High-fidelity noiseless amplification of light and loss compensation via noiseless amplification

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european social fund in the czech republic

EUROPEAN UNION

MINISTRY OF EDUCATION, YOUTH AND SPORTS

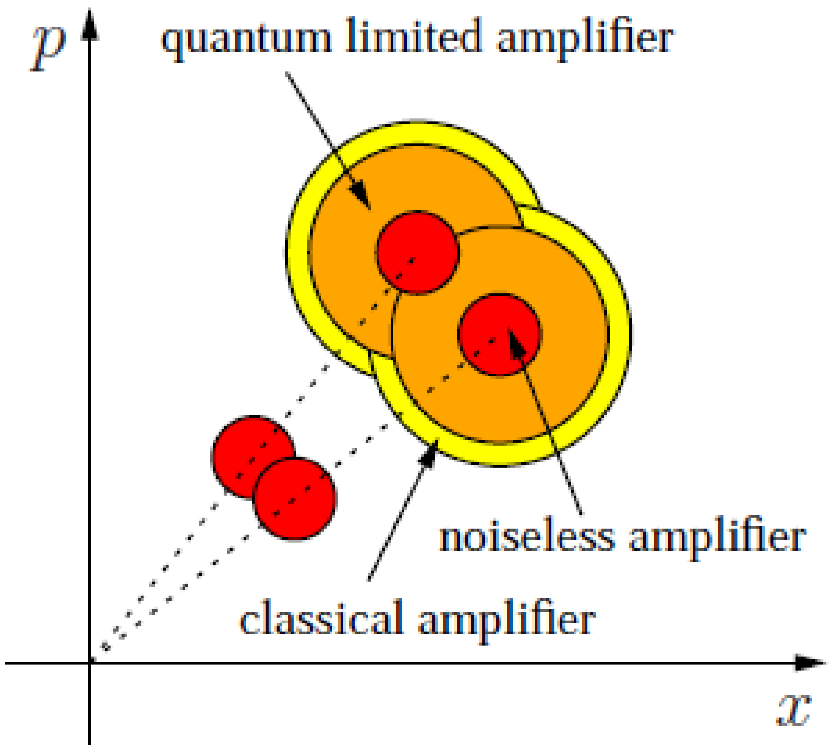
OP Education for Competitiveness

INVESTMENTS IN EDUCATION DEVELOPMENT

Outline

1. Quantum noise limited amplification of light
2. Probabilistic noiseless quantum amplifier
3. Noiseless quantum amplification by photon addition and subtraction
4. Experimental implementation on high fidelity noiseless amplifier
5. Simplified scheme based on addition of thermal noise
6. Emulation of Kerr nonlinearity
7. Loss suppression in quantum optical communication

Amplification of light

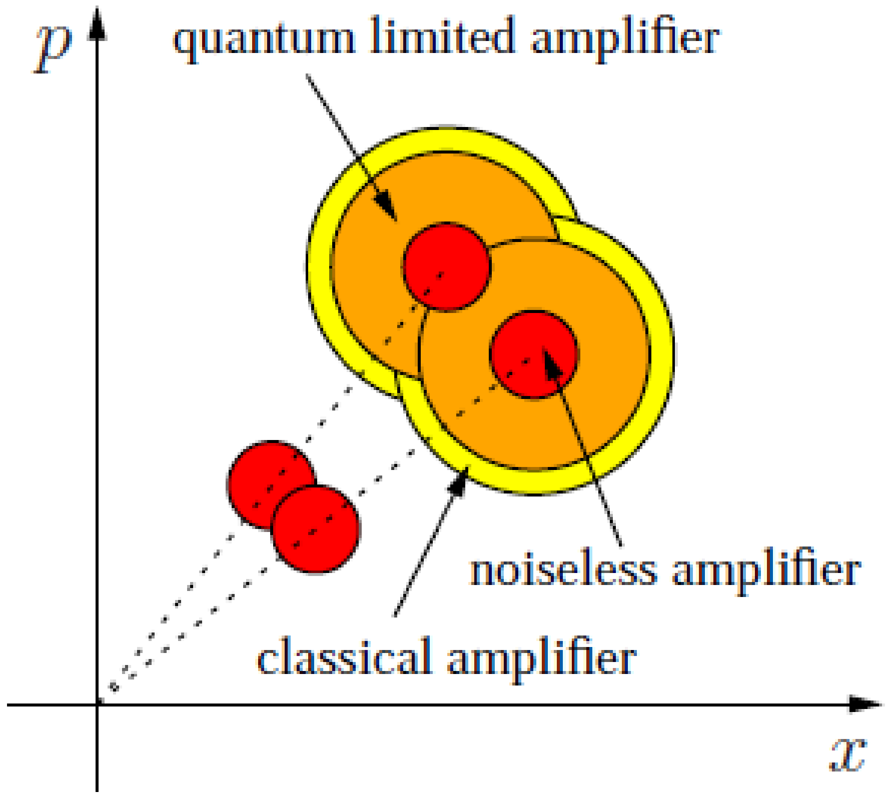


Optimal quantum-noise limited amplifier:

$$\hat{a}_{out} = g\,\hat{a}_{in} + \sqrt{g^2 - 1}\,\hat{b}^{\dagger}$$

g … amplification gain

Noiseless amplification of coherent states



$$|\alpha\rangle \rightarrow |g\,\alpha\rangle$$

Unphysical operation for g>1.

Cannot be implemented exactly and deterministically.

Approximate probabilistic noiseless amplification of coherent states

Physical approximation to the unphysical target operation

$$|\alpha\rangle \rightarrow |g\alpha\rangle$$

Motivation:

- Improved estimation of coherent states
- Probabilistic high-fidelity cloning of coherent states
- Compensation of losses in quantum communication
- Entanglement distillation and concentration
- Breeding of Schrodinger cat-like states
- etc.

Approximate noiseless amplification based on quantum scissors

Truncation to space spanned by vacuum and single-photon state:

$$|\alpha\rangle \rightarrow |0\rangle + \alpha|1\rangle \rightarrow |0\rangle + g\alpha|1\rangle$$

Very crude approximation, works well only for $|\alpha| \ll 1$.

The performance can be improved by a complex interferometric scheme Involving multiple quantum scissors – extremely difficult to implement.

T. C. Ralph and A. P. Lund, arXiv:0809.0326 (2008).

G.Y. Xiang, T.C. Ralph, A. Lund, N. Walk, and G. Pryde, Nature Photonics **4**, 316 - 319 (2010).

T.F. Ferreyrol, M. Barbieri, R. Blandino, S. Fossier, R. Tualle-Brouri, and P. Grangier, Phys. Rev. Lett. **104**, 123603 (2010).

Approximate noiseless amplification based on Fock-state amplitude modulation

Approximation of non-unitary amplification operation

$$g^{\hat{n}}|\alpha\rangle = e^{(g^2-1)|\alpha|^2/2}|g\alpha\rangle$$

This operator is unbounded, cannot be implemented exactly.

Approximate truncated version:

$$g^{\hat{n}} \approx (g-1)\hat{n}+1$$

Higher order approximations:

$$g^{\hat{n}} \approx \sum_{k=0}^{N} \frac{d^k}{k!}\hat{n}^k$$

J. Clausen, L. Knoll, and D.-G. Welsch, Phys. Rev. A **68**, 043822 (2003).

J. Fiurášek, Phys. Rev. A **80**, 053822 (2009).

Implementation of operations diagonal in Fock state basis

$$\sum_{n=0}^{\infty} c_n |n\rangle \rightarrow \sum_{n=0}^{\infty} f_n c_n |n\rangle$$

$$f_n = |f_n| e^{i\Phi_n}$$

Amplitude modulation … $|f_n|$

Phase modulation … Φ_n

J. Fiurášek, Phys. Rev. A **80**, 053822 (2009).

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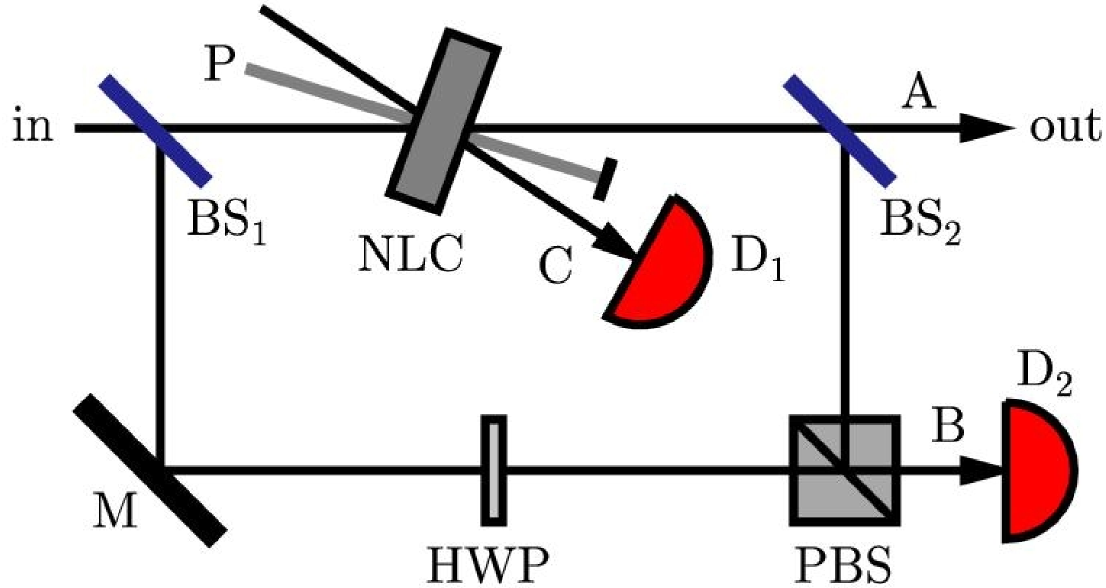
Phase modulation … Φ_n



Combination of multiple photon addition and photon subtraction.

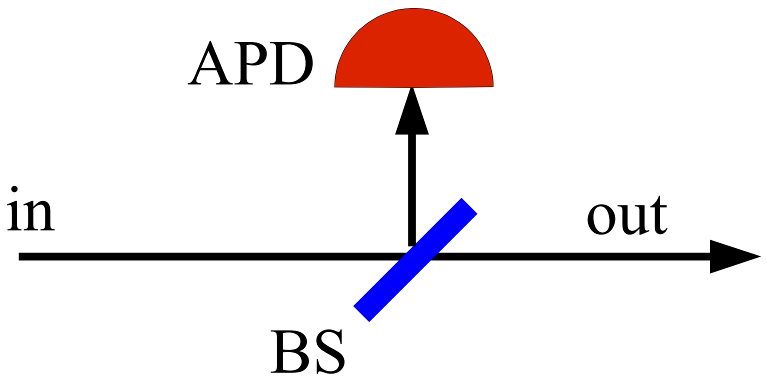
Generalization of a scheme implemented by Bellini et al.:

A. Zavatta, V. Parigi, M. S. Kim, H. Jeong, and M. Bellini, Phys. Rev. Lett. 103, 140406 (2009).

J. Fiurášek, Phys. Rev. A **80**, 053822 (2009).

Elmentary operations

Single-photon subtraction



$$|\psi\rangle \rightarrow \hat{a}|\psi\rangle$$

A. Ourjoumtsev, R. Tualle-Brouri, J. Laurat, and Ph. Grangier, *Science* **312**, 83 (2006).

J.S. Neergaard-Nielsen et al., *Phys. Rev. Lett.* **97**, 083604 (2006).

K. Wakui, H. Takahashi, A. Furusawa, and M. Sasaki, *Opt. Express* **15**, 3568 (2007).

Elmentary operations

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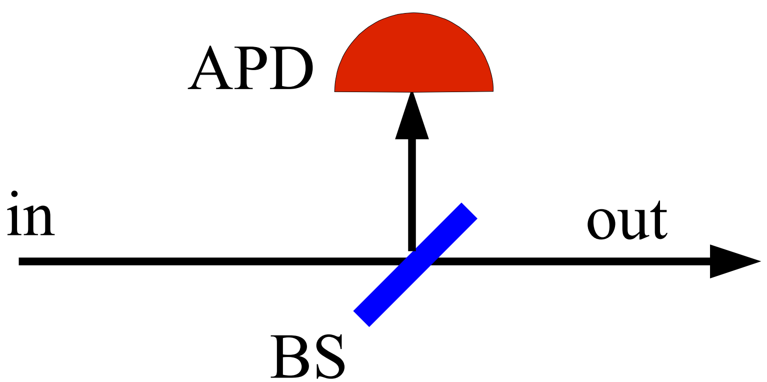

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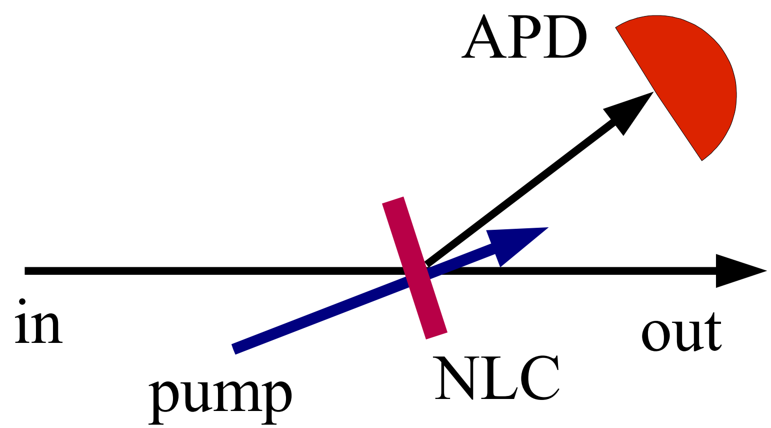
K. Wakui, H. Takahashi, A. Furusawa, and M. Sasaki, *Opt. Express* **15**, 3568 (2007).

Single-photon addition

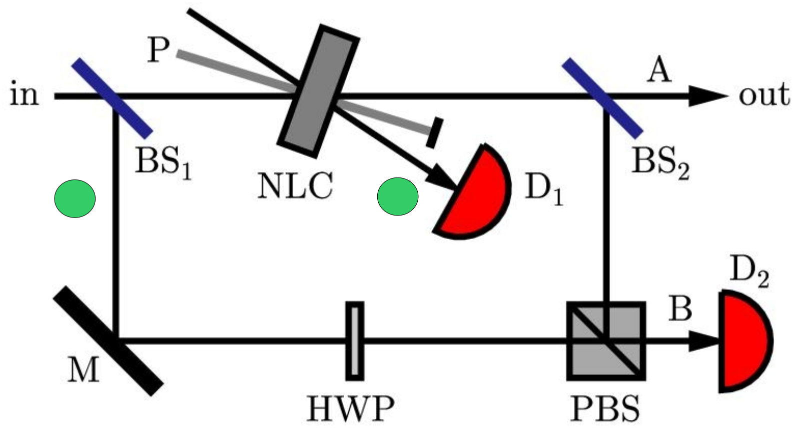


$$|\psi\rangle \rightarrow \hat{a}^{\dagger}|\psi\rangle$$

A. Zavatta, S. Viciani, and M. Bellini, *Science* **306**, 660 (2004).

V. Parigi, A. Zavatta, M. S. Kim, and M. Bellini, Science 317, 1890 (2007)..

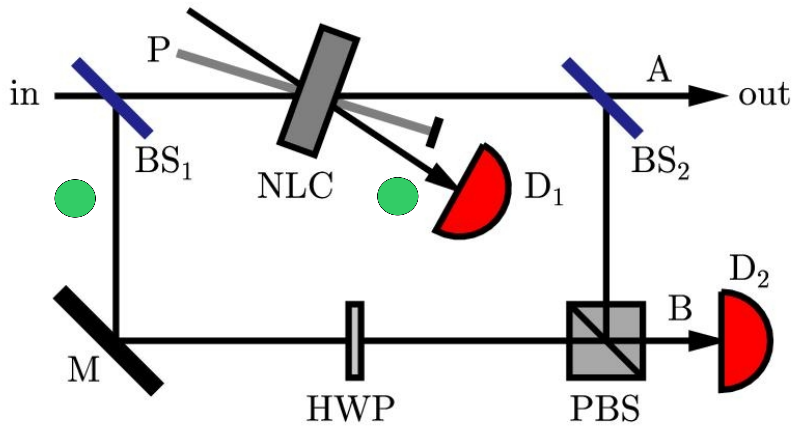
Principle of noiseless amplification



$$\hat{a}^{\dagger}\hat{a}$$

Single-photon addition and single photon subtraction

Principle of noiseless amplification



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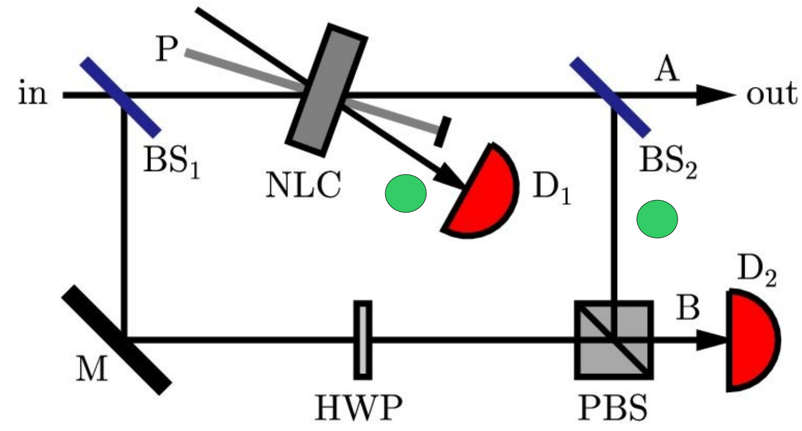
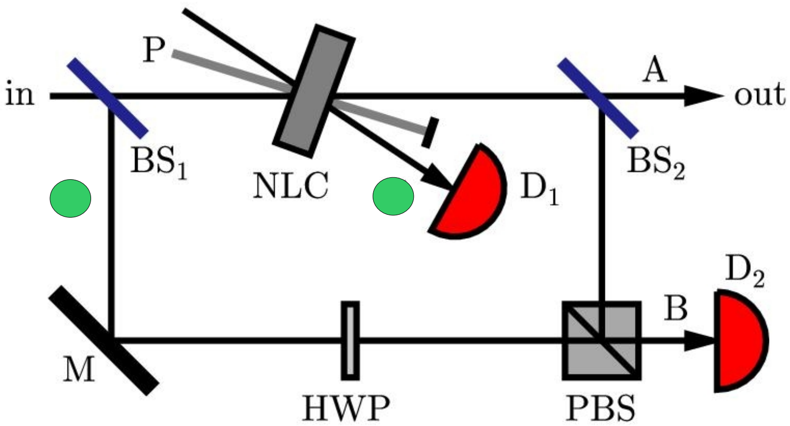


$$\hat{a}^{\dagger}\hat{a} \quad + \quad \hat{a}\,\hat{a}^{\dagger}$$

Single-photon addition and single-photon subtraction

Principle of noiseless amplification



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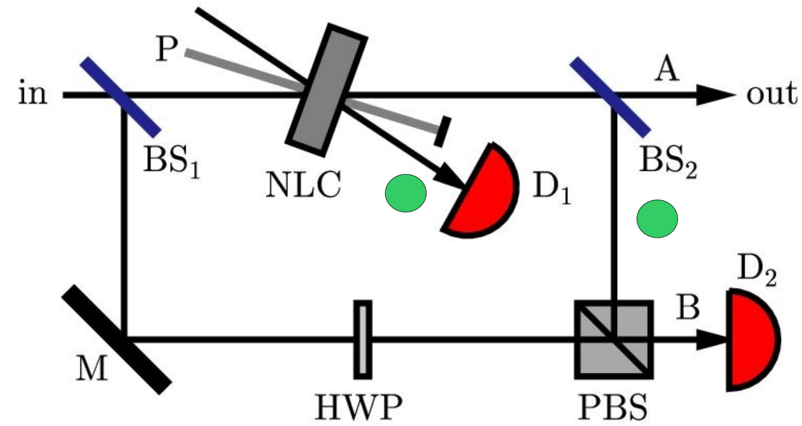


$$\beta \hat{a}^{\dagger} \hat{a} \quad + \quad \gamma \hat{a} \hat{a}^{\dagger}$$

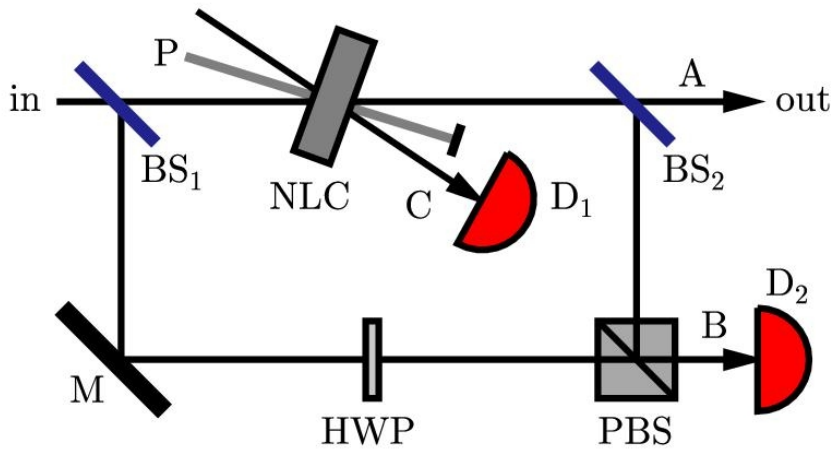
Single-photon addition and single-photon subtraction

Detector D_2 – projection onto polarization state $\beta|H\rangle + \gamma|V\rangle$

Resulting operation on signal mode:

$$\hat{W} = \beta \hat{a}^{\dagger} \hat{a} + \gamma \hat{a} \hat{a}^{\dagger} = (\beta + \gamma) \hat{n} + \gamma$$

Higher-order approximation

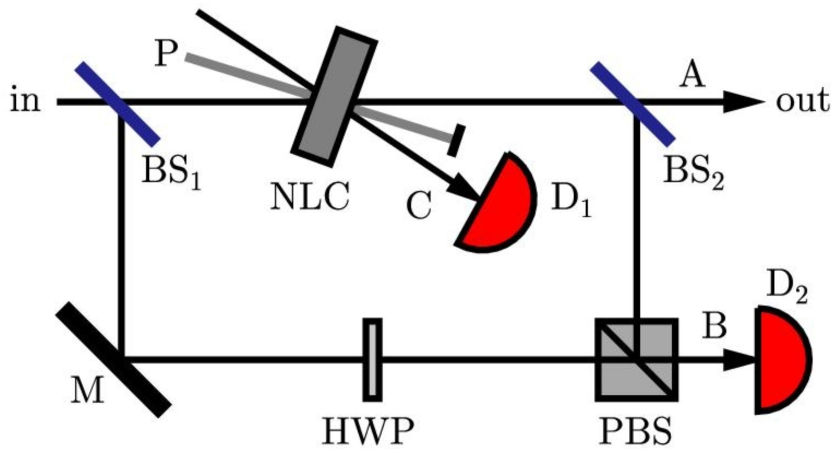


D_1 counts N photons

D_2 projects onto N-photon polarization state:

$$\sum_{n=0}^{N} b_k |k\rangle_{B,H} |N-k\rangle_{B,V}$$

Higher-order approximation



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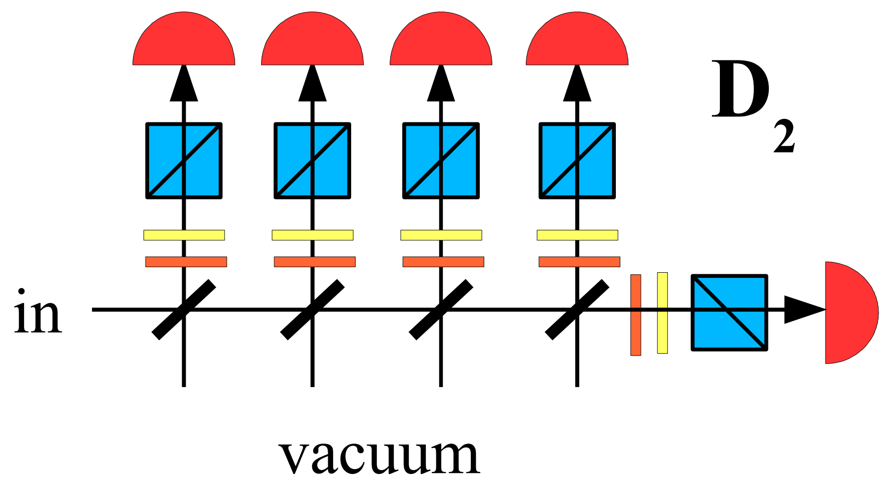
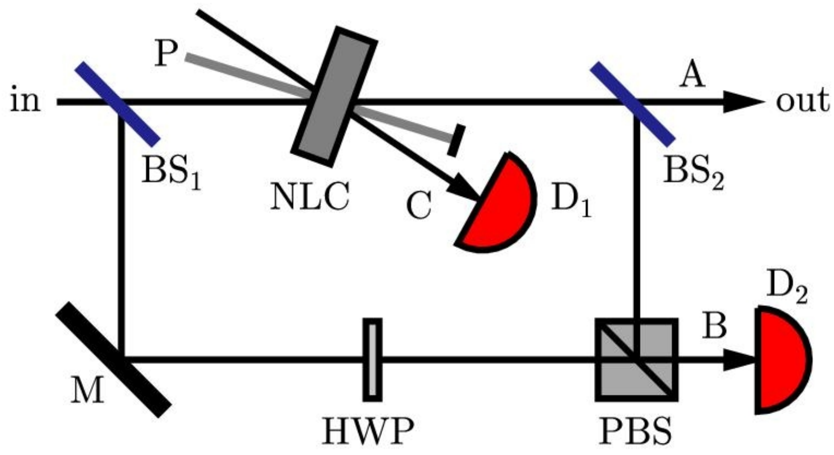


Higher-order approximation



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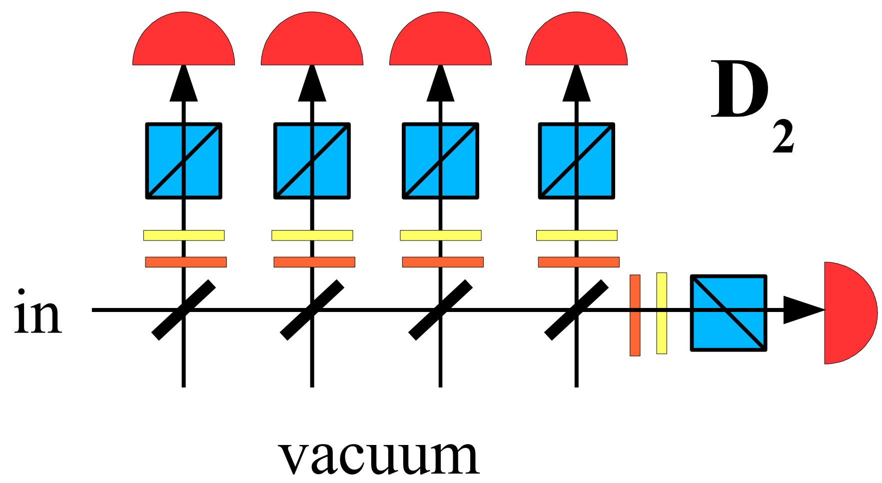
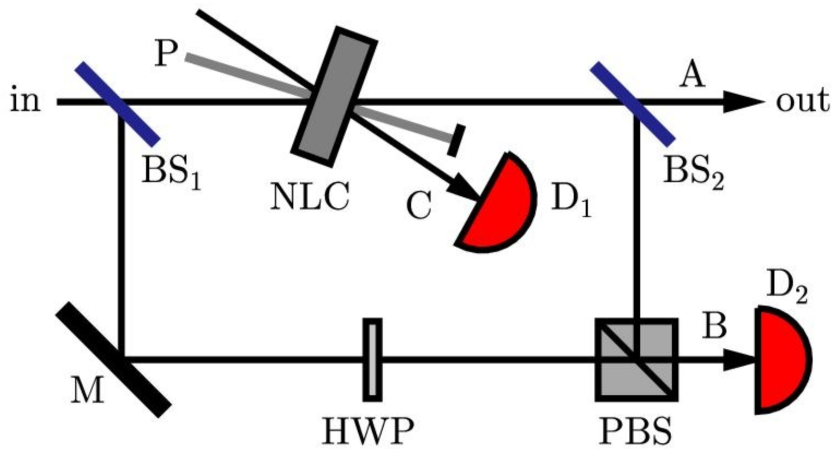


Resulting operation on signal mode:

$$\hat{W}_n = h^{\hat{n}} \sum_{n=0}^{N} b_k d_{N,k} \hat{a}^k \hat{a}^{\dagger N} \hat{a}^{N-k}$$

Higher-order approximations II.



D_1 counts N photons

D_2 projects onto N-photon polarization state:

$$\sum_{n=0}^{N} b_k |k\rangle_{B,H} |N-k\rangle_{B,V}$$

Operation on signal mode expressed in terms of photon number operator:

$$\hat{W}_N = h^{\hat{n}} \prod_{k=1}^{N} (\hat{n} - z_j)$$

$$h = t^2 \sqrt{1-\lambda^2}$$

attenuation factor

N-th order polynomial in photon number operator

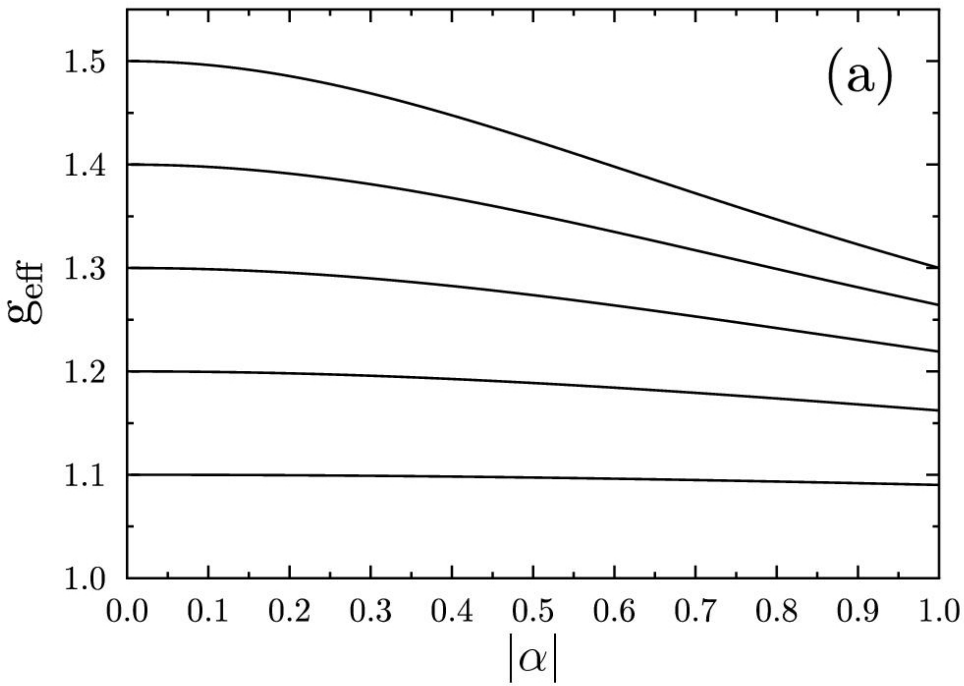
Complex roots z_j are determined by b_j and can be arbitrary

t ... amplitude transmittance of beam splitters BS_1 and BS_2

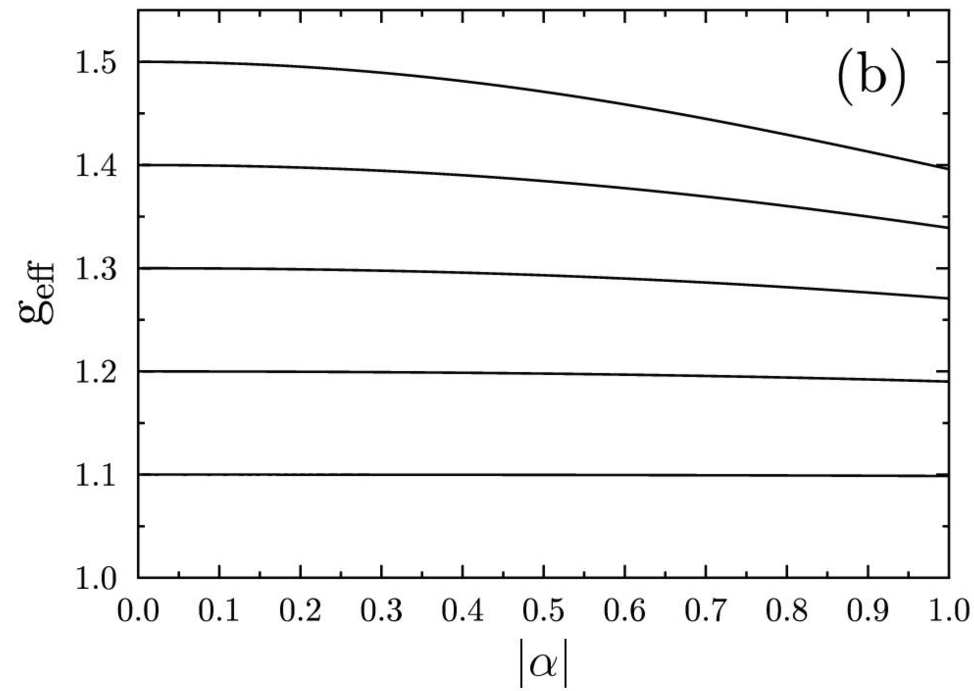
λ ... squeezing parameter of the SPDC process in the nonlinear crystal

Effective gain of the noiseless amplifier

N=1



N=2



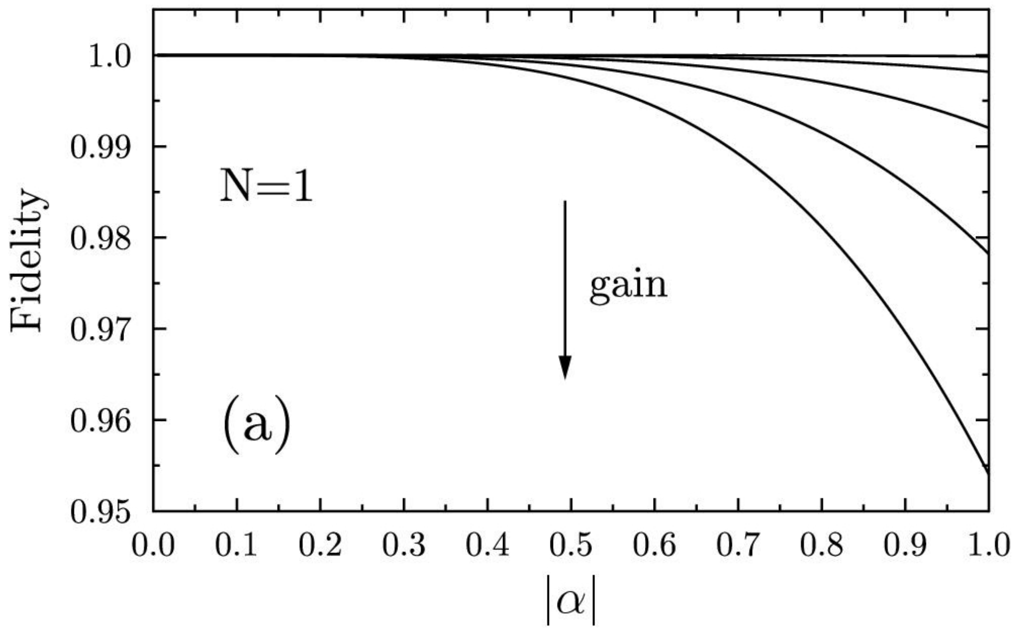
$$\hat{Z}_1 = (g-1)\hat{n}+1$$

$$\hat{Z}_2 = (g-\sqrt{2g-1})\hat{n}^2 + (\sqrt{2g-1}-1)\hat{n}+1$$

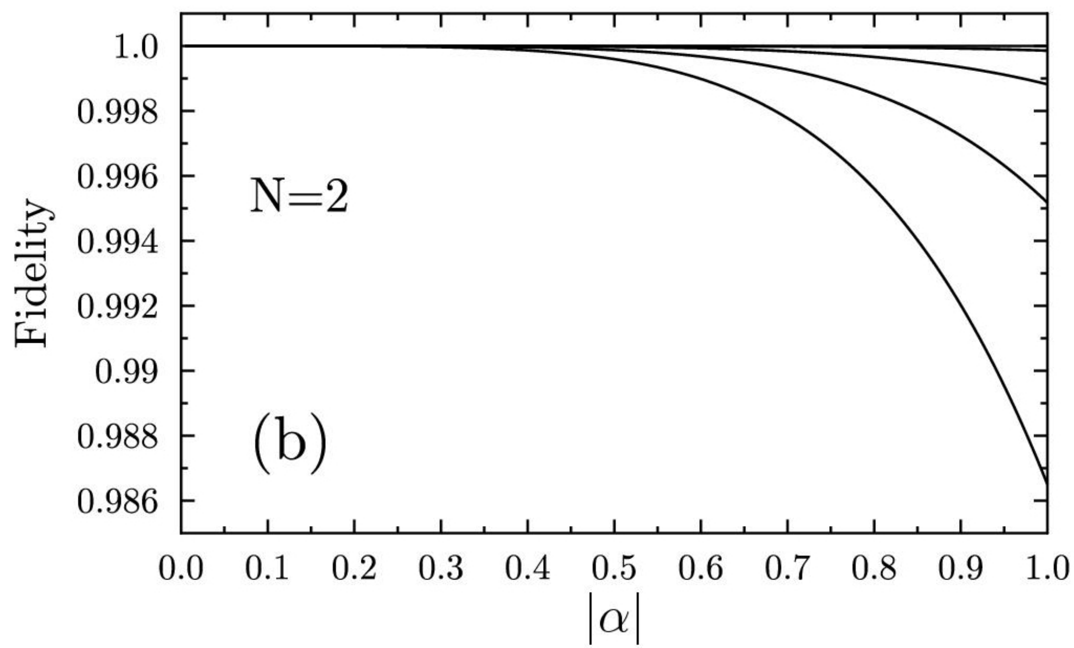
$$g_{eff} = \frac{1}{\alpha}\frac{\langle\alpha|\hat{Z}_N^\dagger \hat{a}\,\hat{Z}_N|\alpha\rangle}{\langle\alpha|\hat{Z}_N^\dagger \hat{Z}_N|\alpha\rangle}$$

$$\lim_{\alpha\to 0} g_{eff} = g$$

Fidelity of amplified coherent states



$$F = \frac{|\langle g\alpha|\hat{Z}_N|\alpha\rangle|^2}{\langle\alpha|\hat{Z}_N^\dagger\hat{Z}_N|\alpha\rangle}$$

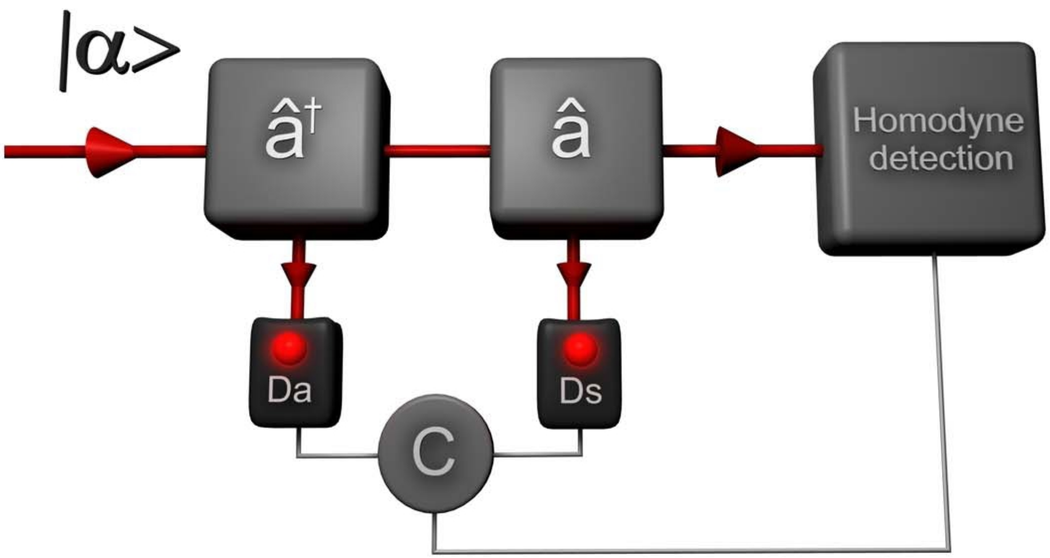


Experimental realization of high-fidelity noiseless amplifier

nature photonics | ARTICLES
PUBLISHED ONLINE: XX XX 2010 | DOI: 10.1038/NPHOTON.2010.260

A high-fidelity noiseless amplifier for quantum light states

A. Zavatta[1,2], J. Fiurášek[3] and M. Bellini[1,2]*

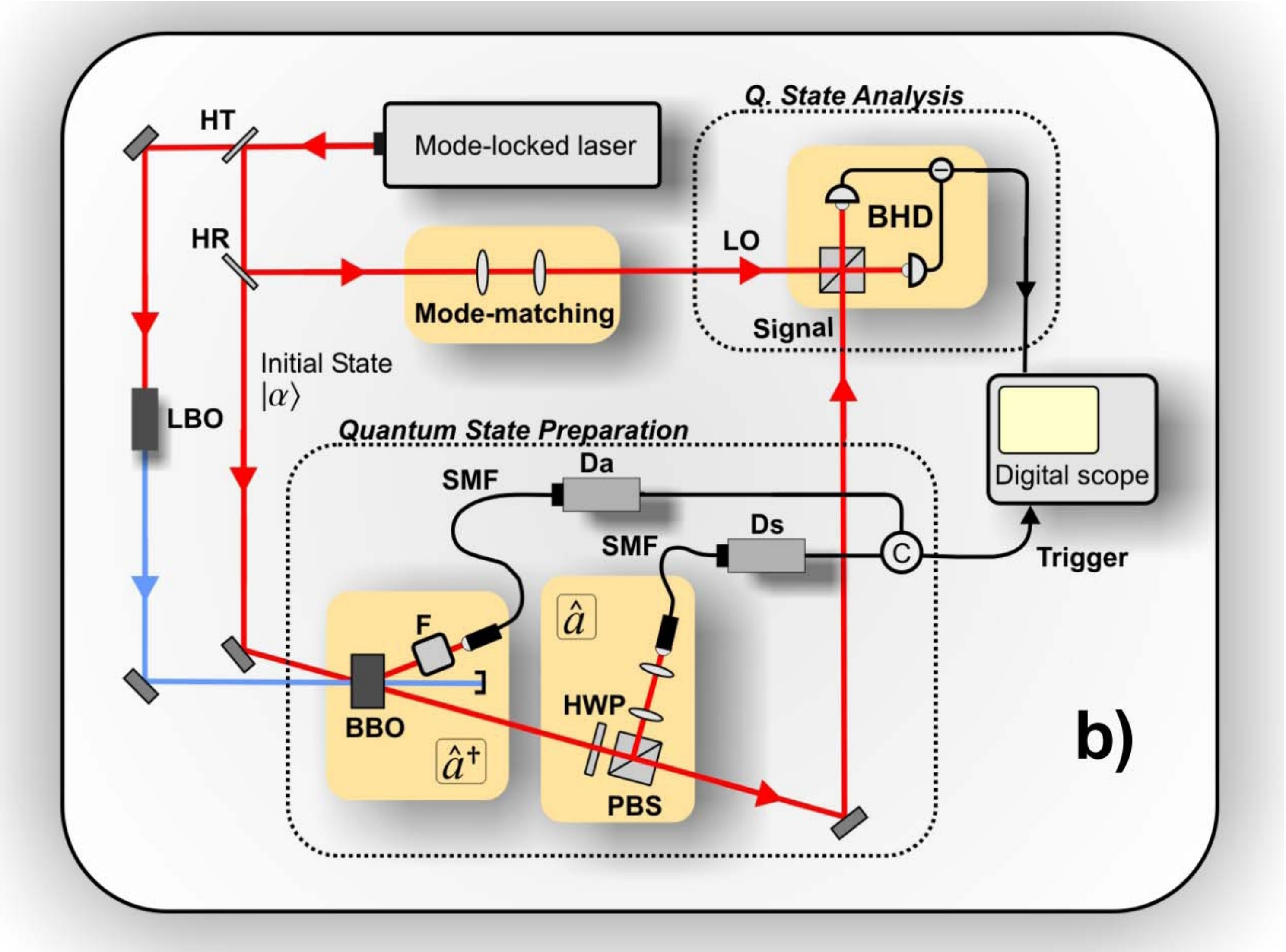


Noiseless amplifier
with nominal gain g=2

Sequence of single-photon addition
and single-photon subtraction.

$$|\alpha\rangle \rightarrow \hat{a}\,\hat{a}^{\dagger}|\alpha\rangle$$

Experimental setup



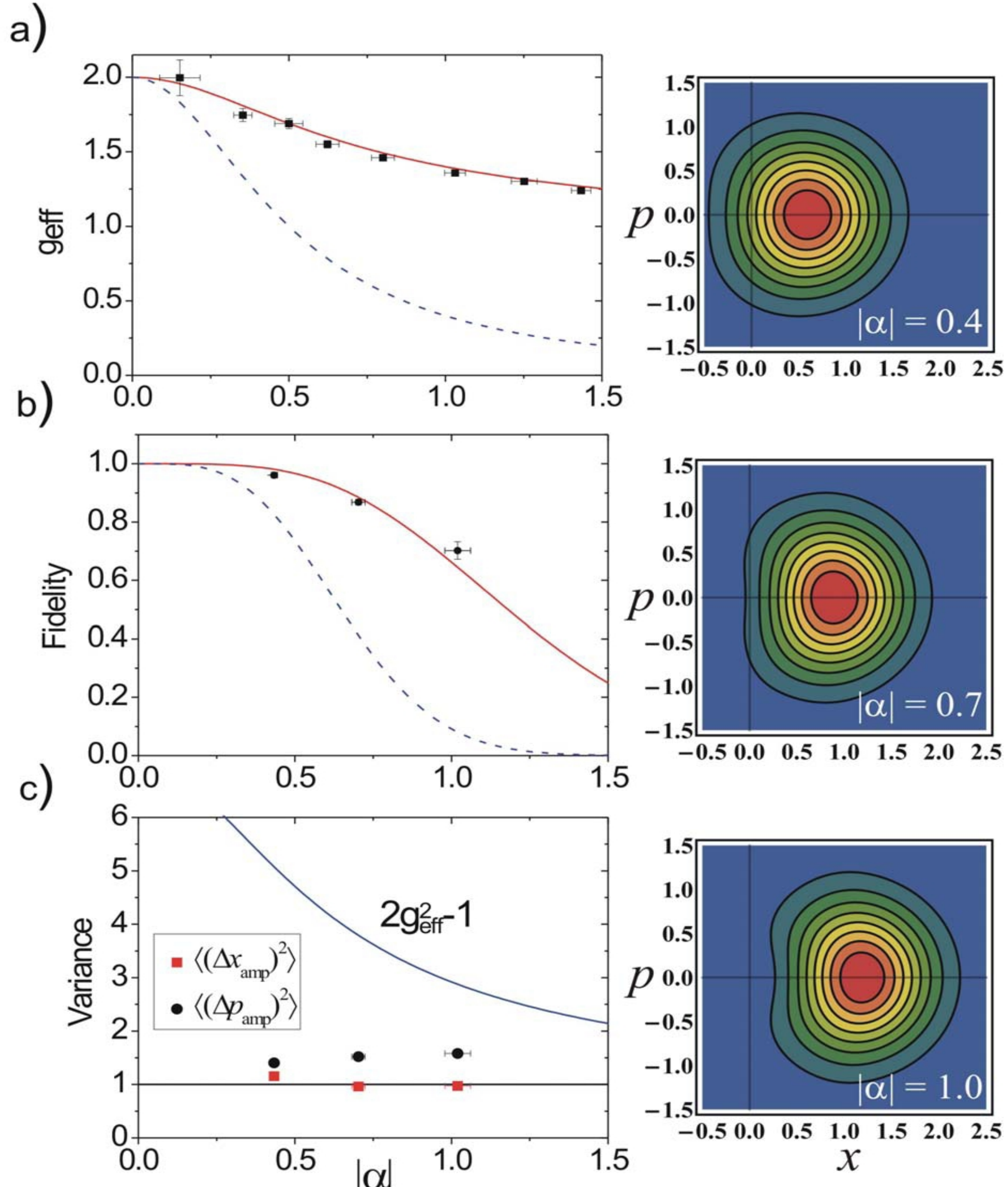
A. Zavatta, J. Fiurášek, and M. Bellini, *A high-fidelity noiseless amplifier for quantum light states*, Nature Photonics **5**, 52–56 (2011).

Experimental results

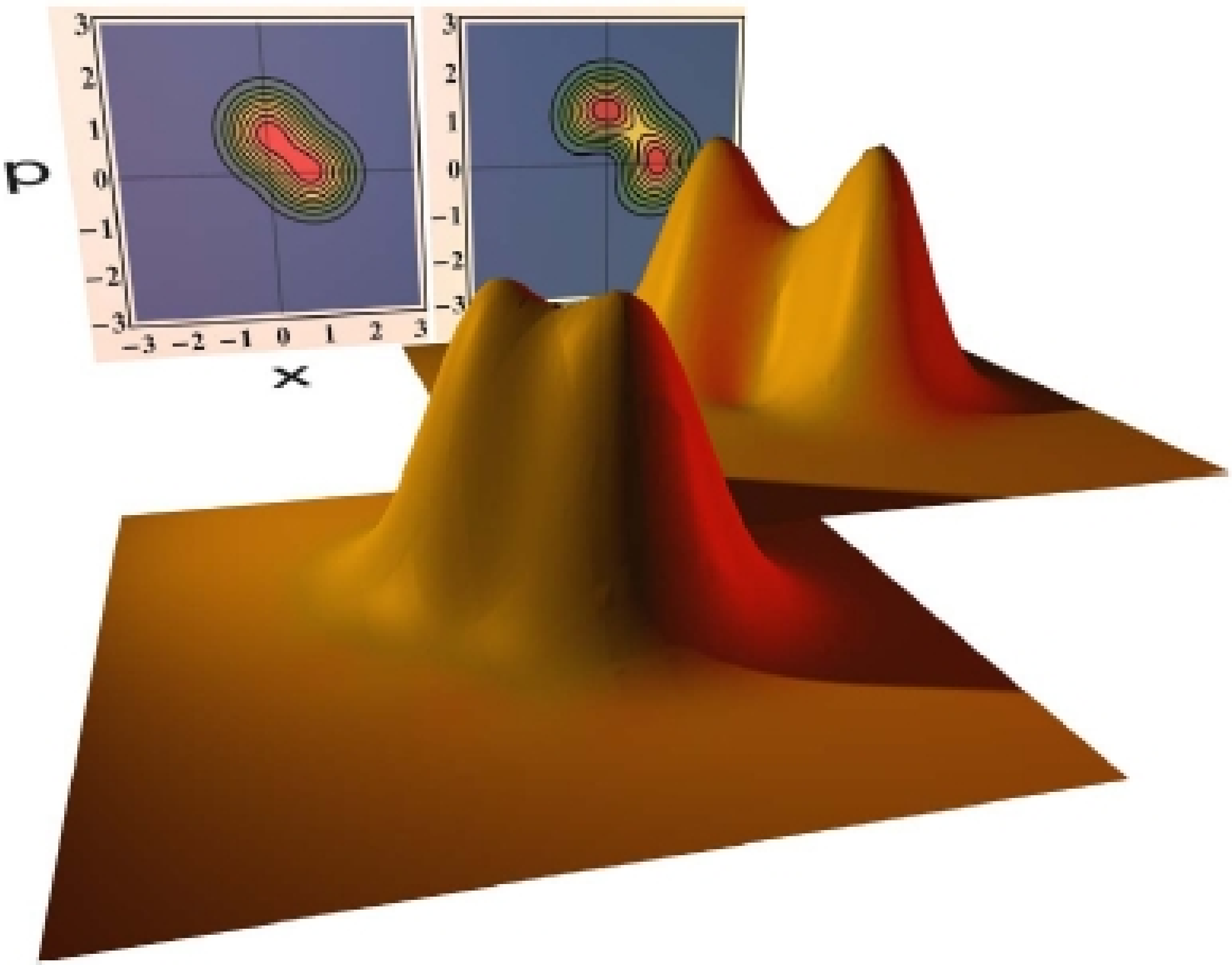
Effective amplification gain

Fidelity of amplified coherent states

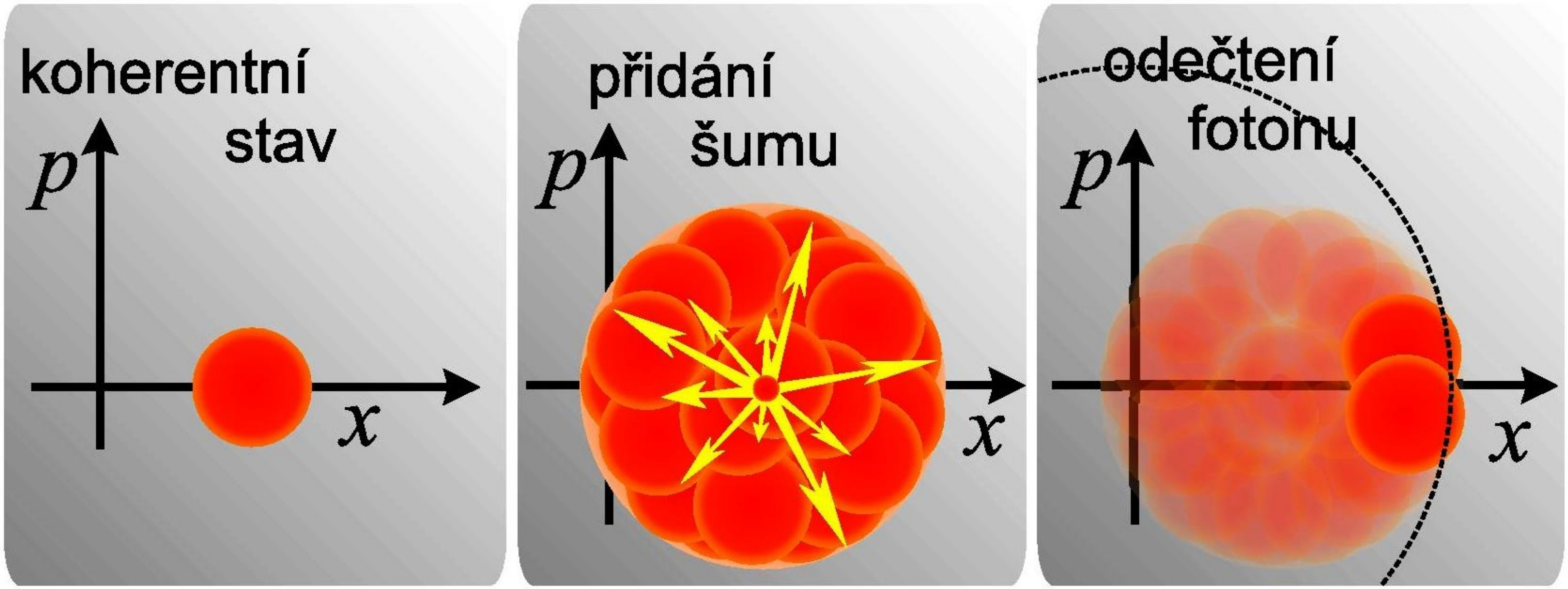
Quadrature variances of amplified coherent states



Enhanced distinguishability of amplified states



Simplified amplification scheme



Single-photon addition replaced with addition of thermal noise.

The scheme can conditionally improve phase resolution.

Its performance can be improved by multiple photon subtractions.

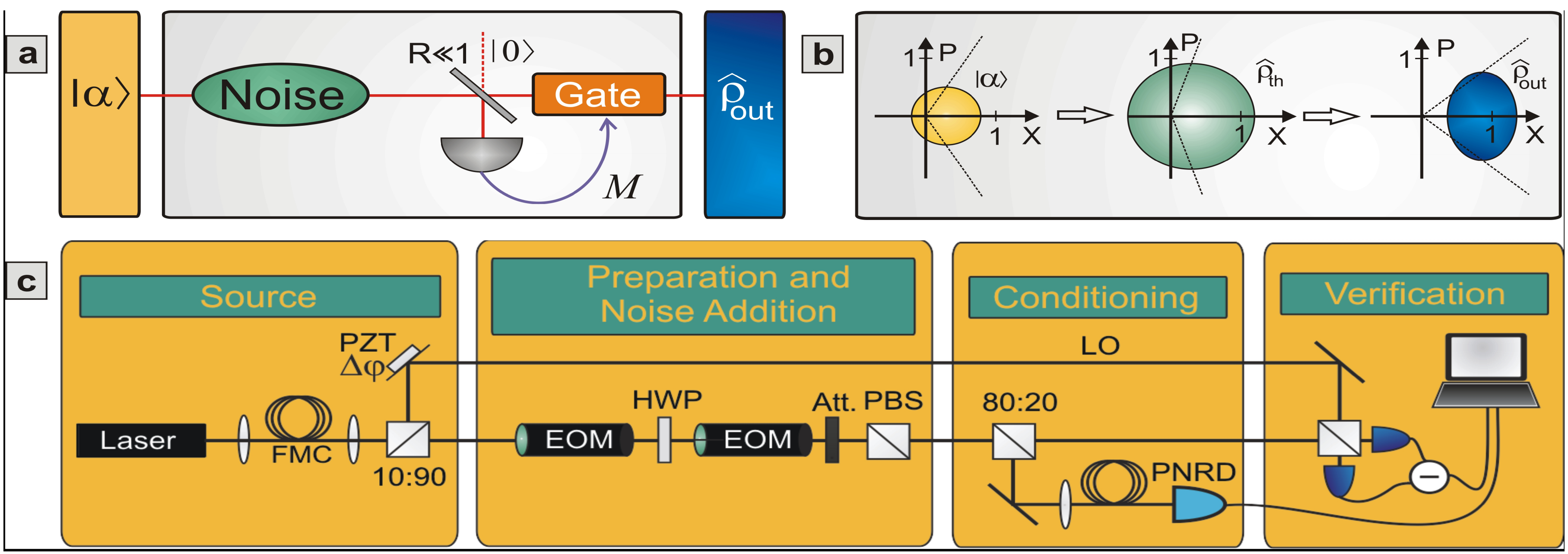
P. Marek and R. Filip, Phys. Rev. A **81**, 022302 (2010).

Noise-powered Noiseless Amplifier

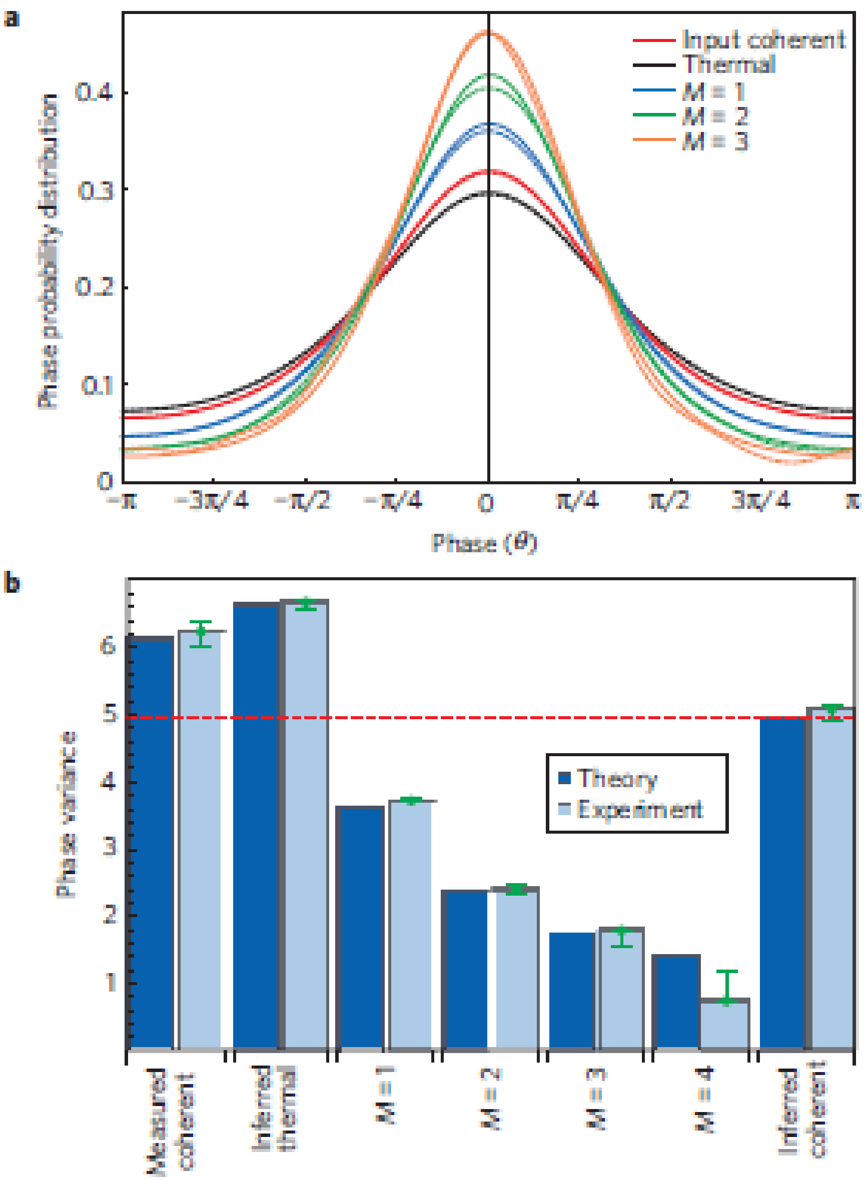
nature physics
LETTERS
PUBLISHED ONLINE: XX MONTH XXXX | DOI: 10.1038/NPHYS1743

Noise-powered probabilistic concentration of phase information

Mario A. Usuga[1,2†], Christian R. Müller[1,3†], Christoffer Wittmann[1,3], Petr Marek[4], Radim Filip[4], Christoph Marquardt[1,3], Gerd Leuchs[1,3] and Ulrik L. Andersen[2*]



Experimental results



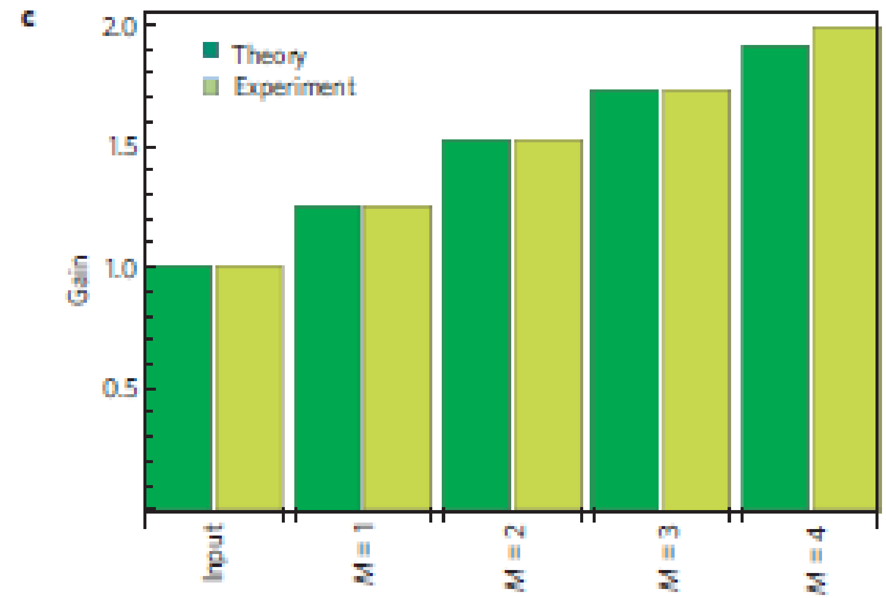
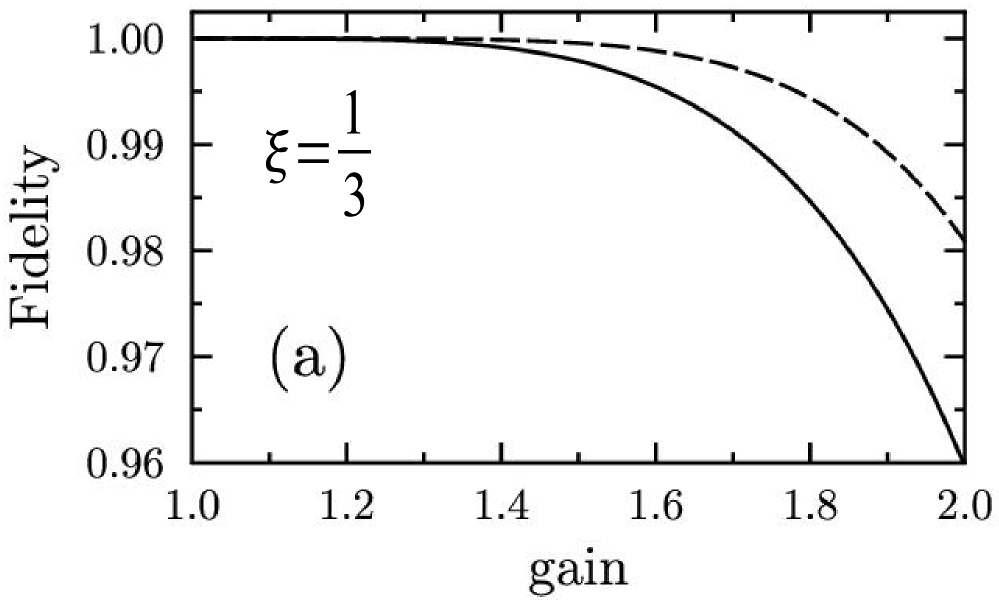
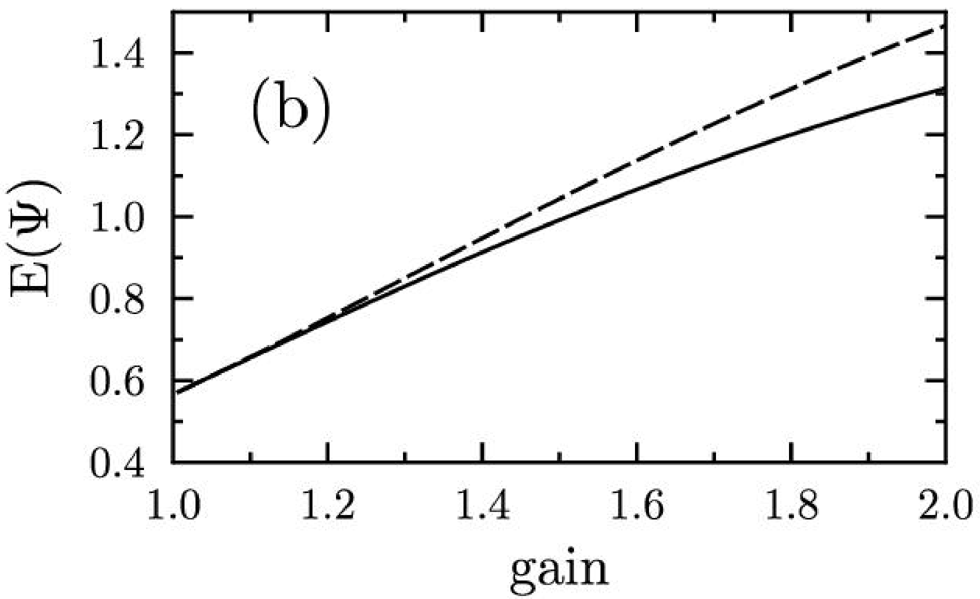


Figure 4 | Comparison between theoretical and experimental results. **a**, Phase probability distribution function derived from the experimental data (solid lines) for the measured coherent, the thermal and the conditioned states. Corresponding theoretical functions (dashed lines) were calculated for states fitting to experimentally derived parameters. **b**, The canonical-phase variance deduced from the experimental data (light blue) and corresponding theoretical values (dark blue) calculated for states fitting to the experimentally derived parameters. The inferred input coherent state serves as the reference value. The error bars represent the statistical deviations over many different realizations of the experiment. **c**, Gain for the input coherent state for different thresholds *M*.

M.A. Usuga, C.R. Mueller, C. Wittmann, P. Marek, R. Filip, C. Marquardt, G .Leuchs, U. L. Andersen, Nature Phys. 6, 767-771 (2010).

CV entanglement concentration by local noiseless amplification





Two-mode squeezed vacuum

$$|\Psi(\xi)\rangle = \sqrt{1-\xi^2}\sum_{n=0}^{\infty} \xi^n |n\rangle|n\rangle$$

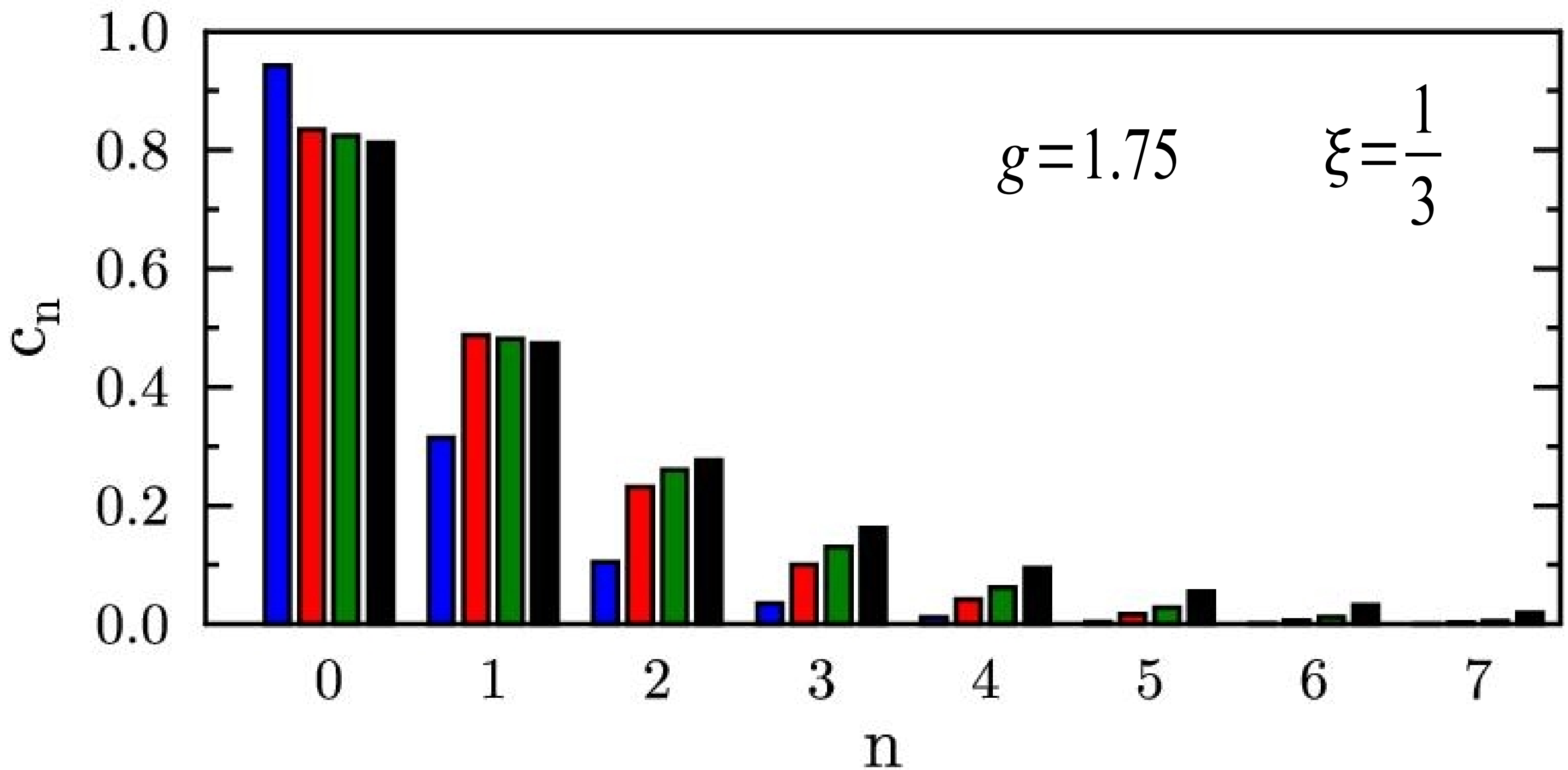
Locally amplified two-mode squeezed vacuum

$$|\Psi(\xi)\rangle \to \hat{I} \otimes \hat{Z}_N |\Psi(\xi)\rangle$$

Large N limit

$$|\Psi(\xi)\rangle \to |\Psi(g\xi)\rangle$$

Modulation of amplitudes of two-mode squeezed vacuum



Amplitude of initial two-mode squeezed vacuum

Amplitude of locally amplified state, N=1

Amplitude of locally amplified state, N=2

Amplitude of target two-mode squeezed vacuum state

Emulation of Kerr nonlinearity

Unitary operation: $\hat{U} = e^{i\phi\hat{n}^2}$

2N-th order approximation: $\hat{U}_{2N} = \sum_{k=0}^{N} \frac{(i\phi)^k}{k!}\hat{n}^{2k}$

First nontirivial approximation requires addition and subtraction of two photons

$$\hat{U}_2 = 1 + i\phi\hat{n}^2$$

Emulation of Kerr nonlinearity

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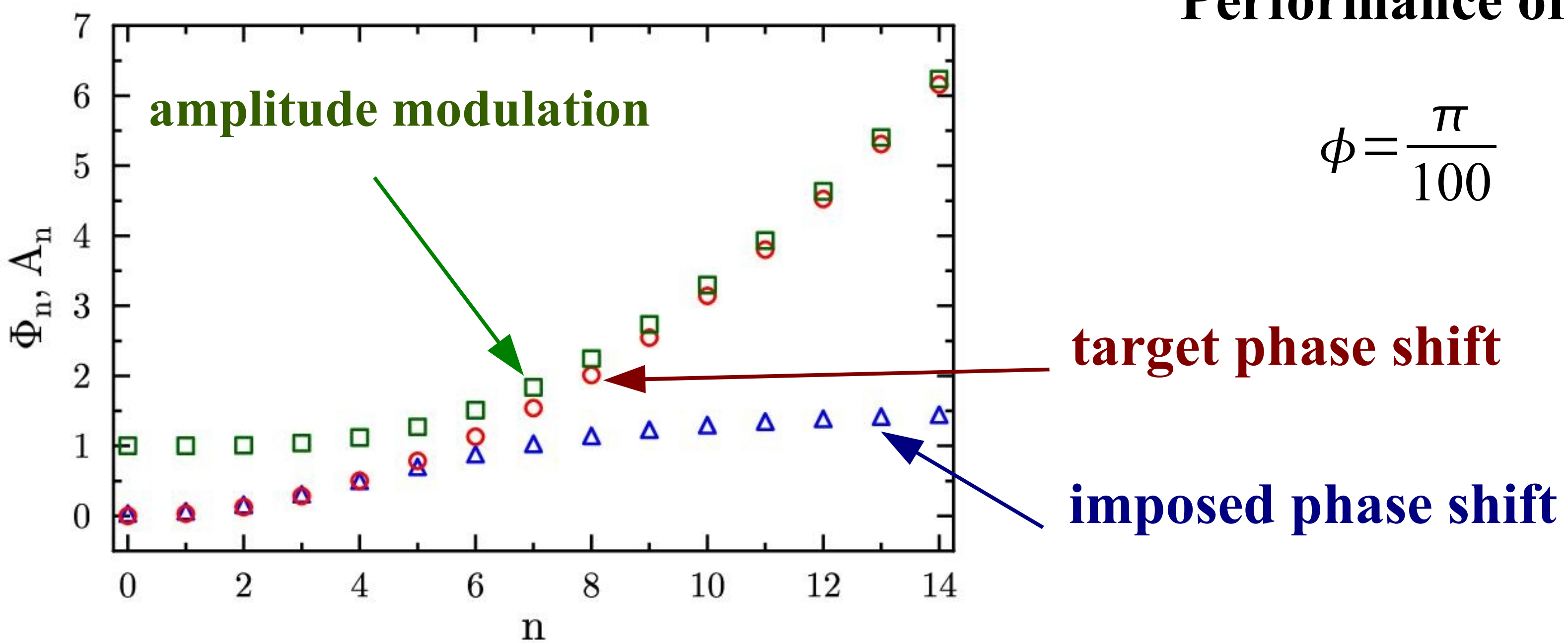
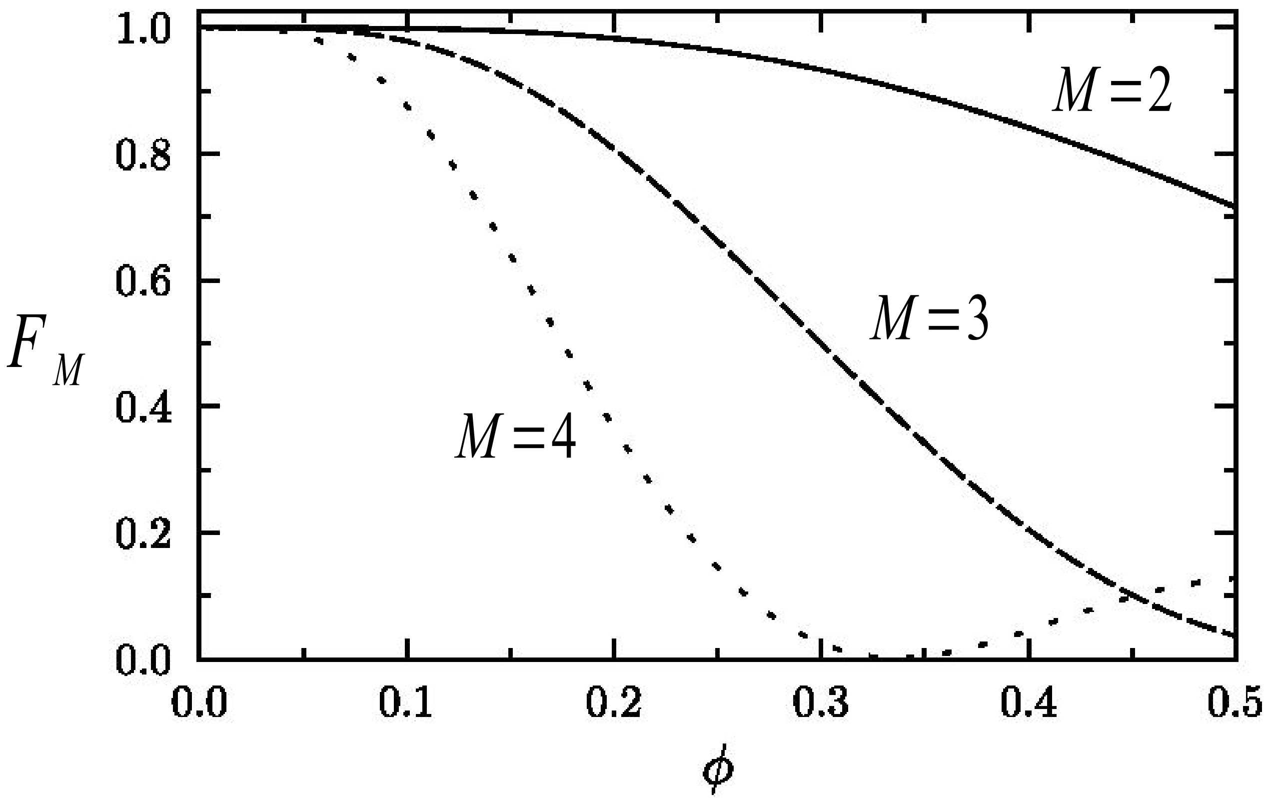


Performance of U_2

$$\phi = \frac{\pi}{100}$$

Fidelity of Kerr-nonlinearity emulation



$$|\psi_M\rangle = \frac{1}{\sqrt{M+1}} \sum_{n=0}^{M} |n\rangle$$

$$F_M = \frac{|\langle \psi_M | \hat{U}^\dagger \hat{U}_2 | \psi_M \rangle|^2}{\langle \psi_M | \hat{U}_2^\dagger \hat{U}_2 | \psi_M \rangle}$$

Perfect emulation on a finite-dimensional subspace

Exact probabilistic implementation of operations on Hilbert subspace spanned by the first N+1 Fock states:

$$\sum_{n=0}^{N} c_n |n\rangle \rightarrow \sum_{n=0}^{N} f_n c_n |n\rangle$$

Construction of the polynomial operator:

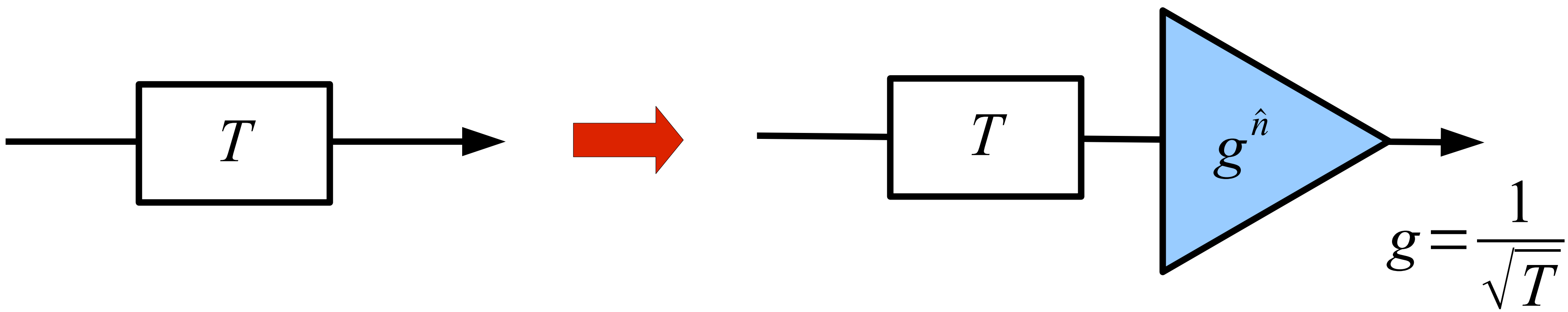
$$\hat{Z}_n = \sum_{k=0}^{N} \frac{f_k}{h^k} \prod_{j=0,\, j\neq k}^{N} \frac{\hat{n}-j}{k-j}$$

Built-in compensation for the attenuation factor h.

Requires addition and subtraction of N photons.

Loss compensation by noiseless amplification

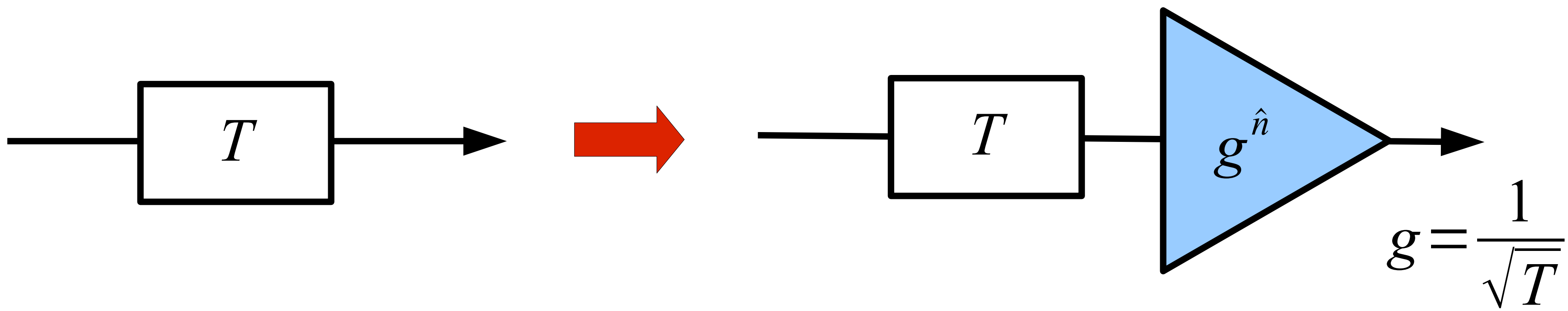
Naive approach: insert noiseless amplifier after a lossy channel



This method does not fully elliminate loss and noise

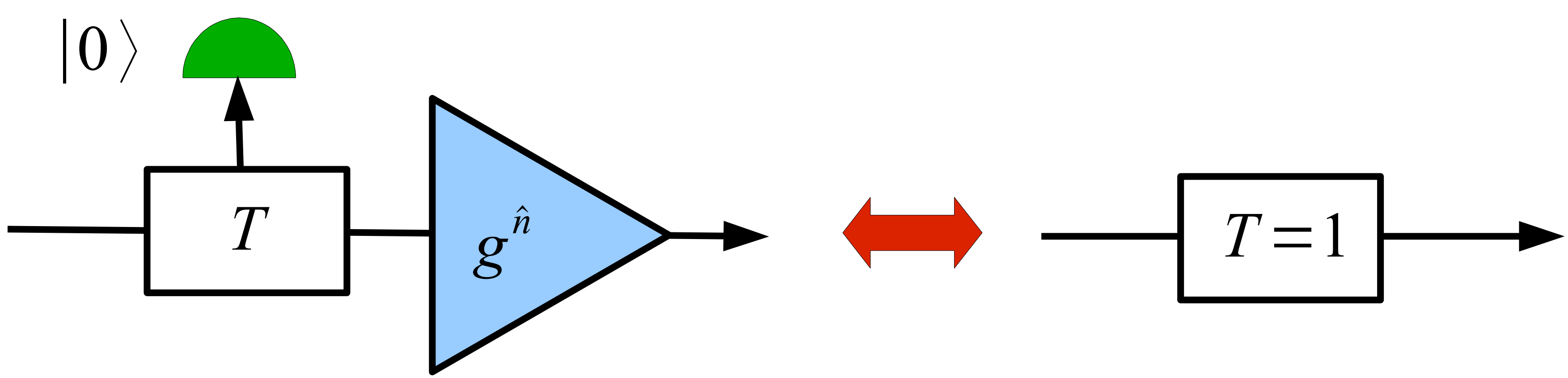
Loss compensation by noiseless amplification

Naive approach: insert noiseless amplifier after a lossy channel



This method does not fully elliminate loss and noise

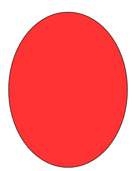
This approach compensates the effect of a lossy channel only in the case that no photon was actually lost in the channel



Faithful entanglement distribution over a lossy channel

Scheme proposed by Tim Ralph:

- prepare weakly squeezed (or entangled) state

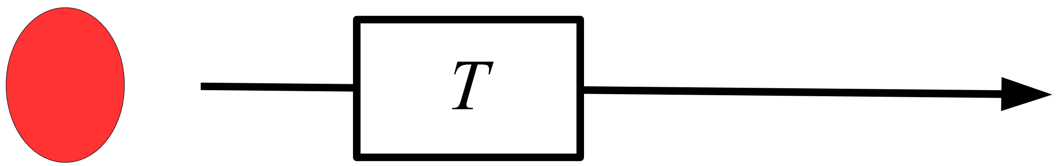


T. C. Ralph, Phys. Rev. A **84**, 022339 (2011).

Faithful entanglement distribution over a lossy channel

Scheme proposed by Tim Ralph:

- prepare weakly squeezed (or entangled) state
- send the state over a lossy channel

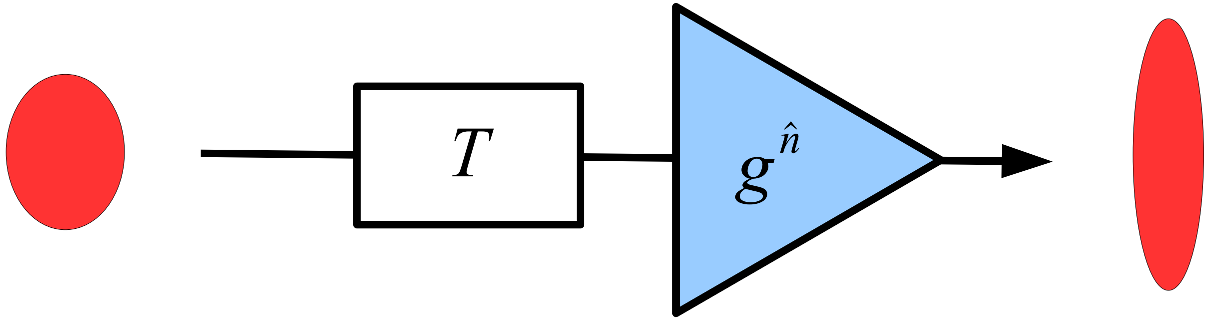


T. C. Ralph, Phys. Rev. A **84**, 022339 (2011).

Faithful entanglement distribution over a lossy channel

Scheme proposed by Tim Ralph:

- prepare weakly squeezed (or entangled) state
- send the state over a lossy channel
- noiselessly amplify the output state



$$g > \frac{1}{\sqrt{T}}$$

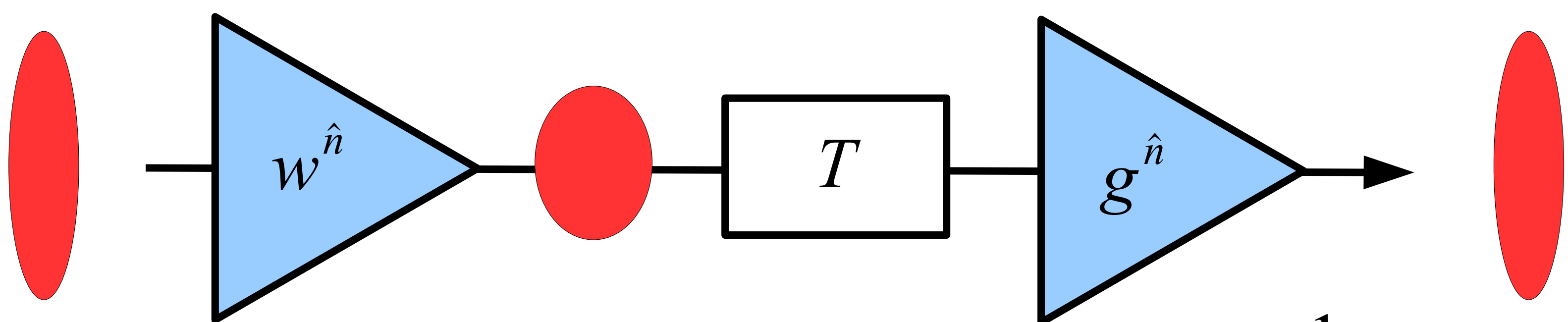
Highly pure strongly squeezed/entangled states can be conditionally distributed through a lossy channel

T. C. Ralph, Phys. Rev. A **84**, 022339 (2011).

Towards complete suppression of losses

Equivalent version of Ralph's protocol:

- prepare strongly squeezed (or entangled) state
- noiselessly attenuate it
- send the state over a lossy channel
- noiselessly amplify the output state

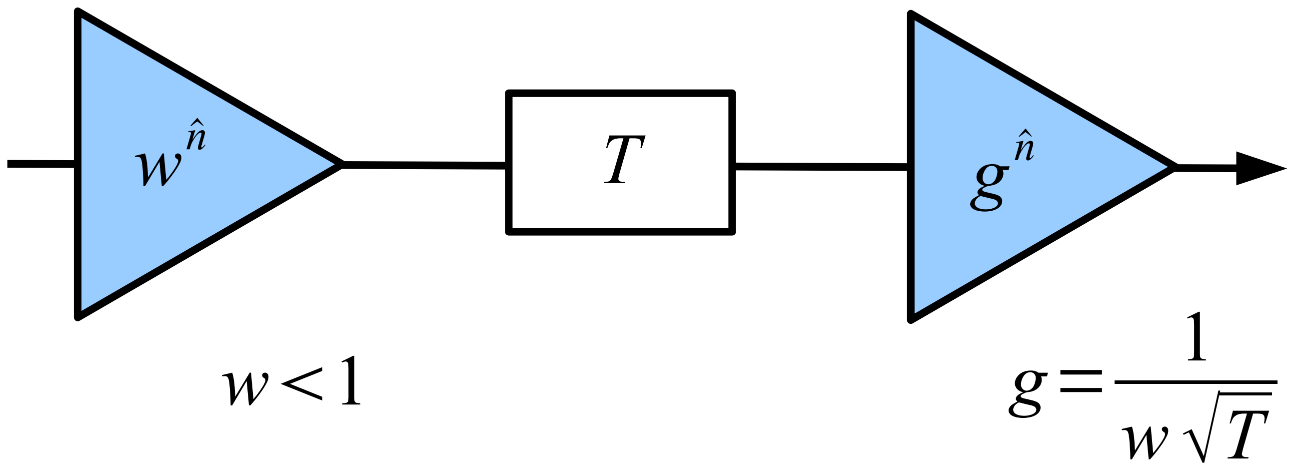


$w < 1$ $\qquad$ $g = \frac{1}{w\sqrt{T}}$

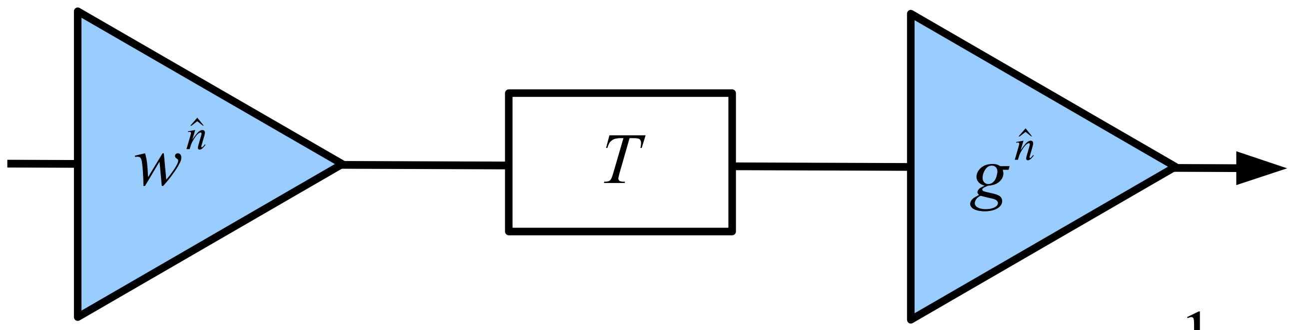
Universal scheme, works for any input state.

The channel becomes identity channel in the limit $w \to 0$

Towards complete suppression of losses II.

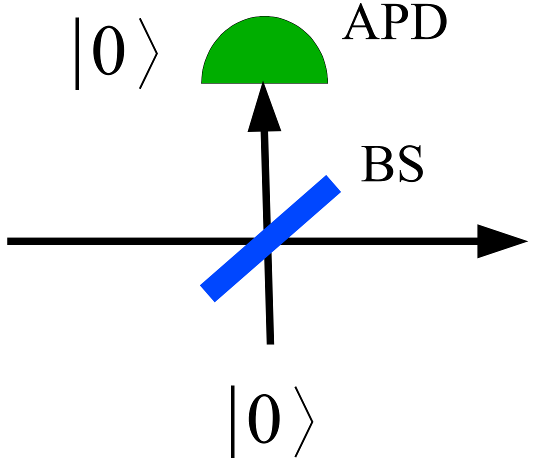


Towards complete suppression of losses II.

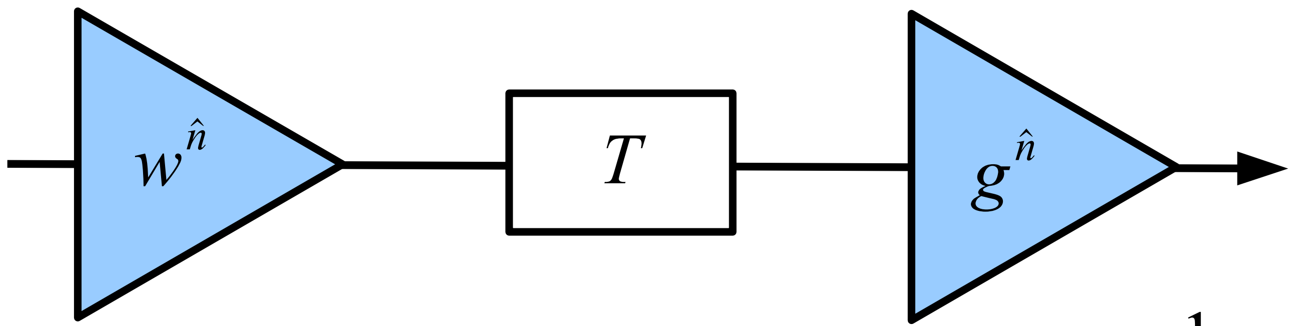


$w < 1$ $\qquad$ $g = \frac{1}{w\sqrt{T}}$

Reversible attenuation:



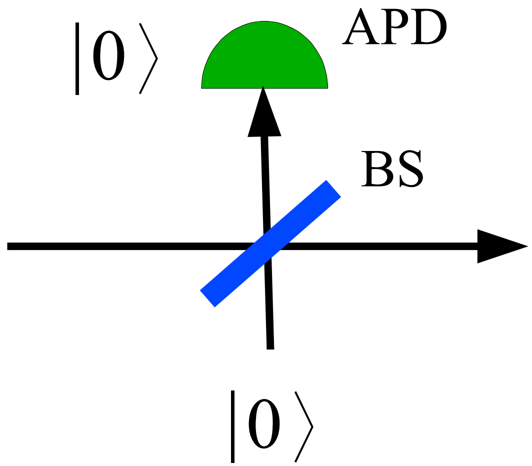
Towards complete suppression of losses II.



$w < 1$

$$g = \frac{1}{w\sqrt{T}}$$

Noiseless attenuation:



Loss compensation for single photons

Assumes input states have the form

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Noiseless amplification

$$g^{\hat{n}} \rightarrow (g-1)\hat{n} + 1$$

Scheme yields identity channel in the subspace |0>, |1> in the limit w->0

Lossy quantum channel representation

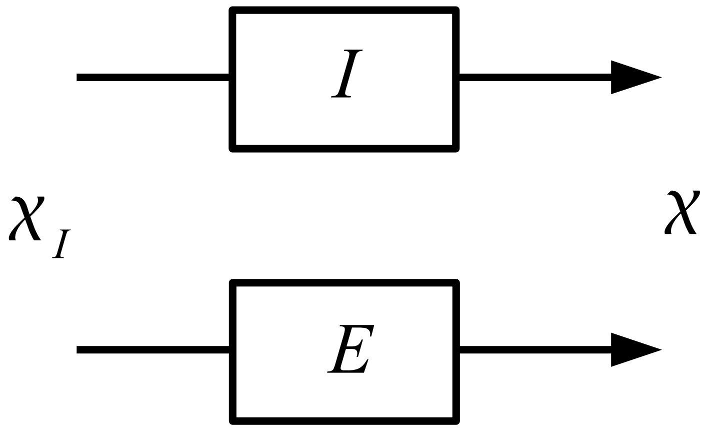
- Choi-Jamiolkowski isomorphism
- Single-qubit channel

  no more than one input photon, Hilbert space spanned by |0> and |1>
- Identity single-qubit channel is isomorphic to a two-qubit Bell state

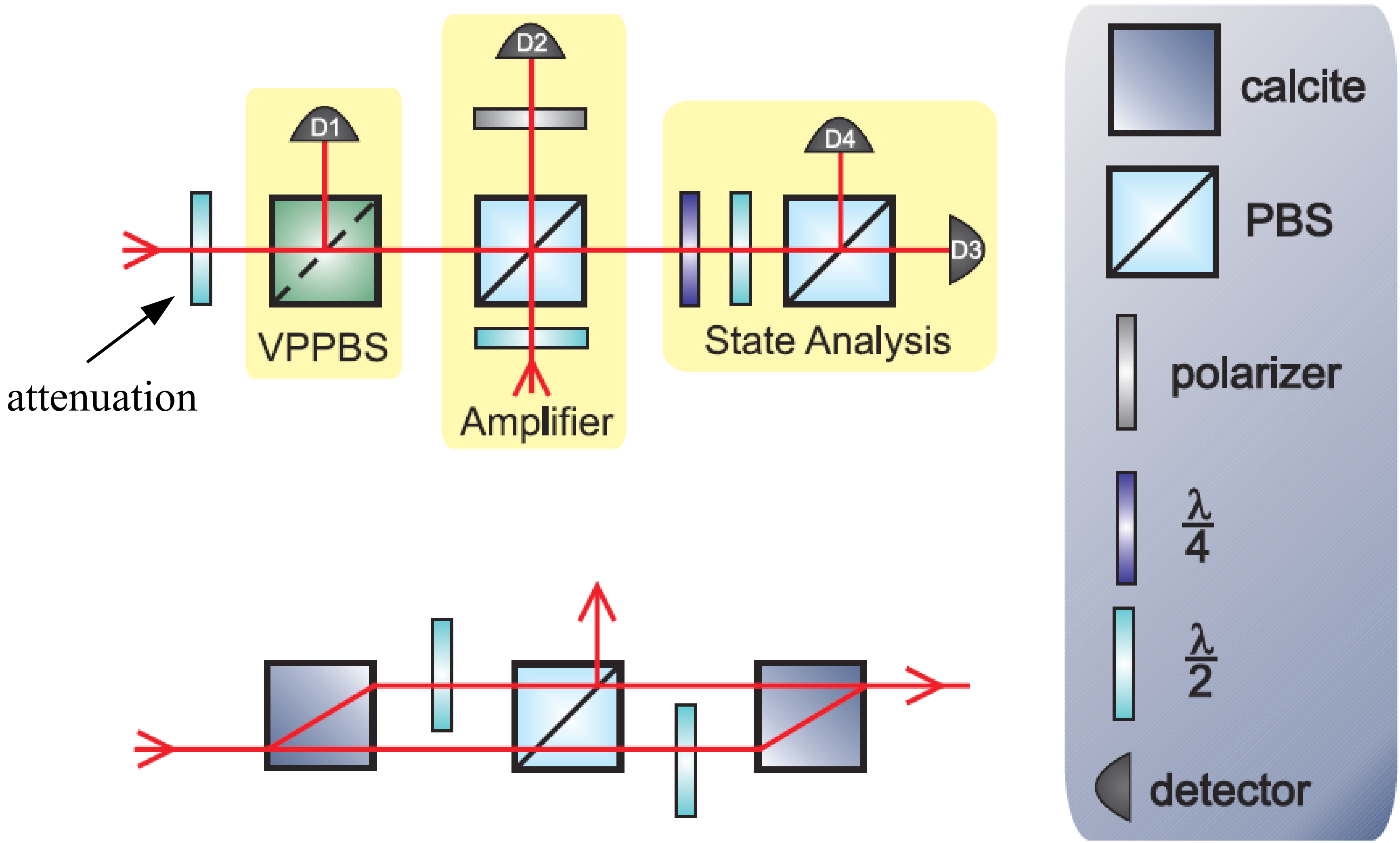
$$\chi_I = |\chi_I\rangle\langle\chi_I| \qquad |\chi_I\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle + |1\rangle|0\rangle\right)$$

Generic quantum channel E:

$$\chi = E \otimes I\,(\chi_I)$$

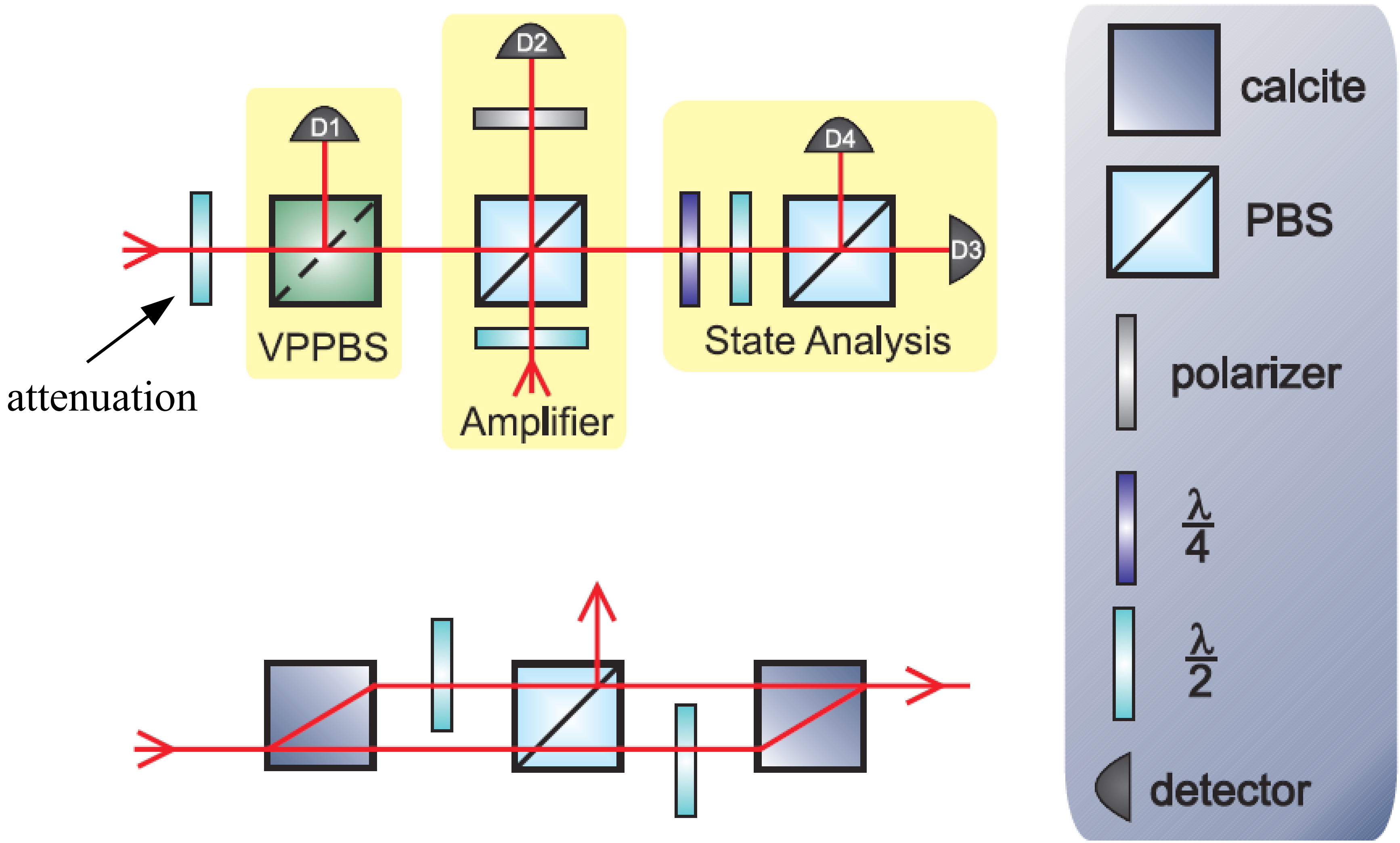


Experimental setup



M. Mičuda, I. Straka, M. Miková, M. Dušek, N. J. Cerf, J. Fiurášek, and M. Ježek, submitted (2011).

Experimental setup

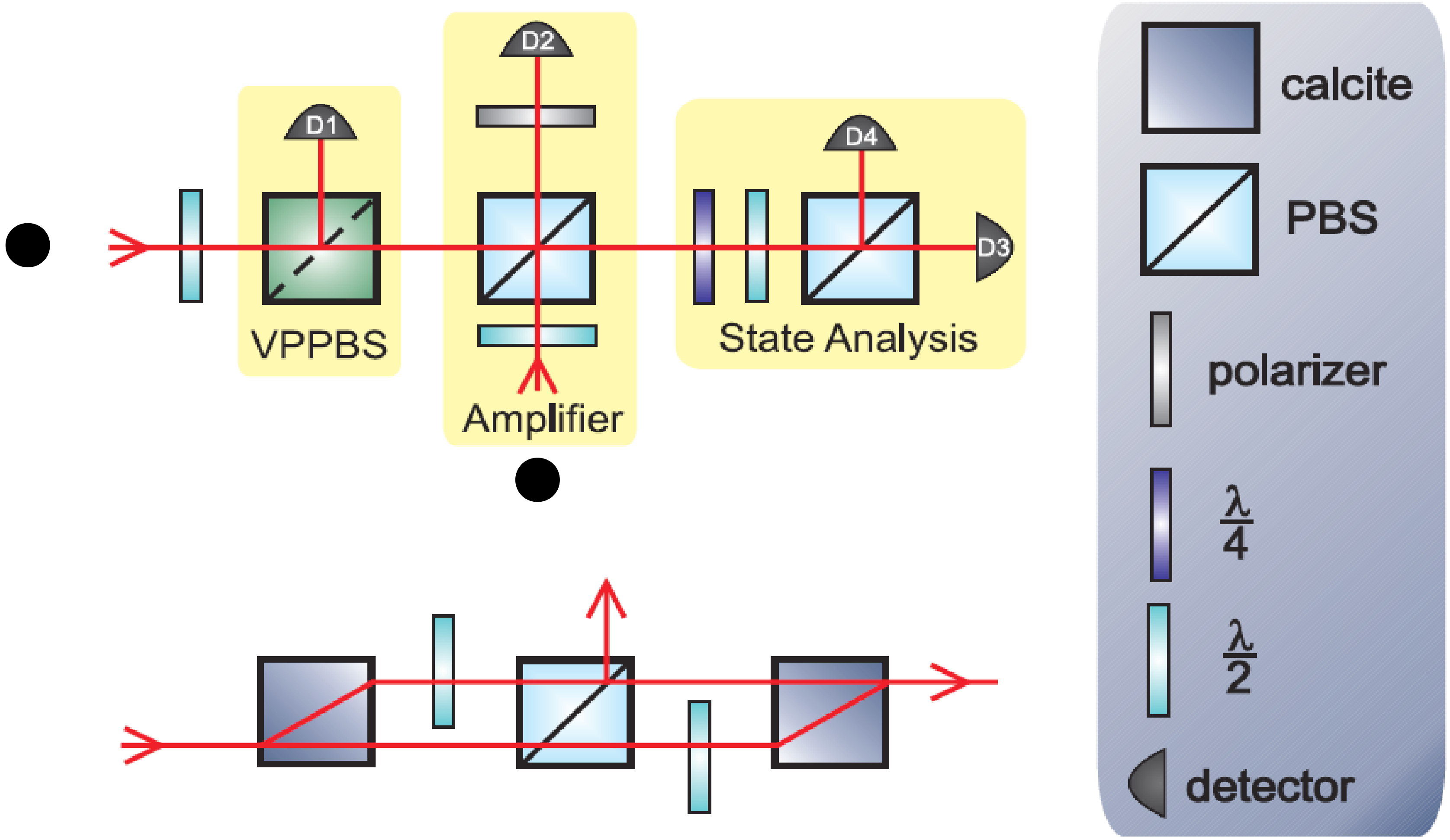


Polarization degree of freedom is explored:

Vertical polarization – lossy channel.

Horizontal polarization – reference identity channel.

Noiseless amplification

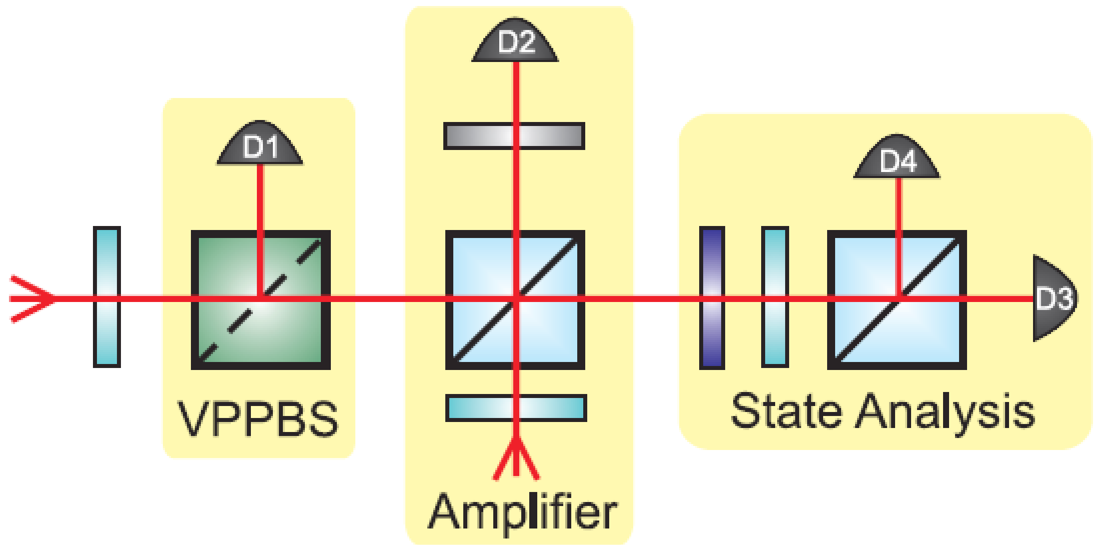


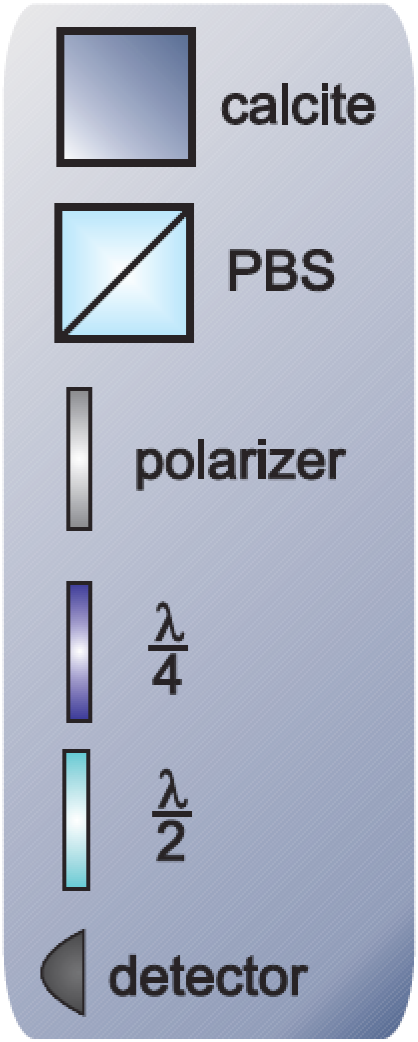
Time-correlated photon pairs are generated by SPDC (not shown).

Idler photon is used for noiseless amplification – interference on PBS.

G.Y. Xiang, T.C. Ralph, A. Lund, N. Walk, and G. Pryde, Nature Photonics **4**, 316 - 319 (2010).
M. Mičuda, M. Ježek, M. Dušek, and J. Fiurášek, Phys. Rev. A **78**, 062311 (2008).

Coincidence measurements





Coincidence measurements:

D2&D1 – indicates photon loss

D2&D3, D2&D4 – no photon loss

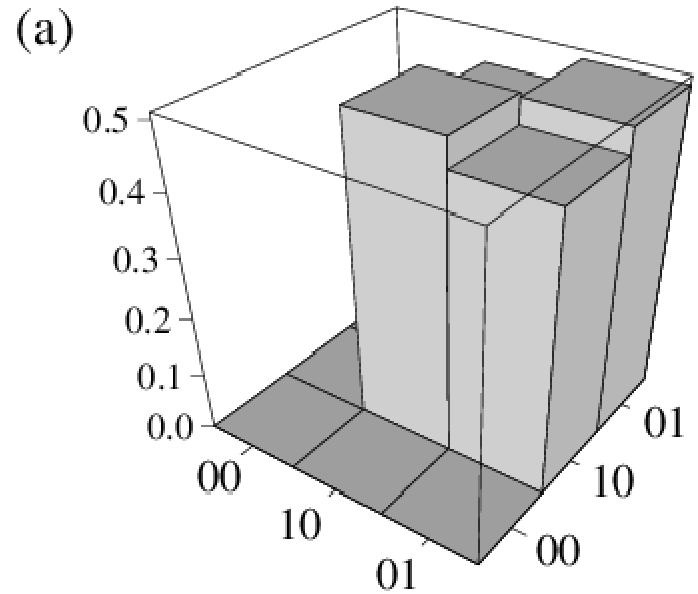
Coincidences D2&D3, D2&D4:

Full tomographic reconstruction of output single photon polarization state – complete quantum channel tomography

Lossy quantum channel tomography

Identity channel

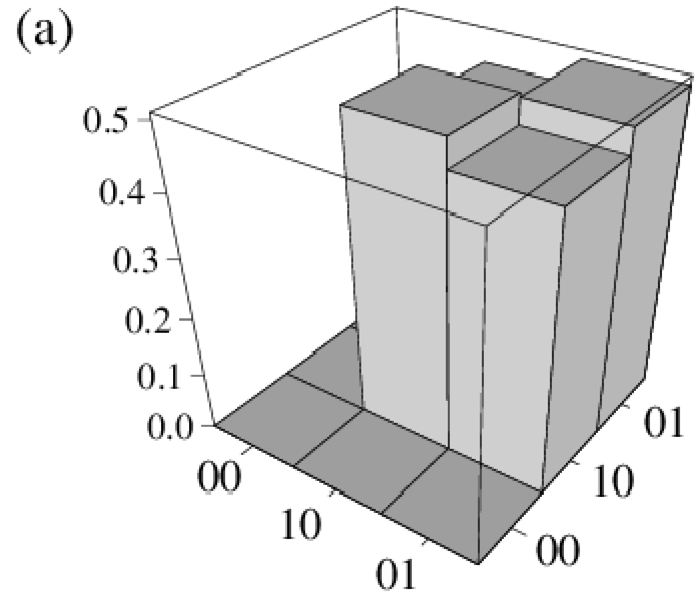
$$|\chi_I\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle + |1\rangle|0\rangle)$$



Lossy quantum channel tomography

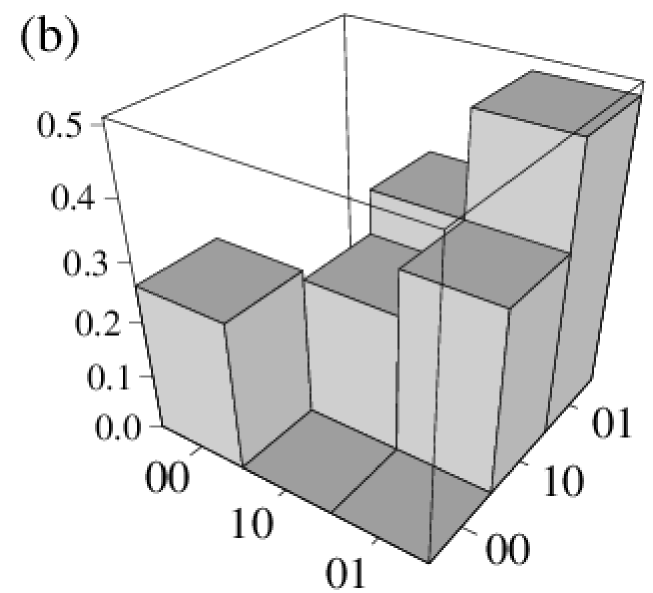
Identity channel

$$|\chi_I\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle + |1\rangle|0\rangle)$$



Lossy channel, T=0.5

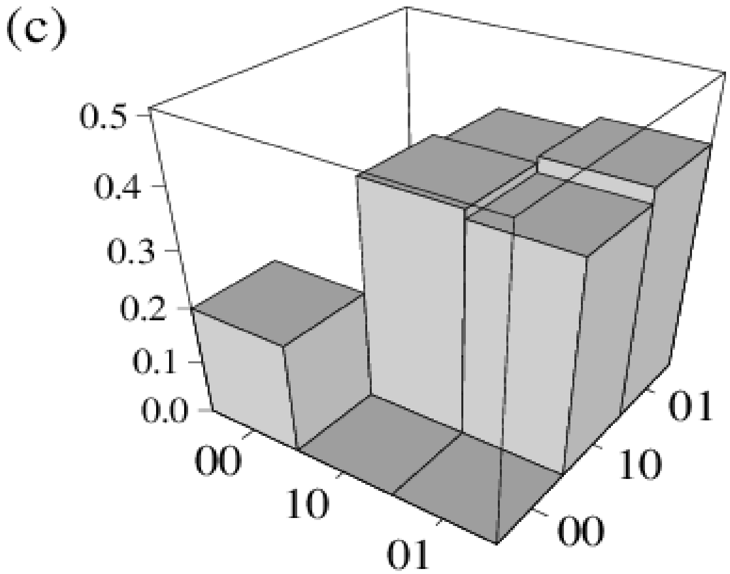
$$\chi \propto |01\rangle\langle 01| + T|10\rangle\langle 10| + (1-T)|00\rangle\langle 00| + \sqrt{T}(|01\rangle\langle 10| + |10\rangle\langle 01|)$$



Lossy quantum channel tomography

Lossy channel, T=0.5
Noiseless amplification G=1/T

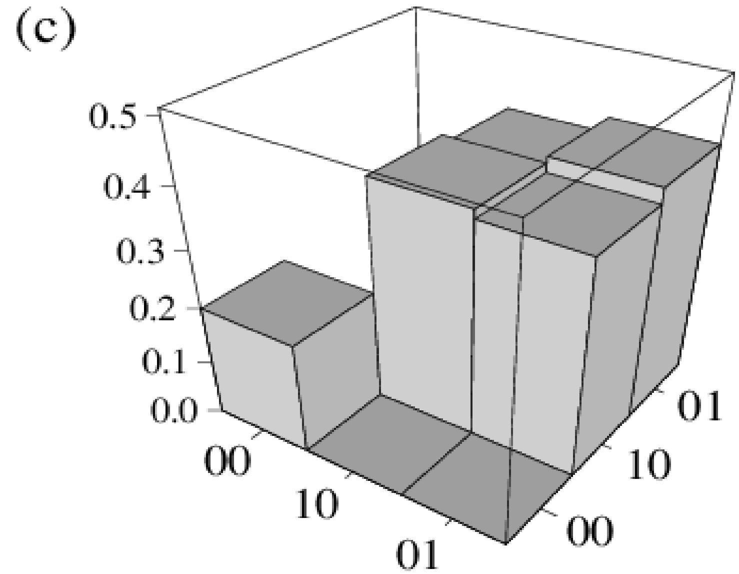
$$\chi \propto |01\rangle\langle 01| + |10\rangle\langle 10| + (1-T)|00\rangle\langle 00| + |01\rangle\langle 10| + |10\rangle\langle 01|$$



Lossy quantum channel tomography

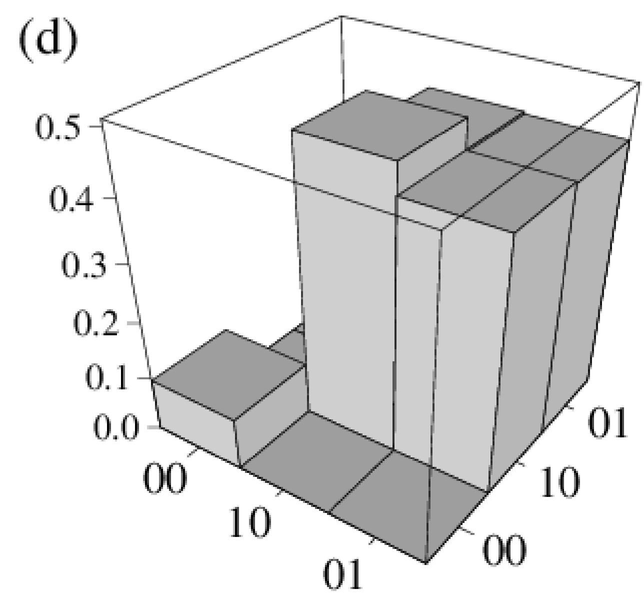
Lossy channel, T=0.5
Noiseless amplification G=1/T

$$\chi \propto |01\rangle\langle 01| + |10\rangle\langle 10| + (1-T)|00\rangle\langle 00| + |01\rangle\langle 10| + |10\rangle\langle 01|$$

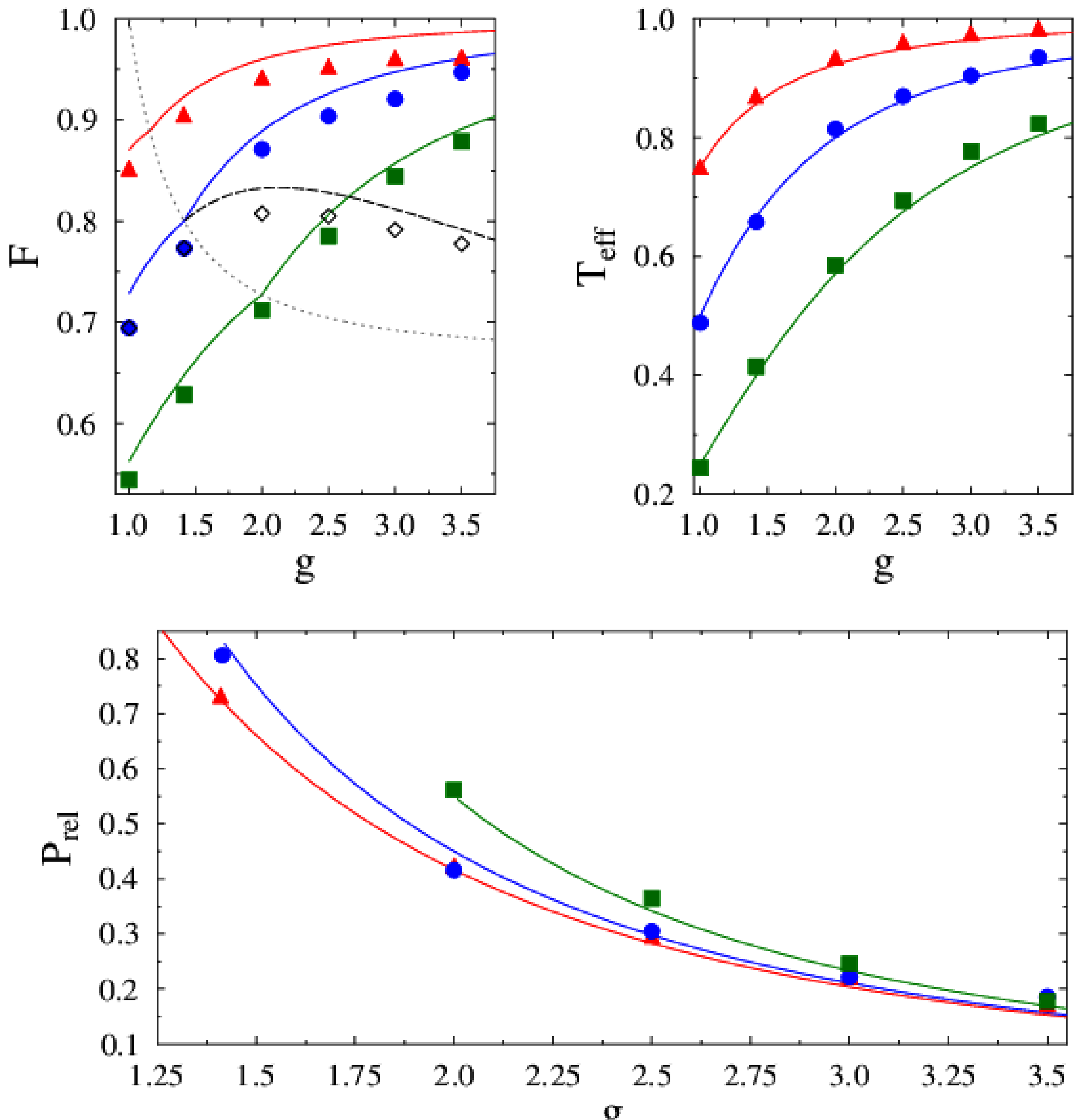


Lossy channel, T=0.5
Attenuation W=0.5
Noiseless amplification G=1/(TW)

$$\chi \propto |01\rangle\langle 01| + |10\rangle\langle 10| + W(1-T)|00\rangle\langle 00| + |01\rangle\langle 10| + |10\rangle\langle 01|$$



Further experimental results



Thank you for your attention!