Process-fidelity estimation of a linear optical quantum CZ gate

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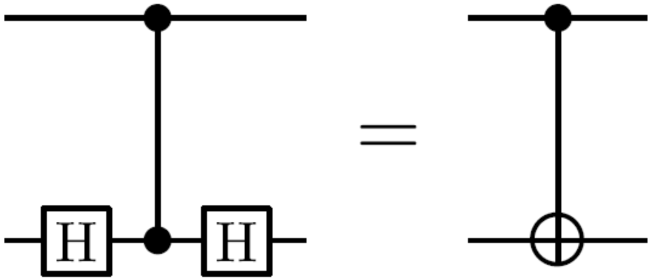
Outline of the talk

- Linear optical quantum CZ gate
- Quantum gate fidelity
- Quantum process tomography
- Hofmann bound on gate fidelity
- Three-qubit linear optical Toffoli gate
- Hofmann-like bounds from a minimum number of measurements

Linear optical quantum CZ/CNOT gate

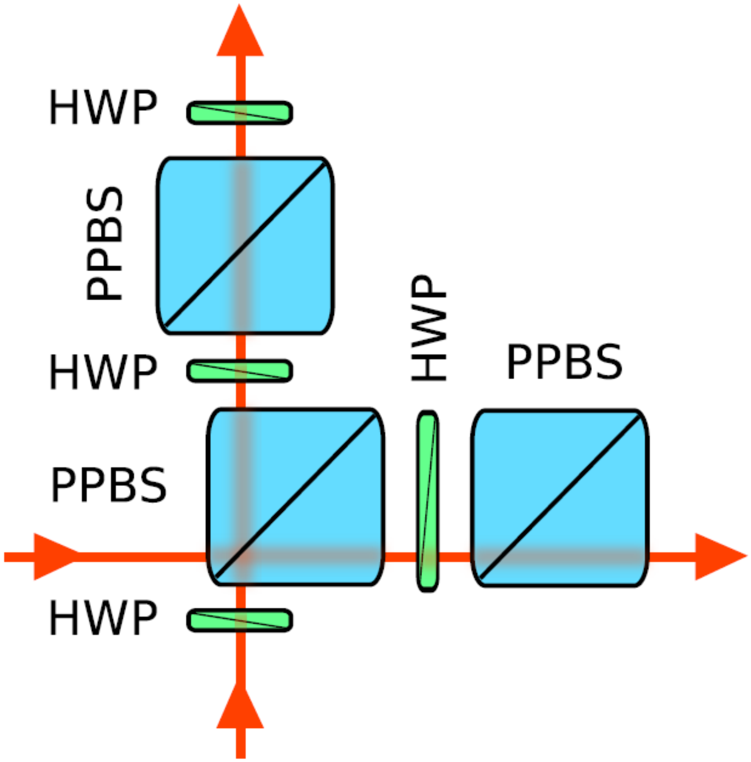
$$U_{\mathrm{CZ}} = \mathbb{I} - 2|11\rangle\langle 11|$$

$$U_{\mathrm{CZ}}|jk\rangle = (-1)^{jk}|jk\rangle, \quad j,k \in \{0,1\}$$



$|0\rangle = |H\rangle$, $|1\rangle = |V\rangle$

PPBS: $T_H = 1$, $T_V = 1/3$

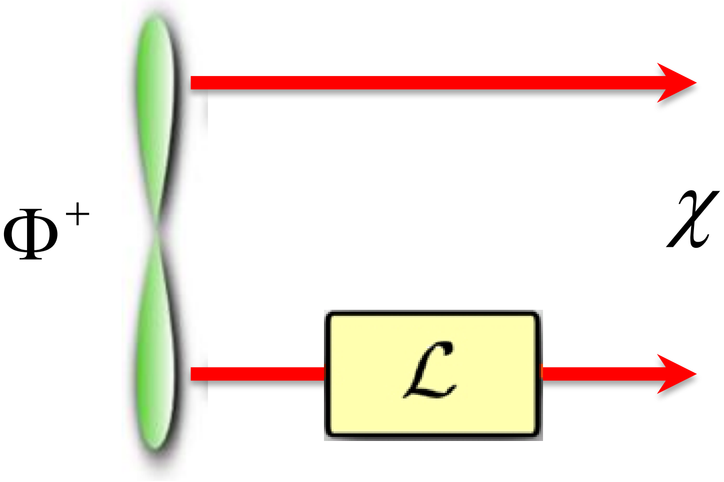


R. Okamoto, H.F. Hofmann, S. Takeuchi, and K. Sasaki, Phys. Rev. Lett. 95, 210506 (2005)

N. K. Langford, T.J. Weinhold, R. Prevedel, K. J. Resch, A. Gilchrist, J. L. OBrien, G. J. Pryde, and A. G. White, Phys. Rev. Lett. 95, 210504 (2005)

N. Kiesel, C. Schmid, U. Weber, R. Ursin, and H. Weinfurter, Phys. Rev. Lett. 95, 210505 (2005)

Choi-Jamiolkowski isomorphism



Maximally entangled probe state

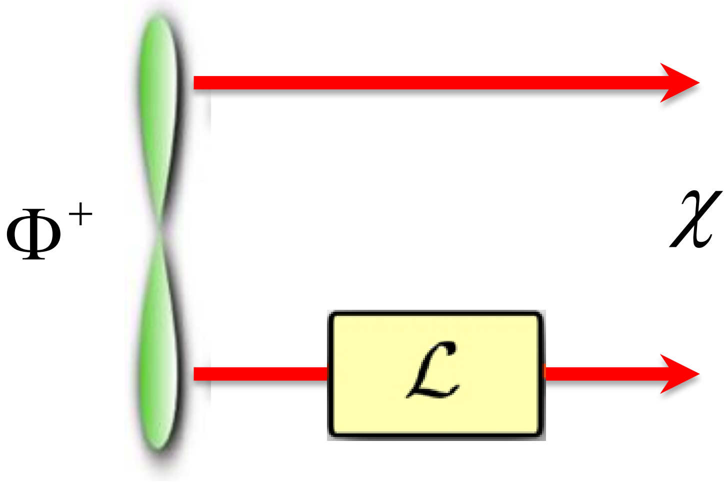
$$\left|\Phi^+\right\rangle = \sum_{j,k=0}^{1} \left|jk\right\rangle\left|jk\right\rangle$$

Choi-Jamiolkowski isomorphism

$$\chi = \mathcal{I} \otimes \mathcal{L}(\Phi^+)$$

A. Jamiolkowski, Rep. Math. Phys. **3,** 275 (1972)**;** M.-D. Choi, Linear Algebra Appl. **10,** 285 (1975).

Choi-Jamiolkowski isomorphism



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Choi-Jamiolkowski isomorphism

$$\chi = \mathcal{I} \otimes \mathcal{L}(\Phi^+)$$

Choi matrix of unitary CZ gate

$$\chi_{\mathrm{CZ}} = (\mathbb{I} \otimes U_{\mathrm{CZ}})|\Phi^+\rangle\langle\Phi^+|(\mathbb{I} \otimes U_{\mathrm{CZ}}^{\dagger})$$

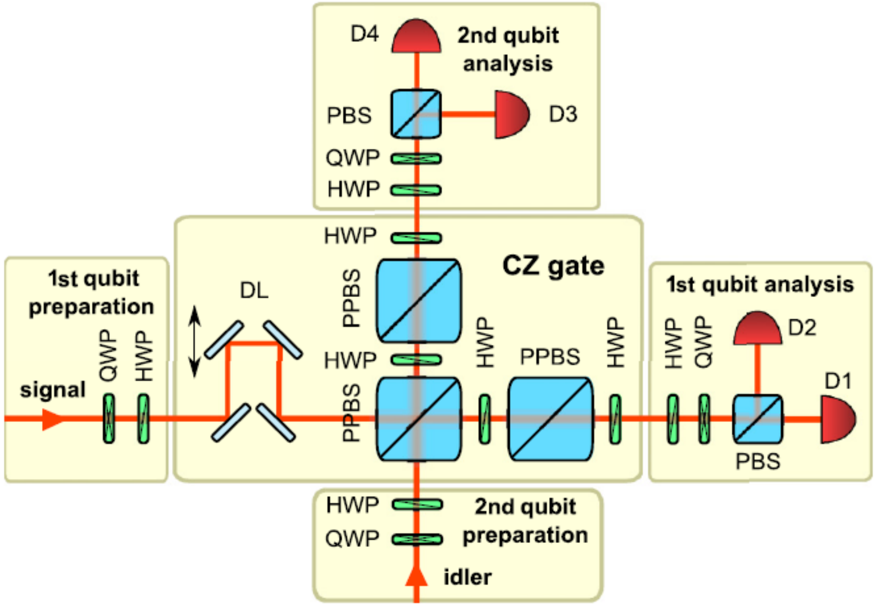
Quantum gate fidelity

$$F_{CZ} = \frac{\mathrm{Tr}[\chi\chi_{CZ}]}{\mathrm{Tr}[\chi]\mathrm{Tr}[\chi_{CZ}]}$$

Normalized overlap of Choi matrices.

A. Jamiolkowski, Rep. Math. Phys. **3,** 275 (1972)**;** M.-D. Choi, Linear Algebra Appl. **10,** 285 (1975).

Quantum process tomography



Preparation of 36 input product states

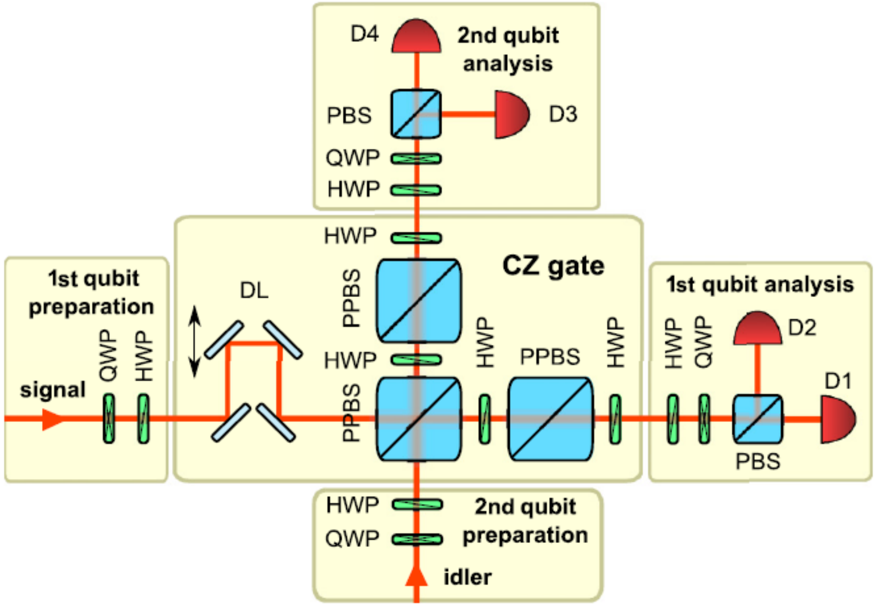
$$|j\rangle|k\rangle, \quad j,k \in \{H,V,D,A,R,L\}$$

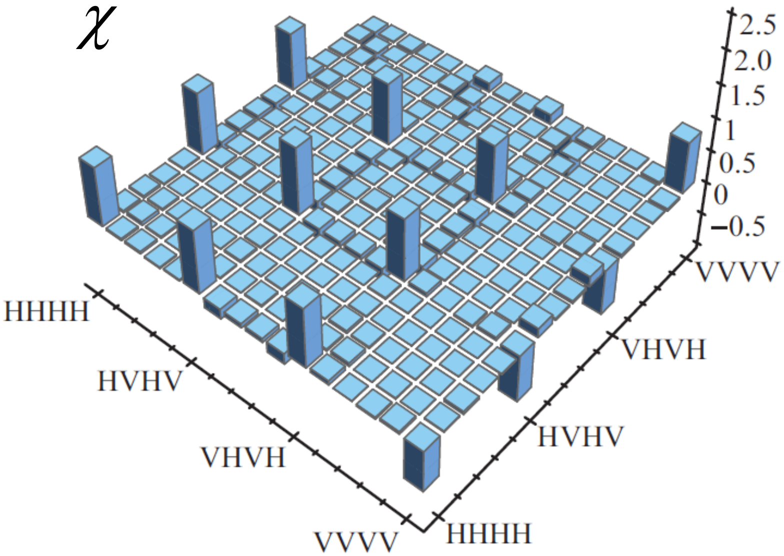
Measurements in 9 combinations of 3 single-qubit bases H/V, D/A, R/L.

Maximum-likelihood estimation of quantum process matrix χ_{CZ} from the experimental data.

J. F. Poyatos, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **78,** 390 (1997); I. L. Chuang and M. A. Nielsen, J. Mod. Opt. **44,** 2455 (1997).
J. L. O'Brien, G. J. Pryde, A. Gilchrist, D. F. V. James, N. K.Langford, T. C. Ralph, and A. G. White, Phys. Rev. Lett. **93,** 080502 (2004).
M. Ježek, J. Fiurášek, and Z. Hradil, Phys. Rev. A **68,** 012305 (2003).

Quantum process tomography





Preparation of 36 input product states

$$|j\rangle|k\rangle, \quad j,k \in \{H,V,D,A,R,L\}$$

$$F_{CZ} \geq 0.860 \pm 0.001$$

Measurements in 9 combinations of 3 single-qubit bases H/V, D/A, R/L.

Maximum-likelihood estimation of quantum process matrix χ_{CZ} from the experimental data.

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J. L. O'Brien, G. J. Pryde, A. Gilchrist, D. F. V. James, N. K.Langford, T. C. Ralph, and A. G. White, Phys. Rev. Lett. **93,** 080502 (2004).
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Hofmann bound on quantum gate fidelity

$$\boxed{F_1 + F_2 - 1 \leq F_{CZ} \leq \min(F_1, F_2)}$$

- F_1 and F_2 denote average state fidelities for two mutually unbiased bases.
- Requires much less measurements than full process tomography -> much faster procedure.
- Suitable basis choice for quantum CZ gate:

Basis #1: HD, HA, VD, VA

Basis #2: DH, DV, AH, AV

H.F. Hofmann, Phys. Rev. Lett. **94**,160504 (2005).
M. Mičuda, M. Sedlák, I. Straka, M. Miková, M. Dušek, M. Ježek, and J. Fiurášek, Phys. Rev. A **89**, 042304 (2014).

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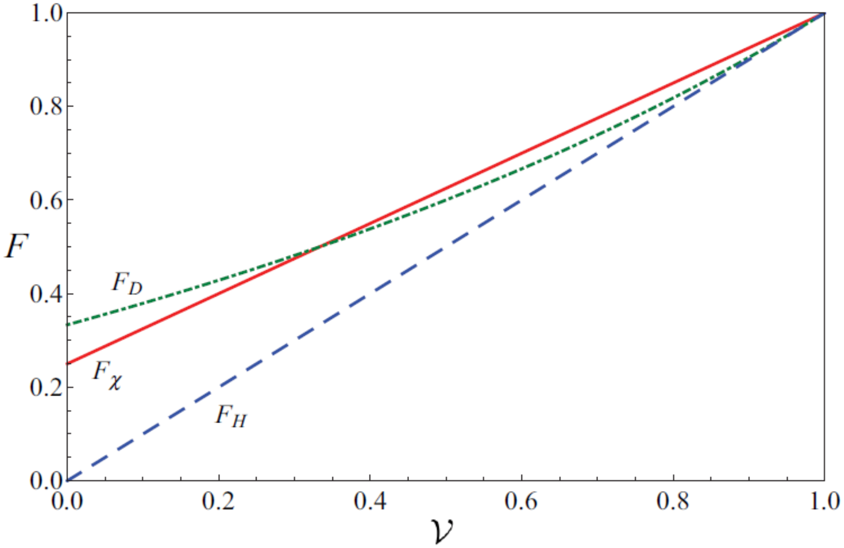
Linear optical CZ gate is probabilistic.

Success probability depends on the input state due to various experimental imperfections.

Average state fidelities have to be calculated as weighted averages with weights given by success probabilities.



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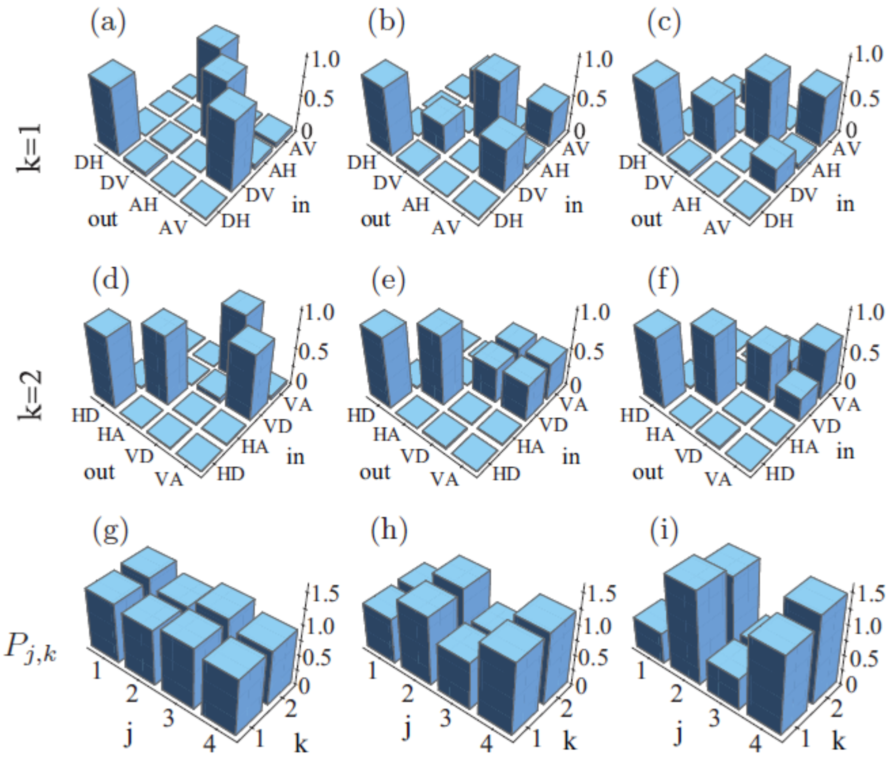
Hofmann bound - experimental results

Basis #1

Basis #2

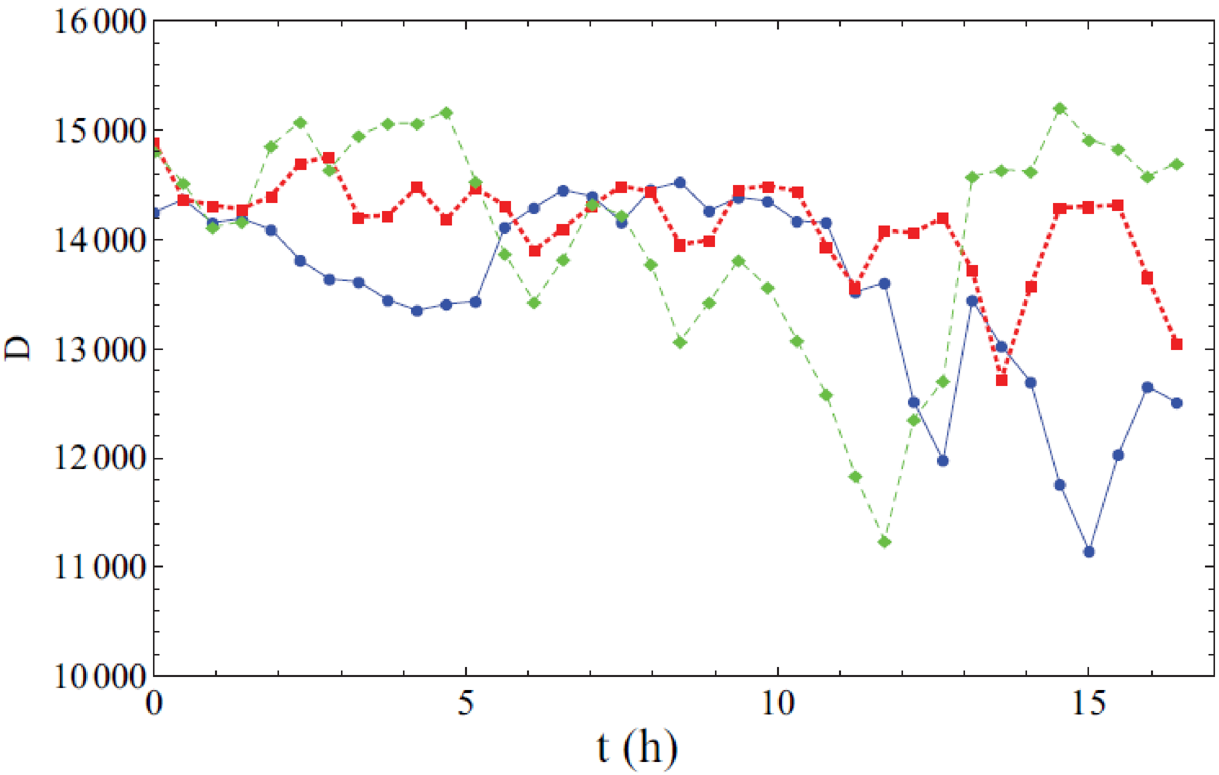
Success probability



$\mathcal{V} = 0.953$ $\mathcal{V} = 0.500$ $\mathcal{V} = 0.022$

$\mathcal{V}$	F_{D}	F_{H}	F_{χ}	$\min(F_1, F_2)$
0.953	0.875(2)	0.877(2)	0.860(1)	0.934(1)
0.500	0.465(2)	0.372(2)	0.531(1)	0.676(1)
0.022	0.253(2)	−0.034(2)	0.232(1)	0.479(1)

Systematic effects and errors



- Long-term fluctuations of the pair generation rate.
- Changes of visibility of two-photon interference during the measurement.
- Imperfections of waveplates and polarizing beam splitters.

M. Mičuda, M. Sedlák, I. Straka, M. Miková, M. Dušek, M. Ježek, and J. Fiurášek, Phys. Rev. A **89**, 042304 (2014).

Monte-Carlo sampling

Linear estimator
of quantum gate fidelity

$$F_{MC} = \frac{\sum_{j,k,l,m} u_{jk,lm} C_{jk,lm}}{\sum_{j,k,l,m} C_{jk,lm}}$$

$$j,k,l,m \in \{H,V,D,A,R,L\}$$

$\mathcal{V}$	σ_0	F_{MC}	$\tilde{F}_{\mathrm{MC}}$
0.953	H/V	0.871(2)	0.861(2)
0.953	D/A	0.882(2)	0.870(2)
0.953	R/L	0.833(1)	0.846(1)
0.500	H/V	0.539(1)	0.533(2)
0.500	D/A	0.521(1)	0.518(2)
0.500	R/L	0.515(1)	0.520(1)
0.022	H/V	0.252(1)	0.240(1)
0.022	D/A	0.245(1)	0.240(1)
0.022	R/L	0.242(1)	0.235(1)

$C_{jk,lm}$ – number of projections onto state |*lm*> for input state |*jk*>

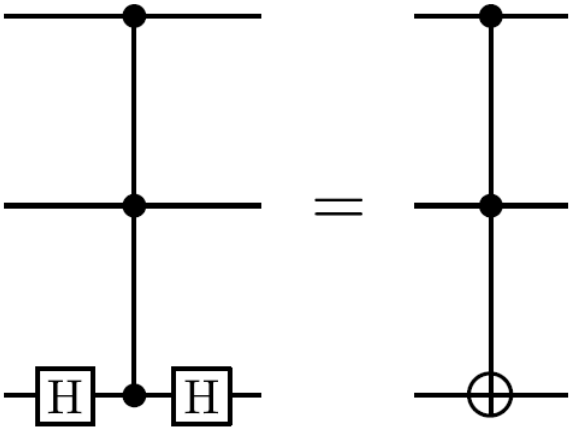
Overdetermined data set - we can construct different estimators depending on the representation of identity operator:

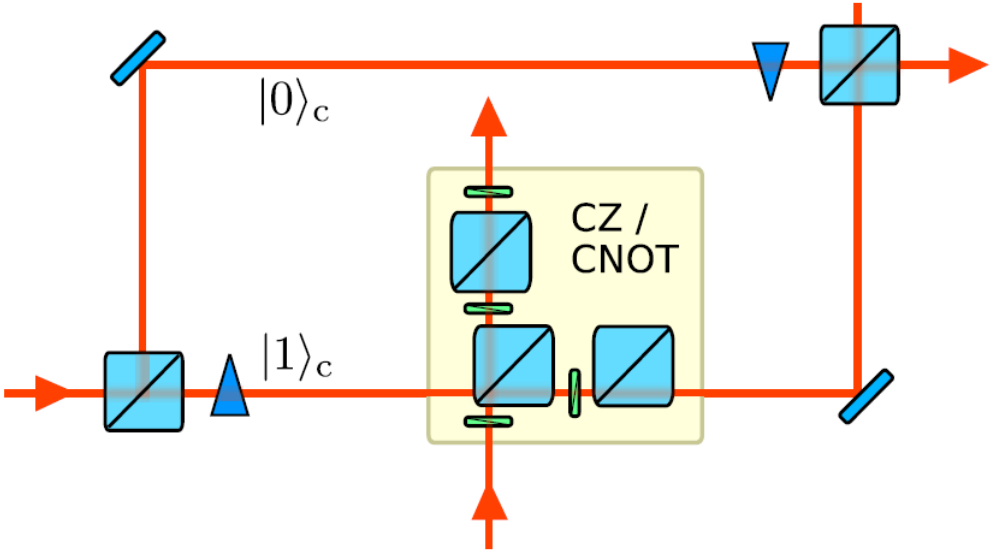
$$\sigma_0 = |H\rangle\langle H| + |V\rangle\langle V|, \quad \sigma_0 = |D\rangle\langle D| + |A\rangle\langle A|, \quad \sigma_0 = |R\rangle\langle R| + |L\rangle\langle L|$$

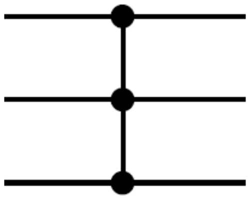
S. T. Flammia and Y.-K. Liu, Phys. Rev. Lett. **106,** 230501 (2011);
M. P. da Silva, O. Landon-Cardinal, and D. Poulin, Phys. Rev. Lett. **107,** 210404 (2011).
M. Mičuda, M. Sedlák, I. Straka, M. Miková, M. Dušek, M. Ježek, and J. Fiurášek, Phys. Rev. A **89**, 042304 (2014).

Linear optical quantum CCZ/Toffoli gate

$$U_{\mathrm{CCZ}} = \mathbb{I} - 2|111\rangle\langle 111| \qquad U_{\mathrm{CCZ}}|jkl\rangle = (-1)^{jkl}|jkl\rangle, \quad j,k,l \in \{0,1\}$$







qubit 1: spatial degree of freedom of the first photon

qubit 2: polarization degree of freedom of the first photon

qubit 3: polarization degree of freedom of the second photon

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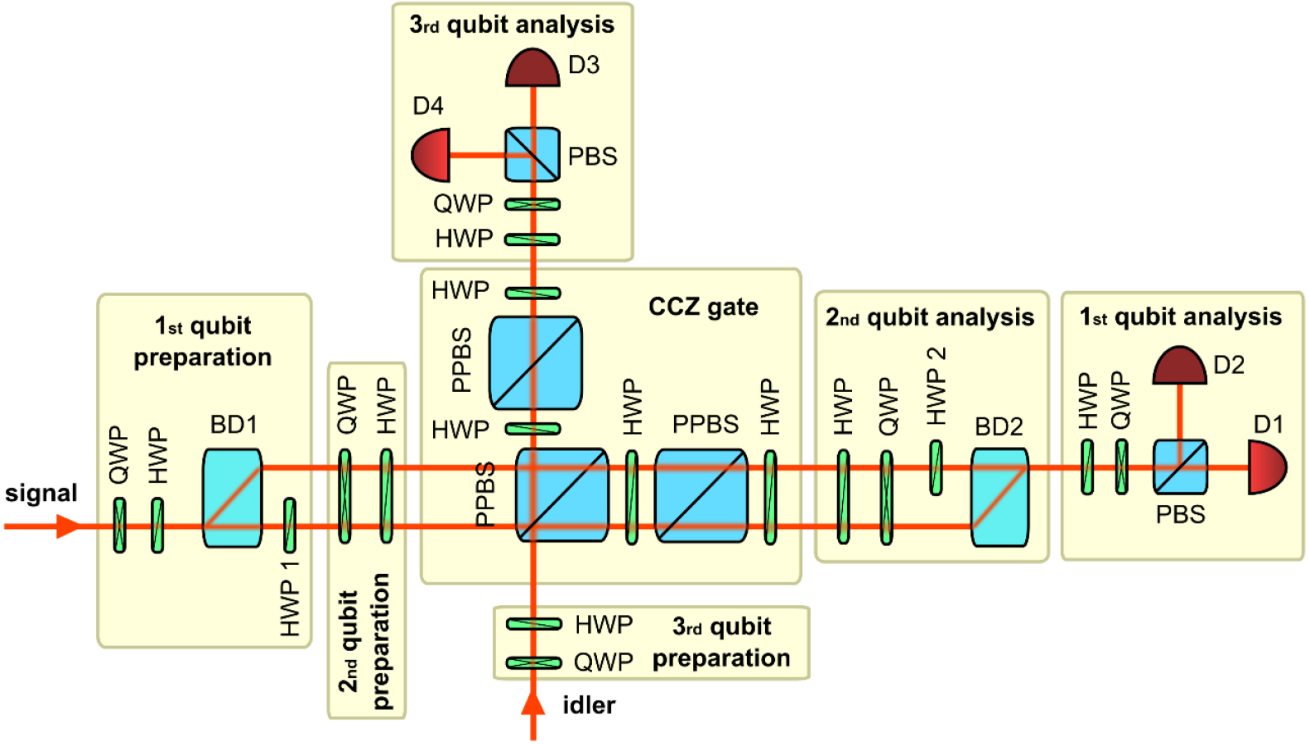
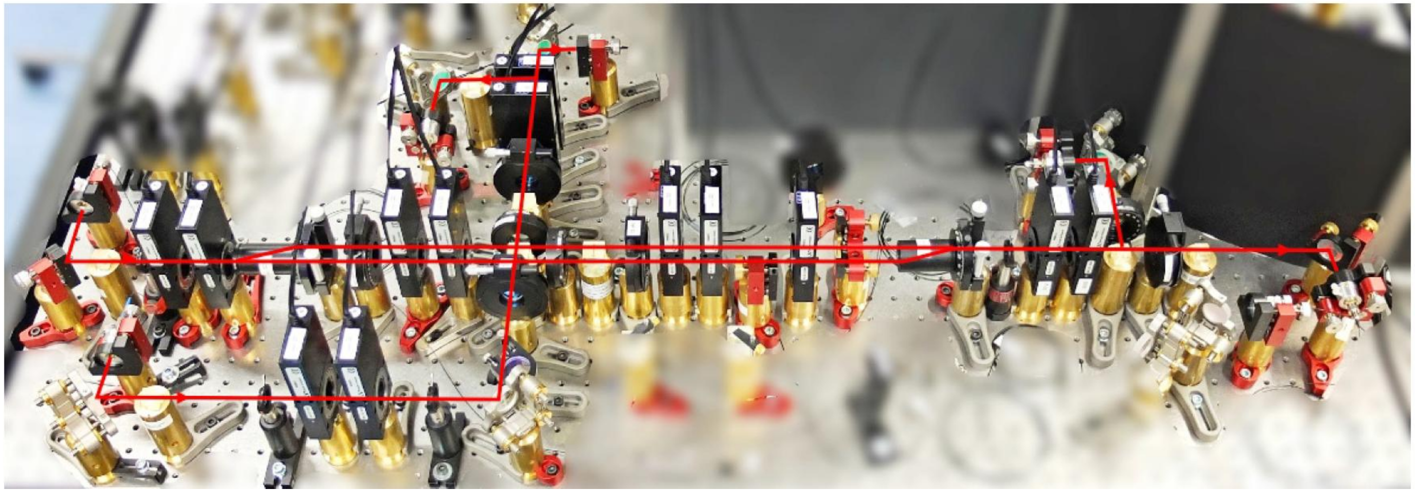
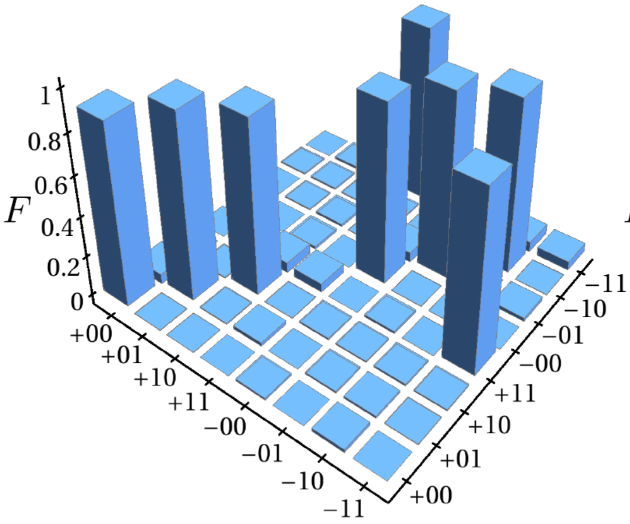
Linear optical CCZ gate – experimental setup

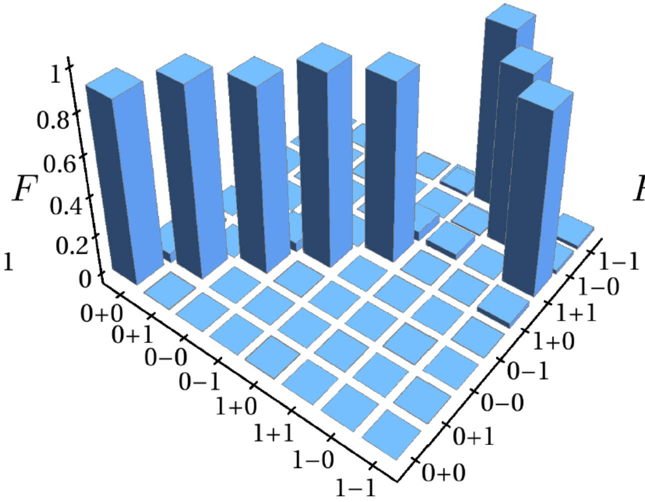




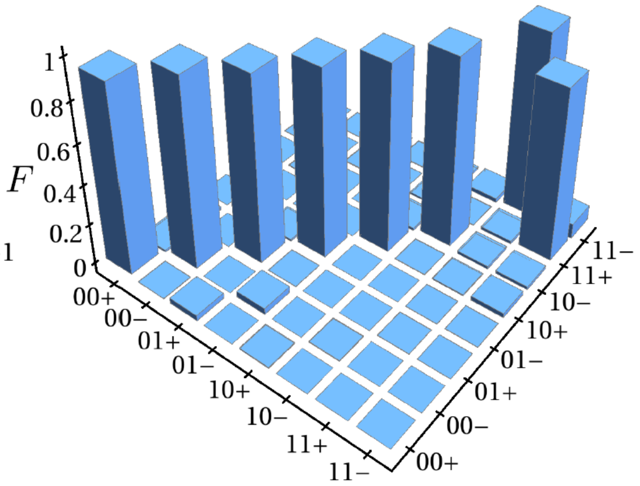
Generalized Hofmann bound on gate fidelity



F_1=0.928(1)



F_2=0.947(1)



F_3=0.955(1)

Truth tables measured for 3 product bases – measurement of fidelities of entangled states is avoided.

Lower bound on gate fidelity in terms of average state fidelities:

$$F_{CCZ} \geq F_1 + F_2 + F_3 - 2$$

$$F_{CCZ} \geq 0.830 \pm 0.002$$

M. Mičuda, M. Sedlák, I. Straka, M. Miková, M. Dušek, M. Ježek, J. Fiurášek, Phys. Rev. Lett. 111, 160407 (2013).

Hofmann-like bound from a minimum number of measurements

To obtain a nontrivial bound on quantum gate fidelity, it suffices to probe the quantum gate with computational basis states and a single superposition state.

Computational basis

$$|00\rangle, |01\rangle, |10\rangle, |11\rangle$$

Superposition state

$$|+\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

F … average state fidelity for computational basis
G… fidelity of superposition state |+>

D.M. Reich, G. Gualdi, and C.P. Koch, Phys. Rev. A **88**, 042309 (2013).
J. Fiurášek and M. Sedlák, Phys. Rev. A **89**, 012323 (2014).

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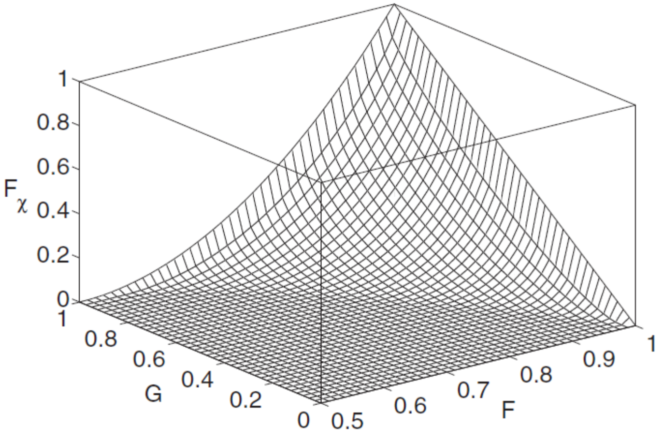
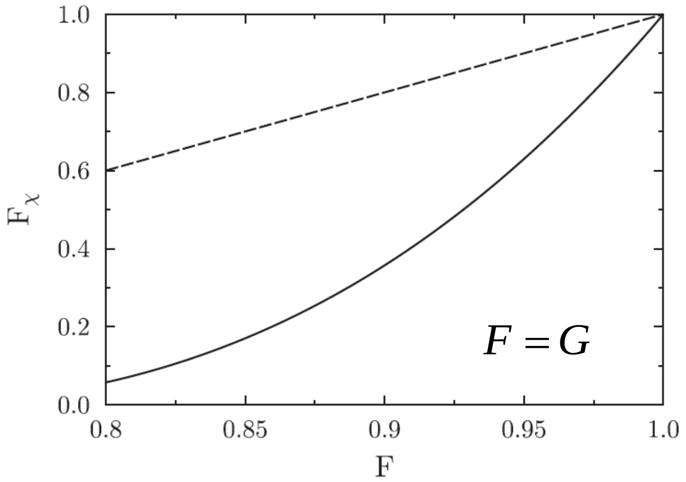
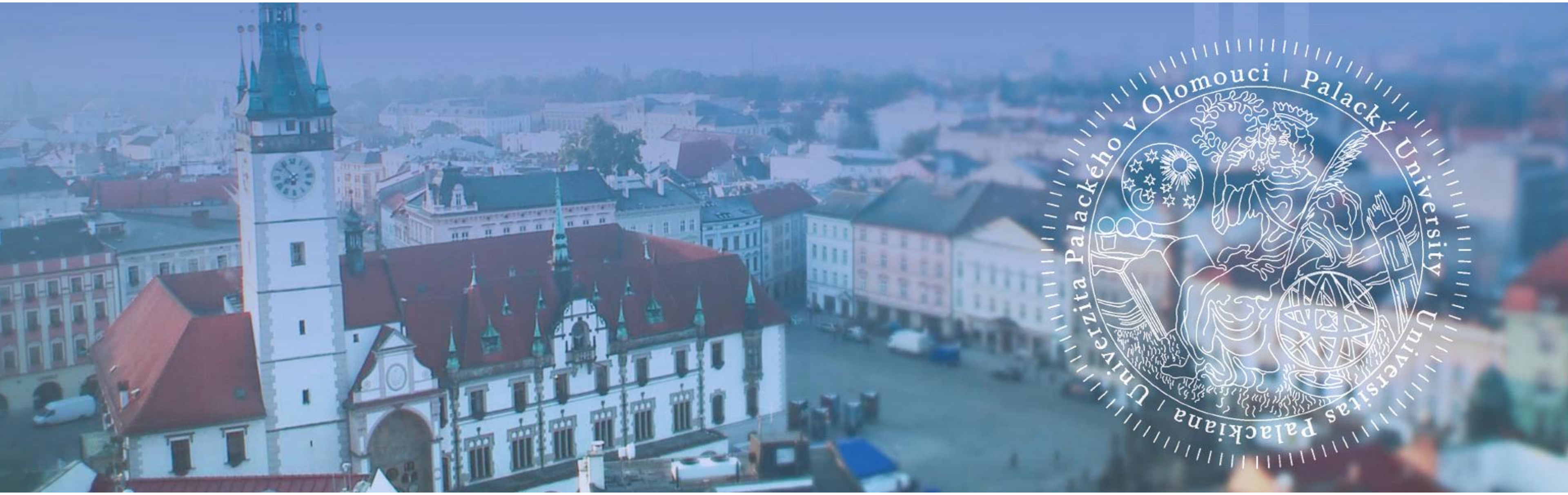




The resulting lower bound on gate fidelity is weak and not practically useful.

D.M. Reich, G. Gualdi, and C.P. Koch, Phys. Rev. A **88**, 042309 (2013).
J. Fiurášek and M. Sedlák, Phys. Rev. A **89**, 012323 (2014).

Thank you for your attention!



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