OPTIMAL PROBABILISTIC MEASUREMENT OF PHASE

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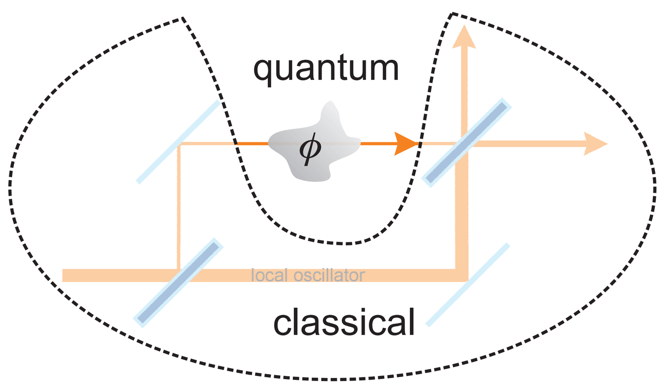
OPTICAL PHASE MEASUREMENT IN CV



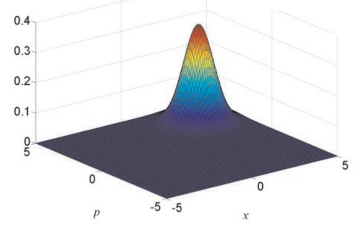
Optical phase:

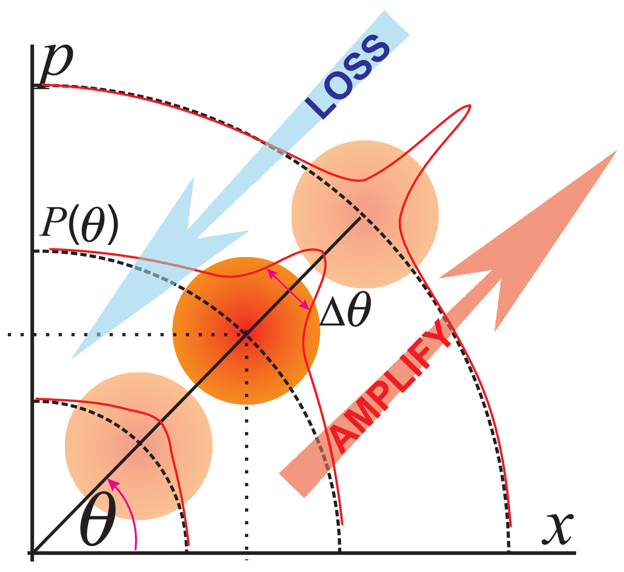
- Path difference between two arms of an interferometer
- In CV, the interferometer is strongly unbalanced and the stronger classical part is often neglected.
- CV quantum states can then have seemingly absolute phase.

Example: Phase of coherent states:

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

Coherent state:
Approximation of single mode laser light
Almost classical quantum state
Usable for quantum communication





θ ... the `absolute' phase

$\Delta\theta$ ... the quality, with which the phase can be extracted
-depends on the quantum state

PHASE MEASUREMENT

- Extracts information about the state from a single copy of the state
- Gives immediate result, but it can be

PHASE ESTIMATION

- Extracts information about the state from many copies of the state
- The quality of the outcome depends on the number of copies

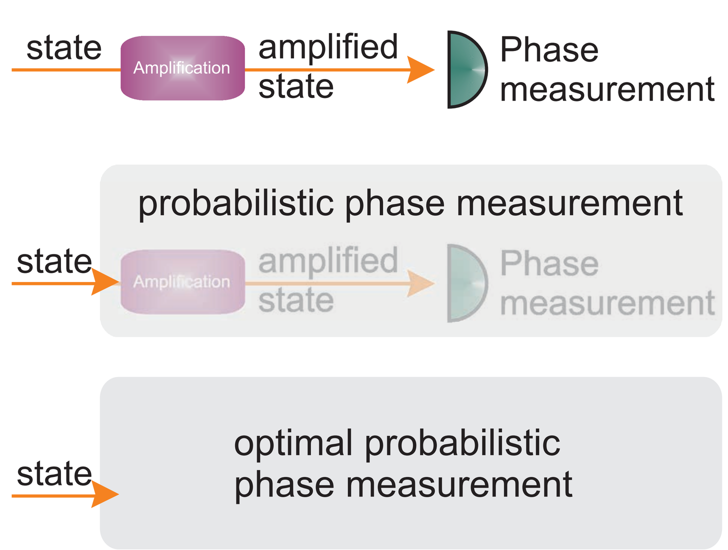
IDEAL PHASE MEASUREMENT

- Represented by POVM with elements $\Pi_\theta = \frac{1}{2\pi}|\theta\rangle\langle\theta|, \quad |\theta\rangle = \sum_{n=0}^{\infty} e^{i\theta n}|n\rangle$
- The probability of obtaining a particular value of θ is given by $P(\theta) = \mathrm{Tr}[\Pi_\theta \rho]$
- The precision of the measurement is given by the phase variance

$$V = \mu^2 - 1 = \left|\langle e^{i\theta}\rangle\right|^{-2} - 1$$

ENHANCING THE PHASE MEASUREMENT

- Prepairing of a specific state before the phase encoding.
  - vulnerable to losses and imperfections
- Amplification of the state after the phase encoding
  -ideal amplification does not exist, but it can be implemented approximatively



Quantum scissors approach
[Xiang et al. Nature Photonics 4, 316 (2010)]
[Ferreyol et al., Phys. Rev. Lett 104, 123603 (2010)]
Photon addition and subtraction
[Marek and Filip, Phys Rev A 81, 022302 (2010)]
[Zavatta et al., Nature Photonics 5, 52 (2011)]
Noise powered amplification
[Marek and Filip, Phys Rev A 81, 022302 (2010)]
[Usuga et al., Nature Phys. 6, 767 (2010)]

Probabilistic phase measurement:

- Represented by POVM elements $\Pi_\theta^{(P)} = \frac{1}{2\pi}|\theta_f\rangle\langle\theta_f|, \quad \Pi_{\text{fail}}^{(P)} = 1 - \int_{-\pi}^{\pi}\Pi_\theta^{(P)} d\theta$

$$|\theta_f\rangle = \sum_{n=0}^{\infty} f_n e^{i\theta n}|n\rangle, \quad |f_n| \in \langle 0, 1\rangle$$

- The parameters f_n characterize a probabilistic filter, a kind of generalized amplifier
- Finding the optimal measurement is therefore reduced to finding the optimal filter

FINDING THE OPTIMAL FILTER

- For any quantum state $|\psi\rangle = \sum_{n=0}^{\infty} c_n|n\rangle$ and any given probability of succcess P, the optimal filter is obtained by fiding the maximum of $|\langle e^{i\phi}\rangle| = \left|\sum_{n=0}^{\infty} f_n f_{n+1}^* c_n c_{n+1}^*\right|$ under the condition $\sum_{n=0}^{\infty}|f_n|^2|c_n|^2 = P$
- We can safely consider $c_n > 0, f_n > 0$.
- The solution is then obtained by solving a set of equations:

$$f_{n-1}a_{n-1} + f_{n+1}a_n = \lambda f_n x_n, \quad n = 0, 1, \ldots, \qquad \sum_{n=0}^{\infty} x_n f_n^2 = P$$

$$a_n = c_n c_{n+1}, \quad x_n = c_n^2, \quad f_{-1} = 0$$

- The solution is state dependent.

SEMI-ANALYTIC SOLUTION

- Can be found under a realistic assumption:
  There is only a finite number of filter parameters, which are not equal to one. $f_n = 1$ for all $n \geqslant N$.
- The optimal filter can be then found as $f_n = f_0 \mathcal{P}_n(\lambda)$, where $\mathcal{P}_n(\lambda)$ is a polynomial defined recursively as

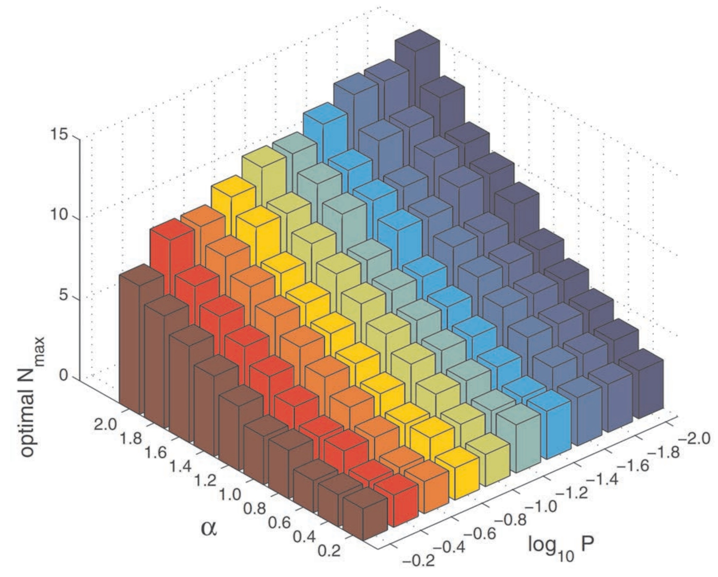
$$\mathcal{P}_{n+1}(\lambda) = \frac{\lambda x_n \mathcal{P}_n(\lambda) - a_{n-1}\mathcal{P}_{n-1}(\lambda)}{a_n}, \qquad \mathcal{P}_0(\lambda) \equiv 1, \quad \mathcal{P}_1(\lambda) = x_0/a_0$$

- Finding the solution is then equivalent to finding the roots of polynomial equation

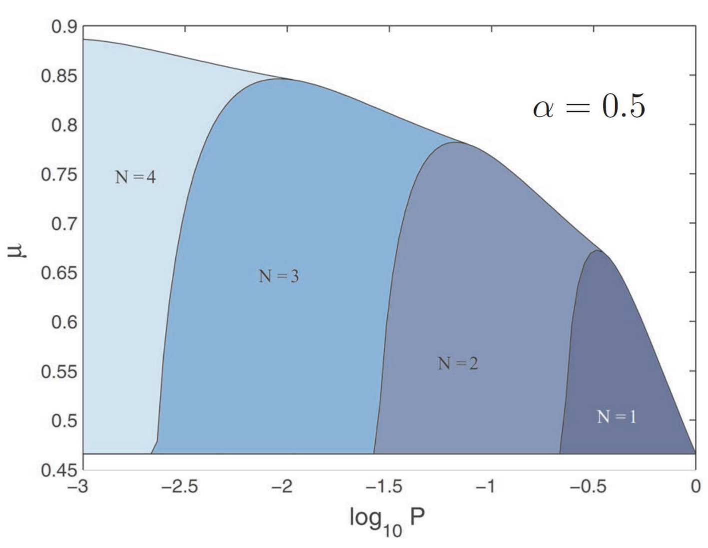
$$\sum_{n=0}^{N} x_n \mathcal{P}_n(\lambda)^2 = \left(P - 1 + \sum_{n=0}^{N} x_n\right)\mathcal{P}_N(\lambda)^2$$

OPTIMAL PHASE MEASUREMENT FOR COHERENT STATES

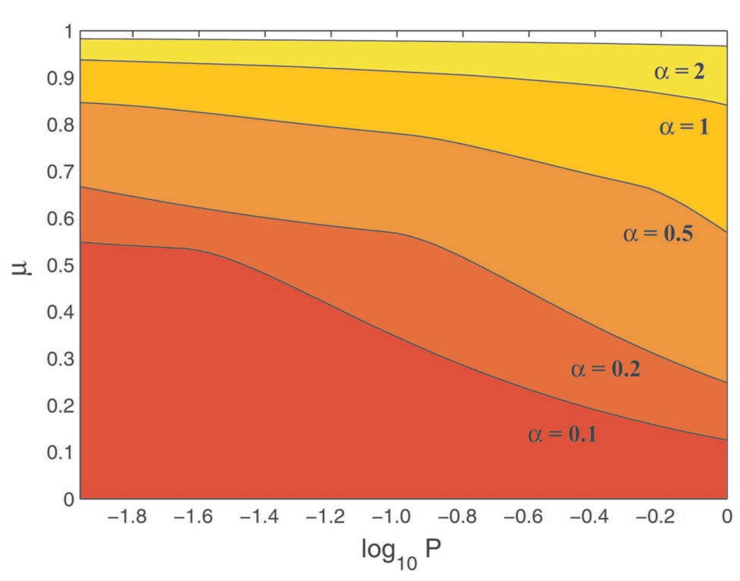
- Coherent states are localized
  - the optimal finite filter always exists
- For some success probabilities there are multiple physical filters with one of them being the optimal one.



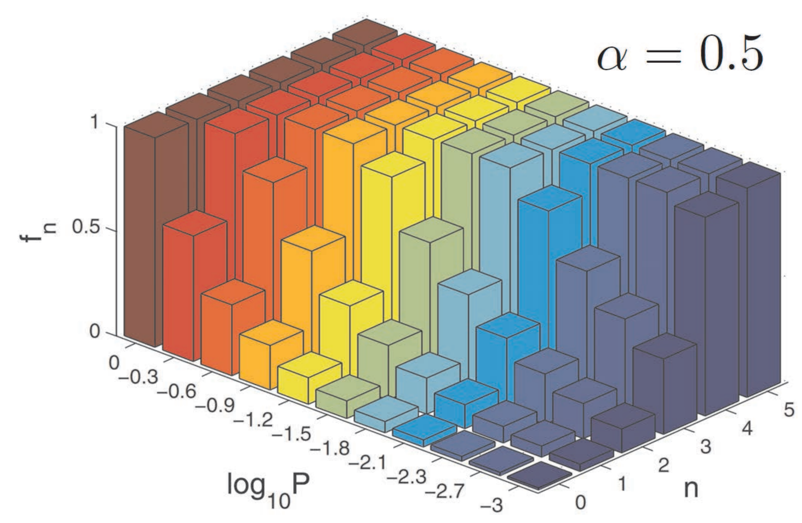
Numbers of non-unit filter parameters.



Effect of filters with different numbers of non-unit filter parameters



Comparison of effects of optimal filters for coherent states with different amplitudes



Explicit values of the filter parameters.

ACKNOWLEDGEMENTS

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