Driving of a mechanical oscillator by a thermomechanical device

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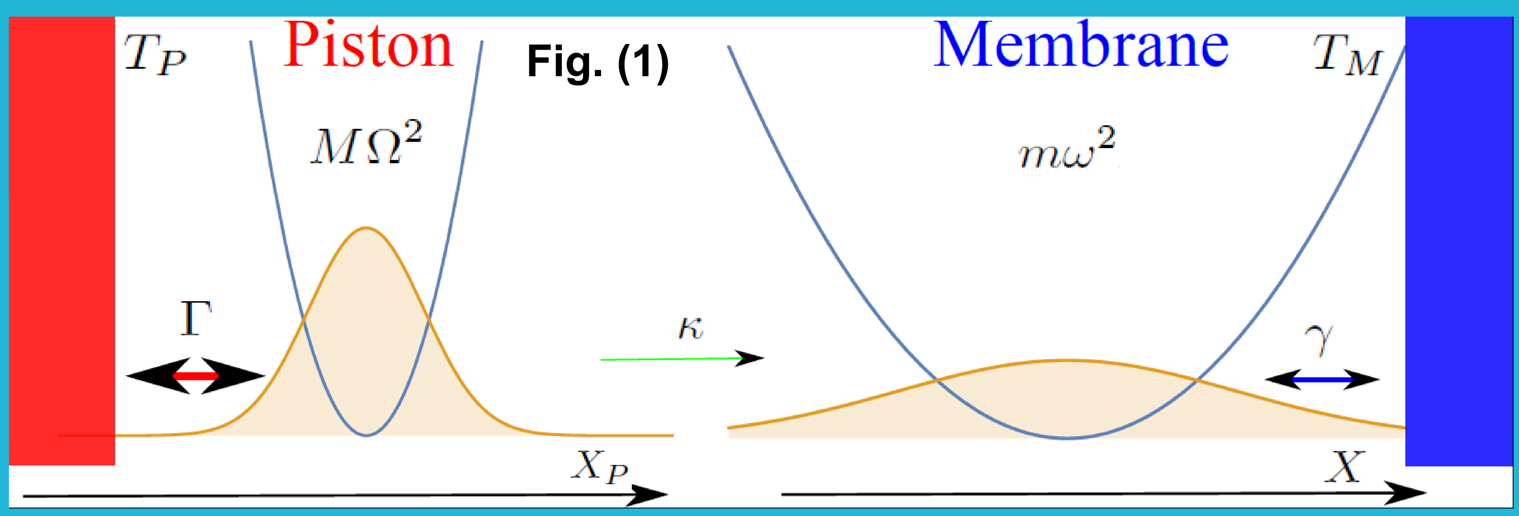
We analyze a situation in which a piston of a thermomechanical device drives a membrane (mechanical oscillator) by a fluctuating mechanical force. By the thermomechanical device we mean an analogue of a classical piston sealing a gas in a cylinder in contact with a heat bath characterized by the temperature T_P. The piston is modeled by an overdamped Brownian particle in a quadratic potential. This combination effectively describes the mechanical behavior of the above mentioned piston+gas. The membrane is directly and linearly coupled to the piston and measured by an auxiliary optical beam to determine its mechanical state after the driving. The membrane is coupled to its heat bath at the temperature T_M. We analyze how the equilibrium dynamics of the piston influences the state and dynamics of the membrane. The goal is to characterize the states of the membrane driven fully mechanically, while the driving is powered purely by the temperature changes of the thermomechanical device.

The scheme:



The notation:

- $X_P \ldots$ piston position
- $M \ldots$ piston mass
- $\Gamma \ldots$ piston damping coefficient
- $\kappa \cdots$ mutual piston-membrane coupling coefficient
- $\Omega \ldots$ piston angular frequency
- $A = c(T_P - T_{P0})$ the displacement caused by the thermal expansion
- $T_P \ldots$ piston bath temperature
- $\xi(t) \ldots$ delta-correlated stationary Gaussian process with zero-mean

- $X \ldots$ membrane position
- $m \ldots$ membrane mass
- $\gamma \ldots$ membrane damping coefficient
- $\omega \ldots$ membrane angular frequency
- $T_M \ldots$ membrane bath temperature

The model: overdamped piston + driven underdamped membrane

$$\begin{aligned} \cancel{M\ddot{X}_P} &= -\Gamma \dot{X}_P - M\Omega^2(X_P - A) + \cancel{\kappa X} + \sqrt{2\Gamma k_B T_P}\,\xi(t) \\ m\ddot{X} &= -\gamma\dot{X} - m\omega^2 X + \kappa X_P + \sqrt{2\gamma k_B T_M}\,\xi(t) \end{aligned}$$

$$\frac{M\Omega^2}{\Gamma\omega} \gg 1$$

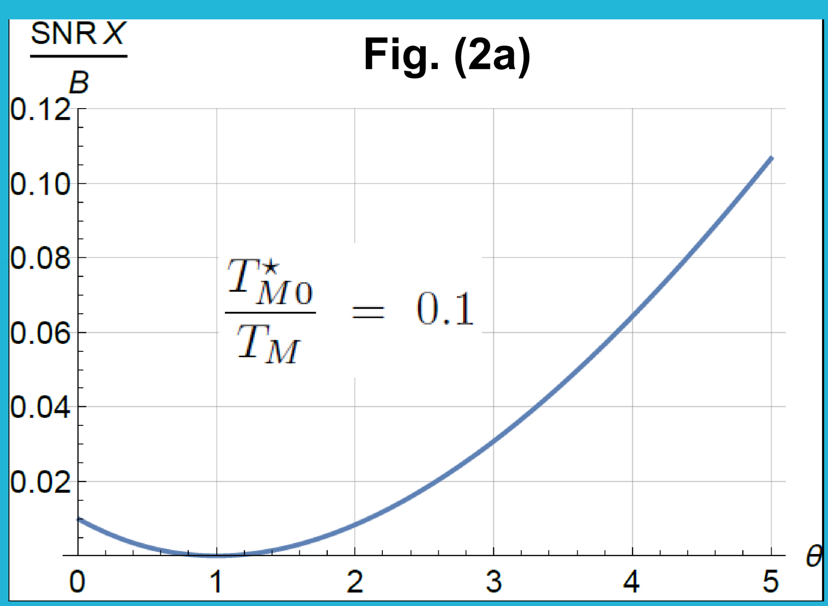
$$m\ddot{X} + \gamma\dot{X} + m\omega^2 X = \kappa\left[A + \frac{\sqrt{2\Gamma k_B T_P}}{M\Omega^2}\xi(t)\right] + \sqrt{2\gamma k_B T_M}\,\xi(t)$$

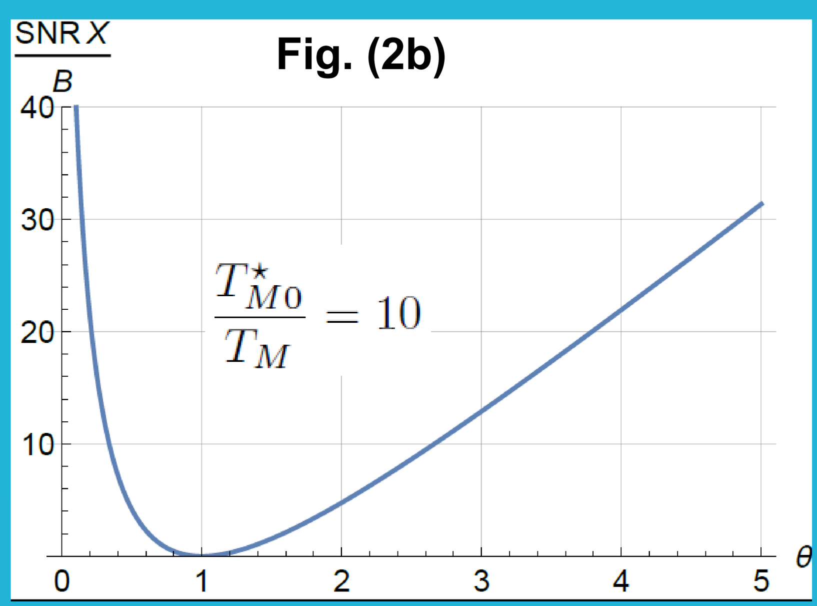
The moments of the stationary process:

$$\begin{aligned} \overline{X} &= \frac{\kappa A}{m\omega^2}, \\ \overline{P} &= m\overline{\dot{X}} = 0, \\ \sigma_X^2 &= \frac{k_B}{m\omega^2}\left(T_M + T_P\frac{\Gamma\kappa^2}{\gamma M^2\Omega^4}\right) \equiv \frac{k_B}{m\omega^2}\left(T_M + T_M^\star\right) \\ \sigma_P^2 &= m k_B\left(T_M + T_P\frac{\Gamma\kappa^2}{\gamma M^2\Omega^4}\right) \equiv m k_B\left(T_M + T_M^\star\right). \end{aligned}$$

The figure of merit for Gaussian states: signal-to-noise ratio (SNR)

$$\mathrm{SNR}(X) \equiv \frac{\overline{X}^2}{\sigma_X^2} = B\left(\frac{T_{M0}^\star}{T_M}\right)^2 \frac{(\theta-1)^2}{\left[1 + \left(\frac{T_{M0}^\star}{T_M}\right)\theta\right]},\quad B = \frac{T_M c^2\gamma^2(M\Omega^2)^4}{m k_B\omega^2\Gamma^2\kappa^2},\quad \theta = \frac{T_P}{T_{P0}}$$



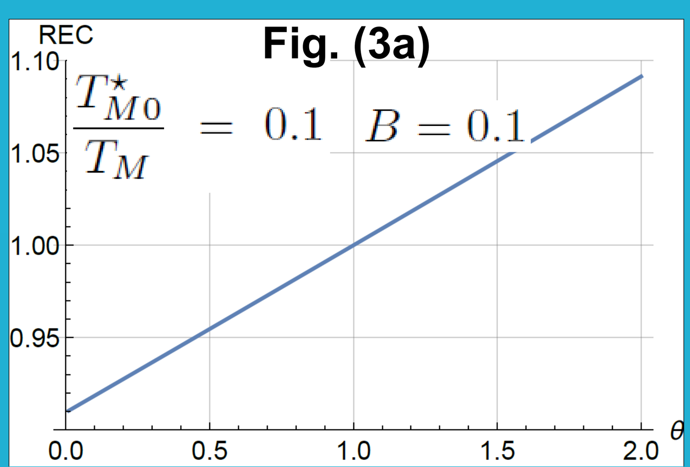


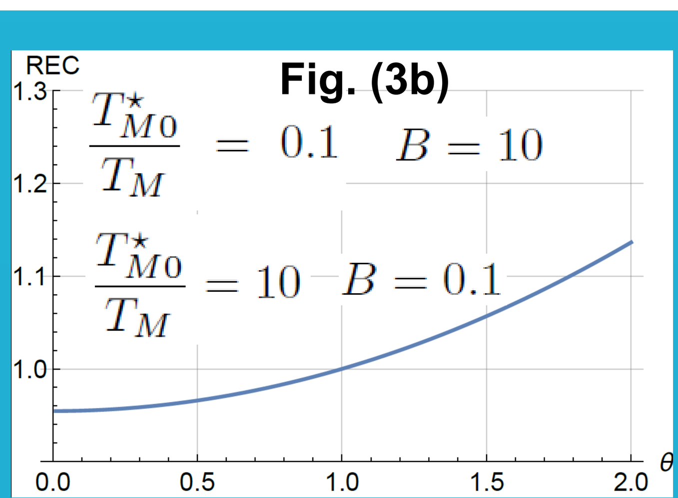
The stationary heat flow through the membrane:

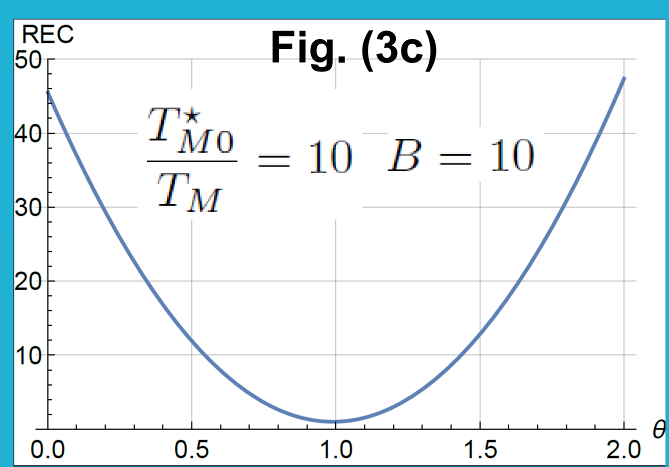
$$\dot{Q}_M = -\frac{\gamma k_B T_M^\star}{m} = -\frac{\gamma k_B T_M}{m}\left(\frac{T_{M0}^\star}{T_M}\right)\theta,\quad \frac{\dot{Q}_M}{\dot{Q}_{M0}} = \theta$$

The relative-energy-change (REC) of the membrane state:

$$\mathrm{REC} \equiv \frac{E}{E_0} = \frac{\frac{1}{2}B\left(\frac{T_{M0}^\star}{T_M}\right)^2(\theta-1)^2 + \left(\frac{T_{M0}^\star}{T_M}\right)\theta + 1}{1 + \left(\frac{T_{M0}^\star}{T_M}\right)}$$







Conclusions:

- **We have described a toy model in which a piston of a thermomechanical device in thermal equilibrium drives a mechanical membrane, Fig. (1).**
- **The piston and membrane were described as classical oscillators under the influence of Langevin forces. The membrane stationary state was characterized by the relevant parameters of the model, namely by its thermodynamic and effective temperatures.**
- **The SNR(X), Figs. (2), of the stationary Gaussian state is analyzed in two regimes. Our results show that: (a) if the thermodynamic temperature of the membrane dominates over the effective one, it is preferable to heat up the driving piston to increase the SNR, (b) if the effective temperature of the membrane dominates over the thermodynamic one, one should better cool down the piston to obtain higher SNR for a given temperature change.**
- **The analysis of the membrane REC, Figs. (3), temperature dependence reveals that one can observe three different regimes: (a) the REC linear dependence on the piston temperature, (b) the REC monotonic quadratic dependence on the piston temperature, (c) the REC non-monotonic quadratic dependence on the piston temperature, where either increase or decrease of the temperature results in the REC increase.**

References: * H.-P. Breuer, F. Petruccione: The Theory of Open Quantum Systems (Oxford, 2002).
* Feynman, Leighton, Sands: The Feynman Lectures on Physics (Addison-Wesley, 1977).

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A model describing the heat powered driving of the stochastic mechanical oscillator (membrane) by another stochastic oscillator was analyzed. Different regimes of the temperature dependence of the signal-to-noise ratio and the relative-energy-change of the membrane stationary state were found. Both the SNR, Fig. (2), and REC, Fig. (3), may non-monotonously depend on the temperature of the driving system.