Relational Division in Rank-Aware Databases: An Overview, Issues, and New Directions

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Rank-Aware Databases

- Rank-aware databases allow imperfect matches.
- Each tuple is annotated by a score – a number quantifying how much a tuple matches the query.

Example

id	*car*	*color*	*price*
1	Honda Civic	blue	$18.500
2	Mazda 323F	dark red	$17.500
3	Toyota Celica	light blue	$21.000
4	Toyota Corolla	blue	$20.000

Intended query: Get all blue cars with price around $20.000.

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Query: Get all blue cars with price between $19.000 and $21.000.

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Example

score	*id*	*car*	*color*	*price*
1.00	4	Toyota Corolla	blue	\$20.000
0.85	1	Honda Civic	blue	\$18.500
0.80	3	Toyota Celica	light blue	\$21.000
0.15	2	Mazda 323F	dark red	\$17.500

Query: Get all blue cars with price around \$20.000.

Note: We call such table a "ranked data table" (RDT).

Approach Based on Residuated Lattices

- There are many approaches to rank-aware databases that differ in the treatment of the scores of tuples.
- In our model the tuple scores come from residuated lattices.
- Our model can be seen as a generalization of Codd's relational model of data.

Definition

Residuated lattice is a general algebra of the form

$$\mathbf{L} = \langle L, \wedge, \vee, \otimes, \rightarrow, 0, 1 \rangle$$

where

- $\langle L, \wedge, \vee, 0, 1 \rangle$ is a bounded lattice,
- $\langle L, \otimes, 1 \rangle$ is a commutative monoid,
- multiplication $\otimes$ and residuum $\rightarrow$ satisfy adjointness property.

Query Systems

Two basic types of query systems:

1. system based on evaluating predicate formulas,
2. system consisting of relational operations.

For convenience:

Abbreviation RS denotes relational schema $R \cup S$ with $R \cap S = \emptyset$.

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Examples of relational operations in our model

- Natural Join of RDTs $\mathcal{D}_1$ (on RS) and $\mathcal{D}_2$ (on ST) is an RDT on RST given by

$$(\mathcal{D}_1 \bowtie \mathcal{D}_2)(rst) = \mathcal{D}_1(rs) \otimes \mathcal{D}_2(st)$$

for all $r \in \mathrm{Tupl}(R), s \in \mathrm{Tupl}(S), t \in \mathrm{Tupl}(T)$.

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- Projection of RDT $\mathcal{D}$ (on R) onto $S \subseteq R$ is defined by

$$(\pi_S(\mathcal{D}))(s) = \bigvee_{t \in \mathrm{Tupl}(R \setminus S)} \mathcal{D}(st)$$

for all $s \in \mathrm{Tupl}(S)$.

"Some φ Is ψ" Query

- Query: Some tuples from $\mathcal{D}_1$ are matching tuples in $\mathcal{D}_2$.
- Such queries can be expressed using joins and projections.

Example

Employees

name	...
Carolyn	...
Amy	...
Jean	...
Elizabeth	...

Teaching assignments

name	*course*
Amy	Database systems
Amy	Algorithms
Jean	Compilers
Peter	Machine Learning

Query: Employees that teach some courses.

- $\pi_{\{\text{name},\ldots\}}(\text{Employees} \bowtie \text{Teaching assignments})$
- $\text{Employees} \bowtie \pi_{\{\text{name}\}}(\text{Teaching assignments})$

"All φ Are ψ" Query

- In the classical model this type of query is connected to relational division.

Example

Supplies

supplier	*product*
Logitech	Keyboard
Logitech	Microphone
Logitech	Webcam
IBM	Keyboard
IBM	Webcam

Required Products

product
Keyboard
Microphone
Webcam

Query: Suppliers that supply all required products.

- Supplies $\div$ Required Products

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IBM	Keyboard
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Required Products

product
Keyboard
Microphone
Webcam

Query: Suppliers that supply all required products.

- Supplies $\div$ Required Products =

supplier
Logitech

Codd-style Division

- The Codd-style division is an initial operation in the class of division-like operations.
- Its presence in the classical relational algebra is not necessary due to the fact that $(\forall x)\varphi \equiv \neg(\exists x)\neg\varphi$.
- In the classical model we consider division as a derived operation expressed by the means of set difference and projection.

Definition

For relation $\mathcal{D}_1$ on RS and relation $\mathcal{D}_2$ on S, the Codd-style division may be introduced as

$$\mathcal{D}_1 \div_{\mathsf{Codd}} \mathcal{D}_2 = \pi_R(\mathcal{D}_1) \setminus \pi_R((\pi_R(\mathcal{D}_1) \bowtie \mathcal{D}_2) \setminus \mathcal{D}_1).$$

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   If φ is $s \in \mathcal{D}_2$ and ψ is $rs \in \mathcal{D}_1$, then

   $$(\forall s)(s \in \mathcal{D}_2 \Rightarrow rs \in \mathcal{D}_1)$$

   is true for all $r \in \mathrm{Tupl}(R)$ if $\mathcal{D}_2$ is empty.

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   The result of Codd-style division is always a subset of $\pi_R(\mathcal{D}_1)$ and can be characterized as

   $$\mathcal{D}_1 \div_{\mathrm{Codd}} \mathcal{D}_2 = \{r \in \pi_R(\mathcal{D}_1) \mid \text{for all } s \in \mathcal{D}_2, \text{ we have } rs \in \mathcal{D}_1\}.$$

Division in Rank-Aware Databases

- Most common approaches to rank-aware databases found in literature introduce the division by

$$(\mathcal{D}_1 \div \mathcal{D}_2)(r) = \bigwedge_{s \in \mathrm{Tupl}(S)} (\mathcal{D}_2(s) \rightarrow \mathcal{D}_1(rs))$$

  for all $r \in \mathrm{Tupl}(R)$, where $\mathcal{D}_1$ is on RS and $\mathcal{D}_2$ is on S.
- This definition solves the second issue of Codd-style division, but it is domain-dependent. If there is an attribute in R, that has an infinite domain, the result of division can be infinite.
- To overcome this issue, some approaches use this definition with additional assumption

$$(\pi_R(\mathcal{D}_1))(r) > 0,$$

  which introduces the second issue again.

Division – Our Approach

- In our setting, $(\forall x)\varphi \equiv \neg(\exists x)\neg\varphi$ does not hold in general. We need to consider the division as a fundamental operation.
- We have proposed domain-independent division, that suffices to establish relational completeness and does not suffer from the second issue of Codd-style division.

Definition

Let $\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3$ be RDTs on R, S and RS. Division $\mathcal{D}_1 \div^{\mathcal{D}_3} \mathcal{D}_2$ of $\mathcal{D}_1$ by $\mathcal{D}_2$ which ranges over $\mathcal{D}_3$ is an RDT on R defined by

$$(\mathcal{D}_1 \div^{\mathcal{D}_3} \mathcal{D}_2)(r) = \bigwedge_{s \in \mathrm{Tupl}(S)} (\mathcal{D}_3(r) \otimes (\mathcal{D}_2(s) \to \mathcal{D}_1(rs)))$$

for each $r \in \mathrm{Tupl}(R)$.

Date's Small Divide

- In order to overcome the second issue of Codd-style division, Date proposed division operation called Small Divide. Later, it has been further extended to eliminate the first issue as well.
- For relations $\mathcal{D}_1$ on R (dividend), $\mathcal{D}_2$ on S (divisor) and $\mathcal{D}_3$ on RS (mediator), the Small Divide can be characterized by

$$\mathcal{D}_1 \div^{\mathcal{D}_3}_{\mathrm{sdo}} \mathcal{D}_2 = \{r \in \mathcal{D}_1 \mid \text{for all } s \in \mathcal{D}_2, \text{ we have } rs \in \mathcal{D}_3\}.$$

- Its generalization in our model is straightforward:

$$(\mathcal{D}_1 \div^{\mathcal{D}_3}_{\mathrm{gsdo}} \mathcal{D}_2)(r) = \mathcal{D}_1(r) \otimes \bigwedge_{s \in \mathrm{Tupl}(S)} (\mathcal{D}_2(s) \rightarrow \mathcal{D}_3(rs)).$$

- If the residuated lattice is prelinear or divisible, division in our style is equivalent to generalized Small Divide.

Todd's Division, Date's Great Divide

- To overcome the first issue of Codd-style division, in the past Todd proposed the following: For $\mathcal{D}_1$ on RS and $\mathcal{D}_2$ on ST

$$\mathcal{D}_1 \div_{\text{Todd}} \mathcal{D}_2 = \{rt \in \pi_R(\mathcal{D}_1) \bowtie \pi_T(\mathcal{D}_2) \mid \forall s\colon \text{ if } st \in \mathcal{D}_2 \text{ then } rs \in \mathcal{D}_1\}$$

- This definition suffers from the second issue. Date proposed Great Divide in similar fashion as Small Divide to overcome this issue. For $\mathcal{D}_1$ on R, $\mathcal{D}_2$ on S, $\mathcal{D}_3$ on RS and $\mathcal{D}_4$ on ST:

$$\mathcal{D}_1 \div_{\text{gdo}}^{\mathcal{D}_3,\mathcal{D}_4} \mathcal{D}_2 = \{rt \in \mathcal{D}_1 \bowtie \mathcal{D}_2 \mid \forall s\colon \text{ if } st \in \mathcal{D}_4 \text{ then } rs \in \mathcal{D}_3\}$$

Darwen's Divide

- Darwen's division is based on Great Divide, but do not pose any requirements on relation schemes.
- For relations $\mathcal{D}_1$ on R_1 (*dividend*), $\mathcal{D}_2$ on R_2 (*divisor*), $\mathcal{D}_3$ on R_3 (*first mediator*), and $\mathcal{D}_4$ on R_4 (*second mediator*), we put

$$\mathcal{D}_1 \div_{\mathrm{ddo}}^{\mathcal{D}_3,\mathcal{D}_4} \mathcal{D}_2 = (\mathcal{D}_1 \bowtie \mathcal{D}_2) \mathbin{\bar{\ltimes}} ((\mathcal{D}_1 \bowtie \mathcal{D}_4) \mathbin{\bar{\ltimes}} \mathcal{D}_3).$$

- Written in set notation:

$$\begin{aligned}\mathcal{D}_1 \div_{\mathrm{ddo}}^{\mathcal{D}_3,\mathcal{D}_4} \mathcal{D}_2 = \big\{& r_1 r_2 \in \mathcal{D}_1 \bowtie \mathcal{D}_2 \mid \\ & \text{for all } r_4 \in \mathcal{D}_4\text{: if } r_1 r_2 \between r_4\text{, then there is } r_3 \in \mathcal{D}_3\text{: } r_1 r_4 \between r_3\big\},\end{aligned}$$

- The relation scheme of result of Darwen's Divide is $R_1 \cup R_2$ since R_1 and R_2 are arbitrary and might have some attributes in common.

Darwen's Divide – graded variant

- We may introduce a graded variant $\div_{\text{gddo}}$ of the Darwen's Divide as follows

$$\big(\mathcal{D}_1 \div_{\text{gddo}}^{\mathcal{D}_3,\mathcal{D}_4} \mathcal{D}_2\big)(r_1 r_2) =$$

$$= \mathcal{D}_1(r_1) \otimes \mathcal{D}_2(r_2) \otimes \bigwedge_{\substack{r_4 \in \text{Tupl}(R_4) \\ r_1 r_2 \between r_4}} \Big(\mathcal{D}_4(r_4) \rightarrow \bigvee_{\substack{r_3 \in \text{Tupl}(R_3) \\ r_1 r_4 \between r_3}} \mathcal{D}_3(r_3)\Big)$$

$$= (\mathcal{D}_1 \bowtie \mathcal{D}_2)(r_1 r_2) \otimes \bigwedge_{\substack{r_4 \in \text{Tupl}(R_4) \\ r_1 r_2 \between r_4}} \Big(\mathcal{D}_4(r_4) \rightarrow \bigvee_{\substack{r_3 \in \text{Tupl}(R_3) \\ r_1 r_4 \between r_3}} \mathcal{D}_3(r_3)\Big)$$

- Semantics of the operation depends on the relations schemes of input relations.

Darwen's Divide – graded variant, contd.

- The condition of joinability is not necessary and can be avoided:

$$\left(\mathcal{D}_1 \div_{\mathrm{gddo}}^{\mathcal{D}_3,\mathcal{D}_4} \mathcal{D}_2\right)(r_1 r_2) =$$

$$= (\mathcal{D}_1 \bowtie \mathcal{D}_2)(r_1 r_2) \otimes \bigwedge_{\substack{r_4 \in \mathrm{Tupl}(R_4) \\ r_1 r_2 \between r_4}} \Big(\mathcal{D}_4(r_4) \to \bigvee_{\substack{r_3 \in \mathrm{Tupl}(R_3) \\ r_1 r_4 \between r_3}} \mathcal{D}_3(r_3)\Big)$$

$$= (\mathcal{D}_1 \bowtie \mathcal{D}_2)(r_1 r_2) \otimes \bigwedge_{r'_4 \in \mathrm{Tupl}(R_{4\setminus 12})} \Big(\mathcal{D}_4(r_{12}^{\between 4} r'_4) \to \bigvee_{r'_3 \in \mathrm{Tupl}(R_{3\setminus 14})} \mathcal{D}_3(r_{14}^{\between 3} r'_3)\Big)$$

$$= (\mathcal{D}_1 \bowtie \mathcal{D}_2)(r_1 r_2) \otimes \bigwedge_{r'_4 \in \mathrm{Tupl}(R_{4\setminus 12})} \Big(\mathcal{D}_4(r_{12}^{\between 4} r'_4) \to \pi_{R_{3\cap 14}}(\mathcal{D}_3)(r_{14}^{\between 3})\Big),$$

- where $r_{12}^{\between 4} = (r_1 r_2)(R_{4\cap 12})$ and $r_{14}^{\between 3} = (r_1 r_4)(R_{3\cap 14})$.

Relationship among the operators – easy part

- If **L** is prelinear or divisible, then division in our sense is equivalent to Date's Small Divide. We have

$$\mathcal{D}_3 \div^{\mathcal{D}_1} \mathcal{D}_2 = \mathcal{D}_1 \div^{\mathcal{D}_3}_{\mathrm{gsdo}} \mathcal{D}_2$$

- As in the classical case, Date's Small Divide is expressible by Date's Great Divide in the following way:

$$\mathcal{D}_1 \div^{\mathcal{D}_3}_{\mathrm{gsdo}} \mathcal{D}_2 = \mathcal{D}_1 \div^{\mathcal{D}_3,\mathcal{D}_2}_{\mathrm{ggdo}} 1_{\emptyset}.$$

- Darwen's Divide is directly applicable to relations that conform to requirements imposed by Date's Great Divide with the same semantics. We have

$$\mathcal{D}_1 \div^{\mathcal{D}_3,\mathcal{D}_4}_{\mathrm{ggdo}} \mathcal{D}_2 = \mathcal{D}_1 \div^{\mathcal{D}_3,\mathcal{D}_4}_{\mathrm{gddo}} \mathcal{D}_2.$$

Relationship among the operators – trickier part

- In order to show further relationships among the division operations we have introduced new relationally complete query language – Pseudo Tuple Calculus
- PTC provides easy way to express any relational operation utilizing its set notation.
- Such PTC expression is semantically equivalent to the original RA operation.
- Using relational completeness we get the equivalent formulation of this operation utilizing just the fundamental RA operations.

Relationship among the operators – trickier part

- Example: Consider the division operation in our sense, i. e., for RDTs $\mathcal{D}_1$, $\mathcal{D}_2$, and $\mathcal{D}_3$ on RS, S, and R, we put

$$\big(\mathcal{D}_1 \div^{\mathcal{D}_3} \mathcal{D}_2\big)(r) = \bigwedge\nolimits_{s \in \mathrm{Tupl}(S)} \big(\mathcal{D}_3(r) \otimes (\mathcal{D}_2(s) \rightarrow \mathcal{D}_1(rs))\big),$$

  for each $r \in \mathrm{Tupl}(R)$.

- Consider relation symbols $\mathbb{D}_1, \mathbb{D}_2$ and $\mathbb{D}_3$ on RS, S, and R, respectively. Then the PTC-expression

$$\mathcal{T}(\mathbb{r}) = \bigwedge\nolimits_{\mathbb{s}} \big(\mathbb{D}_3(\mathbb{r}) \otimes (\mathbb{D}_2(\mathbb{s}) \rightarrow \mathbb{D}_1(\mathbb{r}\mathbb{s}))\big),$$

  is semantically equivalent to the division operation. More precisely, for a database instance $\boldsymbol{\mathcal{D}}$ such that $\mathbb{D}_1^{\boldsymbol{\mathcal{D}}} = \mathcal{D}_1$, $\mathbb{D}_2^{\boldsymbol{\mathcal{D}}} = \mathcal{D}_2$, and $\mathbb{D}_3^{\boldsymbol{\mathcal{D}}} = \mathcal{D}_3$ we have

$$\big(\mathcal{D}_1 \div^{\mathcal{D}_3} \mathcal{D}_2\big)(r) = \mathcal{T}^{\boldsymbol{\mathcal{D}}}(r)$$

  for all $r \in \mathrm{Tupl}(R)$.

Relationship among the operators – trickier part

Let $\mathcal{D}_1$, $\mathcal{D}_2$, and $\mathcal{D}_3$ be RDTs on RS, S, and R, respectively, and let $\div_{\mathrm{gsdo}}$ be Date's Small Divide. For the division operation in our sense we have

$$\big(\mathcal{D}_1 \div^{\mathcal{D}_3} \mathcal{D}_2\big)(r) = (\mathcal{E}_R^{\boldsymbol{\mathcal{D}}} \div_{\mathrm{gsdo}}^{E^{\boldsymbol{\mathcal{D}}}} \mathcal{E}_S^{\boldsymbol{\mathcal{D}}})(r),$$

for all $r \in \mathrm{Tupl}(R)$ where

$$E^{\boldsymbol{\mathcal{D}}} = \mathcal{D}_3 \bowtie \big((\mathcal{D}_2 \bowtie \mathcal{E}_R^{\boldsymbol{\mathcal{D}}})^{\mathcal{E}_{RS}^{\boldsymbol{\mathcal{D}}}} \mathcal{D}_1\big)$$

and the extended active domains $\mathcal{E}_R^{\boldsymbol{\mathcal{D}}}, \mathcal{E}_S^{\boldsymbol{\mathcal{D}}}$, and $\mathcal{E}_{RS}^{\boldsymbol{\mathcal{D}}}$ contain tuples built only from the values from relations $\mathcal{D}_1, \mathcal{D}_2$, and $\mathcal{D}_3$.

Relationship among the operators – trickier part

Let $\mathcal{D}_1$, $\mathcal{D}_2$, and $\mathcal{D}_3$ be RDTs on R, S, and RS, respectively, and let $\div$ be the division operation in our sense. For Date's Small Divide we have

$$\big(\mathcal{D}_1 \div_{\mathrm{gsdo}}^{\mathcal{D}_3} \mathcal{D}_2\big)(r) = \big(\mathcal{D}_1 \bowtie (E^{\boldsymbol{\mathcal{D}}} \div^{\mathcal{E}_R^{\boldsymbol{\mathcal{D}}}} \mathcal{E}_S^{\boldsymbol{\mathcal{D}}})\big)(r),$$

for all $r \in \mathrm{Tupl}(R)$ where

$$E^{\boldsymbol{\mathcal{D}}} = \big((\mathcal{D}_2 \bowtie \mathcal{E}_R^{\boldsymbol{\mathcal{D}}})^{\mathcal{E}_{RS}^{\boldsymbol{\mathcal{D}}}} \mathcal{D}_3\big)$$

and the extended active domains $\mathcal{E}_R^{\boldsymbol{\mathcal{D}}}, \mathcal{E}_S^{\boldsymbol{\mathcal{D}}}$, and $\mathcal{E}_{RS}^{\boldsymbol{\mathcal{D}}}$ contain tuples built only from the values from relations $\mathcal{D}_1, \mathcal{D}_2$, and $\mathcal{D}_3$.

Relationship among the operators – trickier part

Let $\mathcal{D}_1$, $\mathcal{D}_2$, $\mathcal{D}_3$, and $\mathcal{D}_4$ be RDTs on R_1, R_2, R_3, and R_4, respectively, and let $\div$ be the division operation defined by (**??**). For Darwen's Divide we have

$$\big(\mathcal{D}_1 \div^{\mathcal{D}_3,\mathcal{D}_4}_{\mathrm{gddo}} \mathcal{D}_2\big)(r) = \big((\mathcal{D}_1 \bowtie \mathcal{D}_2) \bowtie (E^{\boldsymbol{\mathcal{D}}} \div^{\mathcal{E}^{\boldsymbol{\mathcal{D}}}_{R'_1}} \mathcal{E}^{\boldsymbol{\mathcal{D}}}_{R'_2})\big)(r),$$

for all $r \in \mathrm{Tupl}(R_1 \cup R_2)$ where

$$\begin{aligned} E^{\boldsymbol{\mathcal{D}}} &= \big((\mathcal{D}_4 \bowtie \mathcal{E}^{\boldsymbol{\mathcal{D}}}_{R'_3})^{\mathcal{E}^{\boldsymbol{\mathcal{D}}}_{R'_4}}(\pi_{R'_3}(\mathcal{D}_3) \bowtie \mathcal{E}^{\boldsymbol{\mathcal{D}}}_{R_4})\big), \\ R'_1 &= (R_4 \cap (R_1 \cup R_2)) \cup (R_1 \cap R_3), \\ R'_2 &= R_4 \setminus (R_1 \cup R_2), \\ R'_3 &= R_3 \cap (R_1 \cup R_4), \\ R'_4 &= R_4 \cup (R_1 \cap R_3) \end{aligned}$$

and the extended active domains contain tuples built only from the values from relations $\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3$, and $\mathcal{D}_4$.

Note on Graded Subsethood

- Graded subsethood can be seen as a relational operation on two RDTs $\mathcal{D}_1, \mathcal{D}_2$ on the same relation scheme R.
- The result is an RDT on empty relation scheme containing an empty tuple with score that is equal to the degree of $\mathcal{D}_1$ being a subset of $\mathcal{D}_2$. It is given by

$$(\mathcal{D}_1 \subseteq \mathcal{D}_2)(\emptyset) = \bigwedge_{r \in \mathrm{Tupl}(R)} (\mathcal{D}_1(r) \to \mathcal{D}_2(r))$$

- Graded subsethood can be seen as a special case of division in our style, generalized Small Divide, Great Divide or Darwen's Divide.
- The generalized division operations can be expressed using the graded subsethood and the notion of image relations.

Conclusions

- We have presented an overview of various relational division operations and their generalized counterparts.
- Furthermore, we have investigated their relationship with the help of newly introduced relationally complete query language – Pseudo Tuple Calculus.
- We have answered an open question regarding the relationship of Date's Great Divide and Darwen's Divide in the classical model.
- Future research will focus on a model with graded subsethood as a fundamental operation alongisde with the imaging operation.