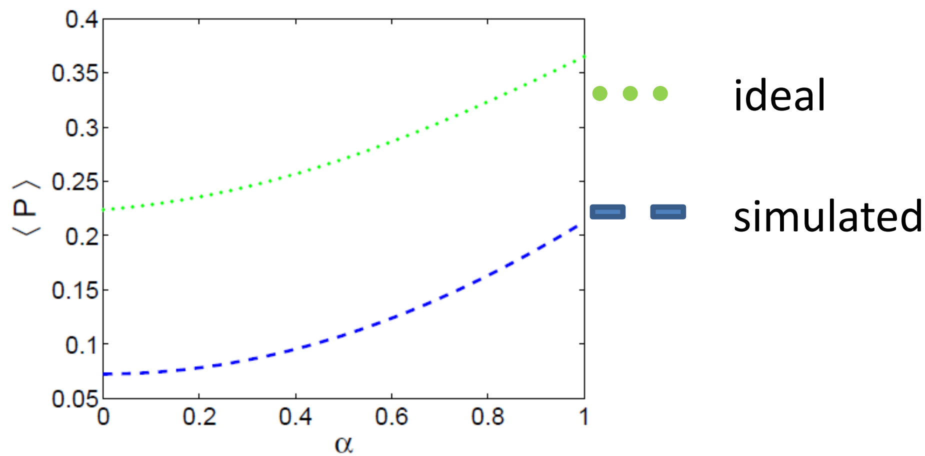
Verification by conditional operation:



Expected behavior:

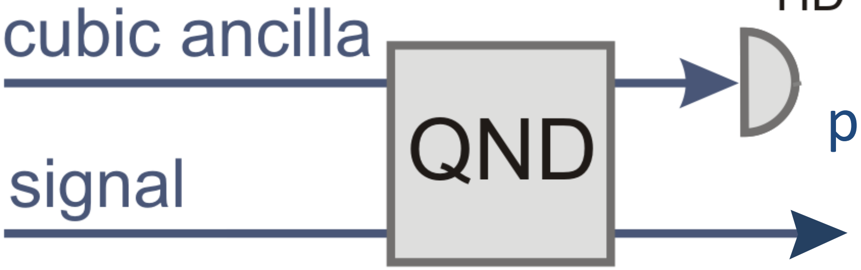
$$\hat{x} \to \hat{x}$$
$$\hat{p} \to \hat{p} + \chi\hat{x}^2$$

Transformation of coherent states:



M. Yukawa, K. Miyata, H. Yonezawa, P. Marek, R. Filip, and A. Furusawa, PRA **88**, 053816 (2013);

Quick comparison with probabilistic method



HD $x = 0$

post-selection

$$|\mathcal{A}_{x^3}\rangle = \int_{-\infty}^{\infty} e^{-i\chi x^3}|x\rangle dx$$

$$|\mathcal{A}_{x^3,\approx}\rangle = \hat{S}\sum_{k=0}^{N_{\max}} c_k|k\rangle$$

$$\int \psi(x)\mathcal{A}(y)|x,y\rangle dxdy \to \int \psi(x)\mathcal{A}(x)|x\rangle dx \qquad \hat{p} \to \hat{p} + \chi\hat{x}^2$$

- In the ideal case, the ancillas are identical
- In the approximate case they differ
  - – different figures of merit:

    shape of wave function instead of operator moments

$$\langle[\Delta(\hat{p} - \widehat{x^2})]^2\rangle \to 0$$

In summary:

- Two ways to view quantum operations:
  - manipulation of states VS manipulation of operators
  - Schröedinger VS Heisenberg
  - probabilistic VS deterministic
  - ancilla/measurement-driven VS feed-forward driven
- Deterministic operations are feed-forward driven
  - they add noise
  - nonlinear operations add nonlinear noise
- Nonlinear squeezing can be used to reduce this noise

Two ways to view quantum operation

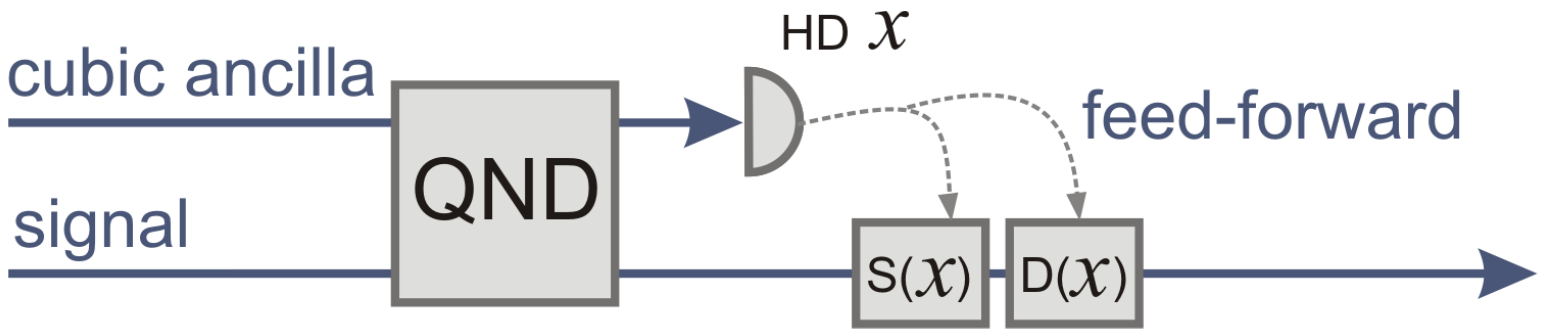
- manipulation of states
- Schröedinger
- probabilistic
- ancilla/measurement-driven
- only for some states

- manipulation of operators
- Heisenberg
- deterministic
- feed-forward driven
- add noise

- Nonlinear squeezing can be used to reduce the noise of nonlinear deterministic operations

Thank you for the attention!

Different angle: nonlinear squeezing



- The role of the ancilla is to reduce the noise

$$\langle [\Delta(\hat{p} - \chi \hat{x}^2)]^2 \rangle \to 0$$

- For any given dimension of the Hilbert space we can look for states that minimize this variance