

Proof-of-principle Test of Continuous-Variable QKD in Free-Space Atmospheric Channel

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Outline

- Continuous-variable QKD
- Fading channels
- Effect on security of CS CV QKD
- Post-selection
- Proof-of-principle test
- Further plans
- Summary

Continuous-variable (CV) QKD



ALICE

Share a secret key between
Alice & Bob for the one-time pad
(Vernam, 1919; Shannon, 1949)

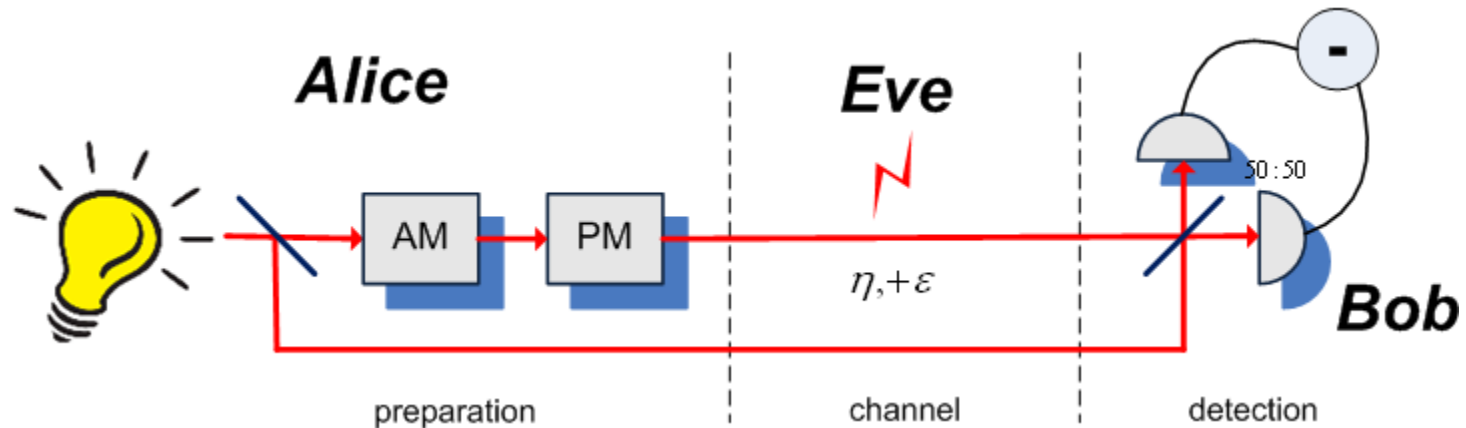


BOB



CV protocols – attempt to go beyond the single photon statistics and use the wave properties of light employing the multiphoton quantum states.

CV Quantum Key Distribution

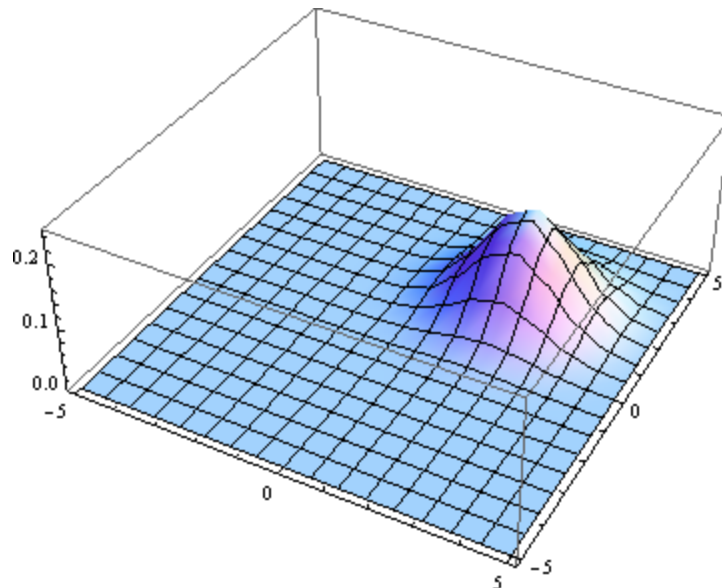
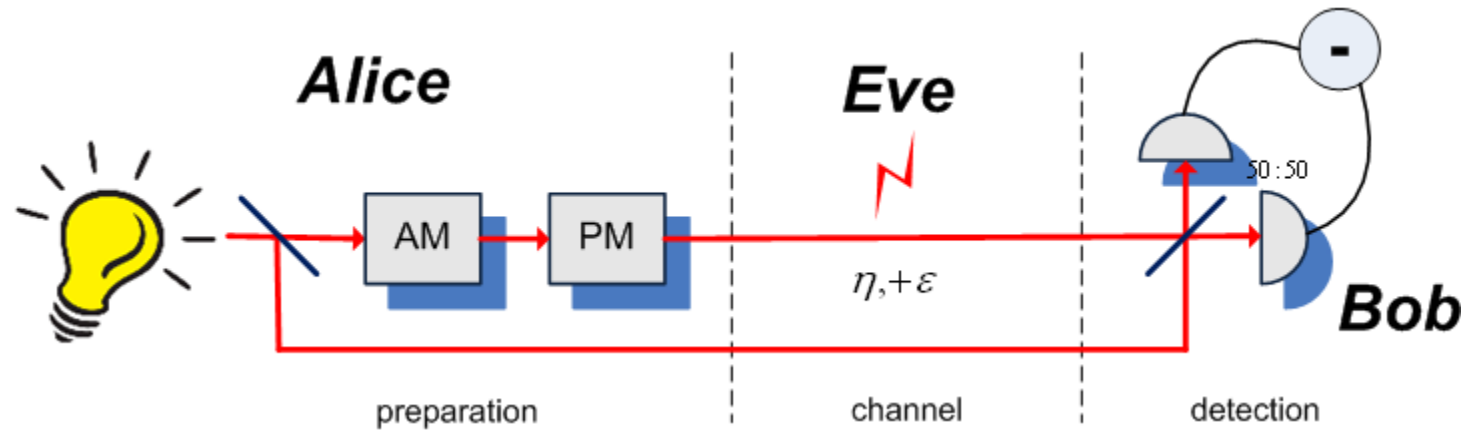


Coherent states-based protocol (GG02)

- Laser source
- Gaussian quadrature modulation
- Homodyne detection

*F. Grosshans and P. Grangier. PRL 88, 057902 (2002);
F. Grosshans et al., Nature 421, 238 (2003)*

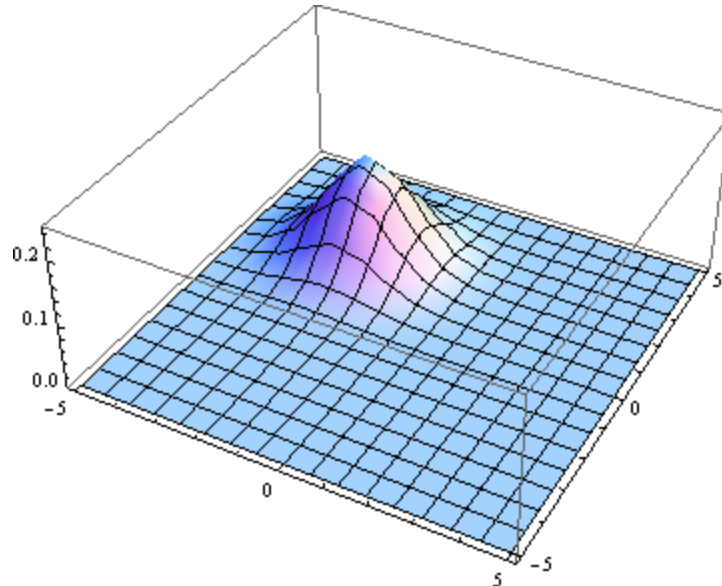
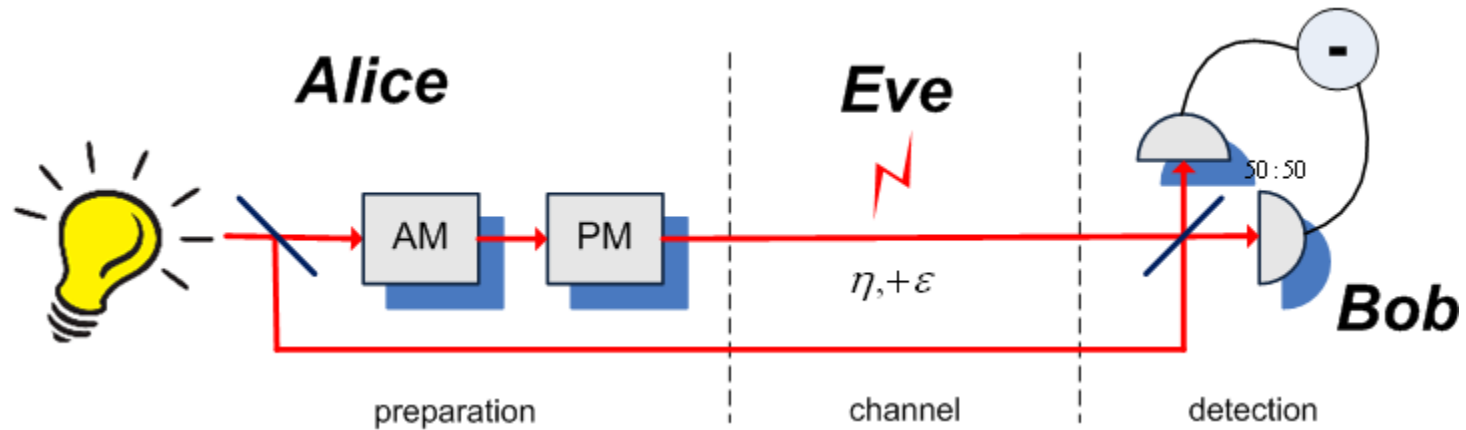
CV Quantum Key Distribution



Coherent states-based protocol

- Alice generates two Gaussian random variables $\{a, b\}$
- Alice prepares a coherent state, displaced by $\{a, b\}$
- Bob measures a quadrature, obtaining a or b
- Bases reconciliation
- Error correction, privacy amplification

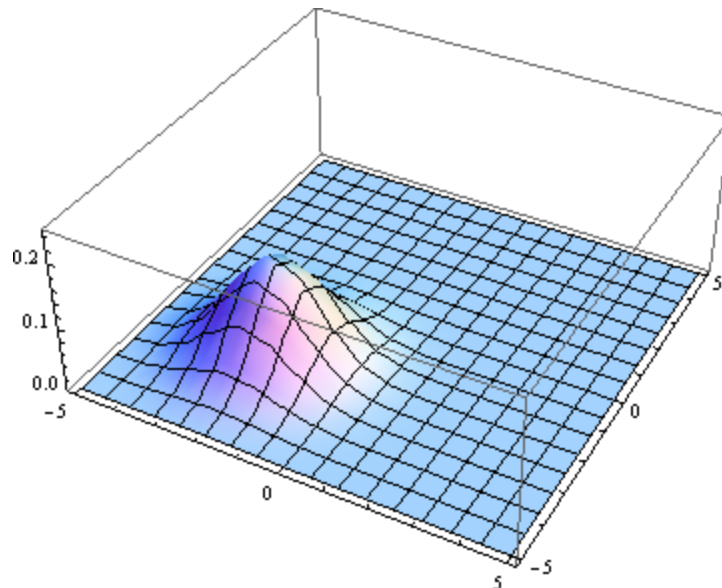
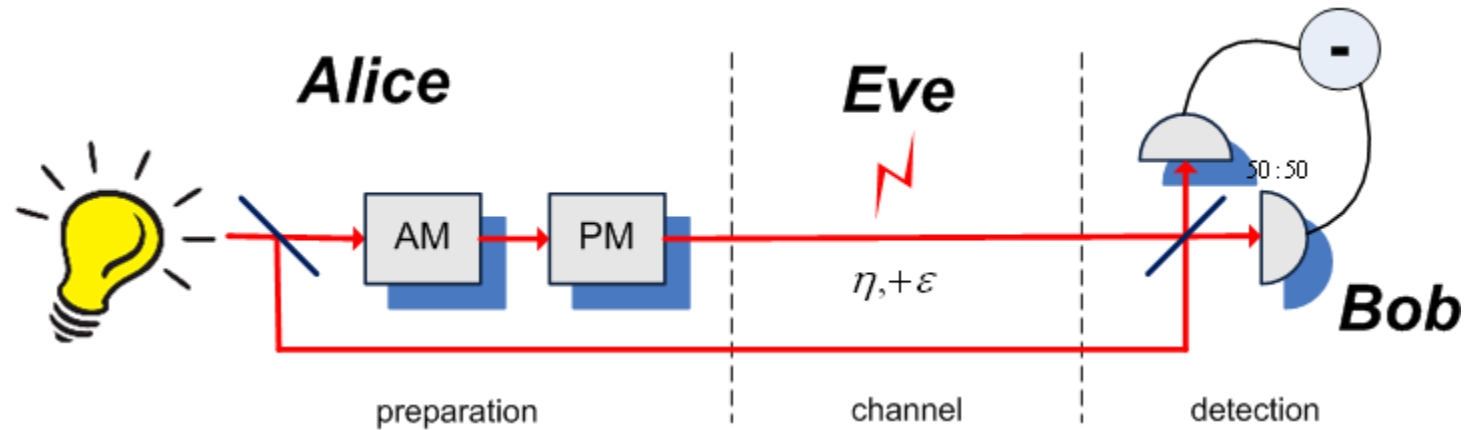
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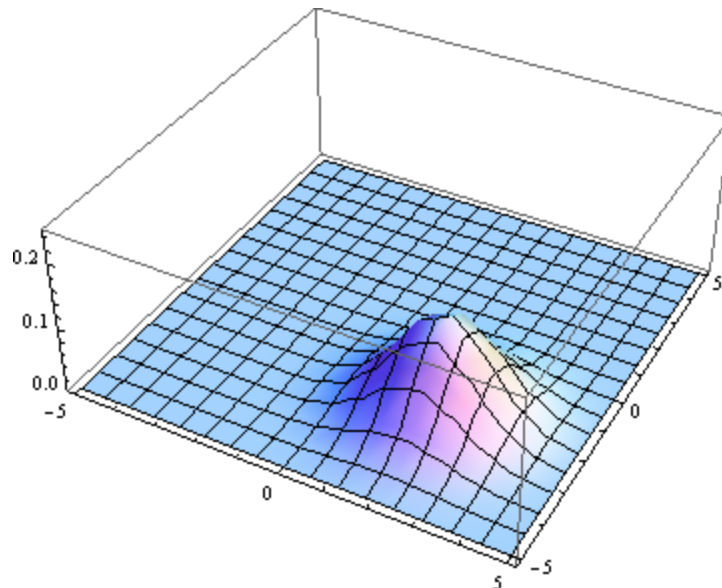
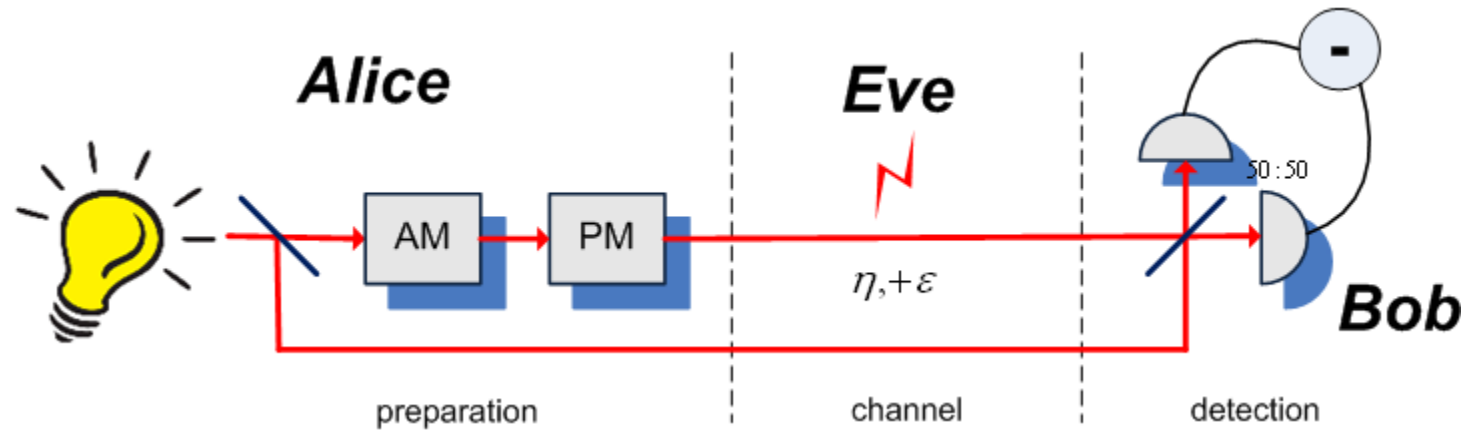
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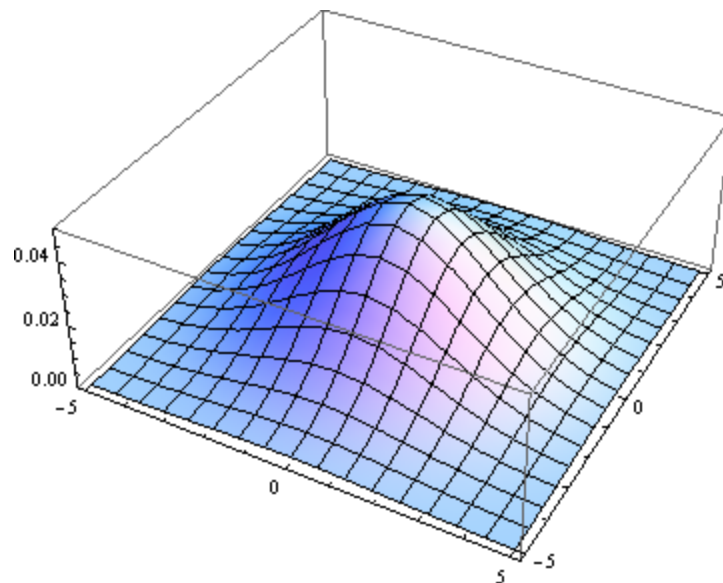
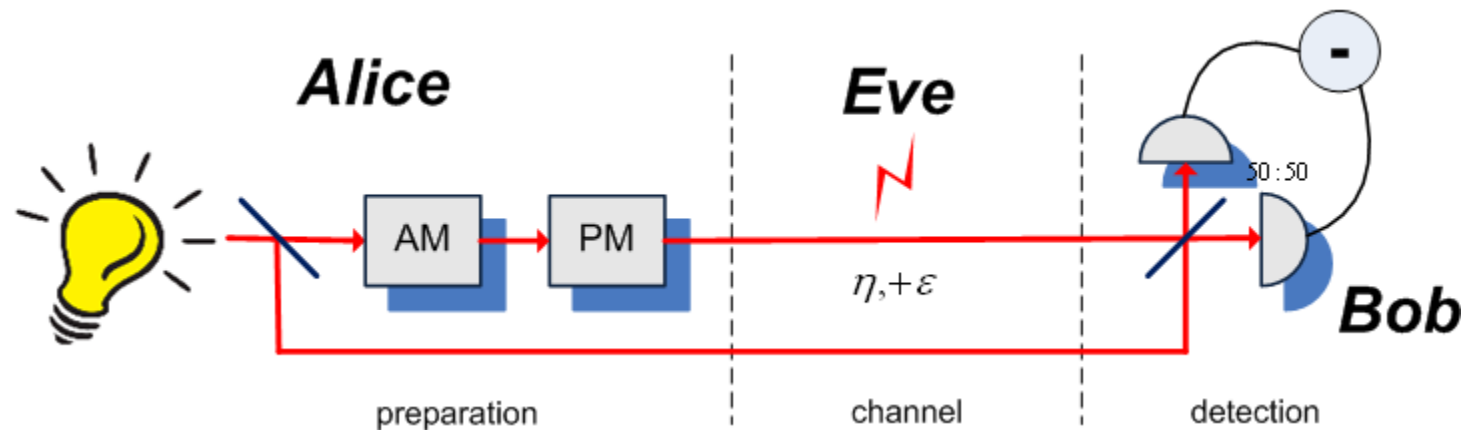
CV Quantum Key Distribution



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CV Quantum Key Distribution

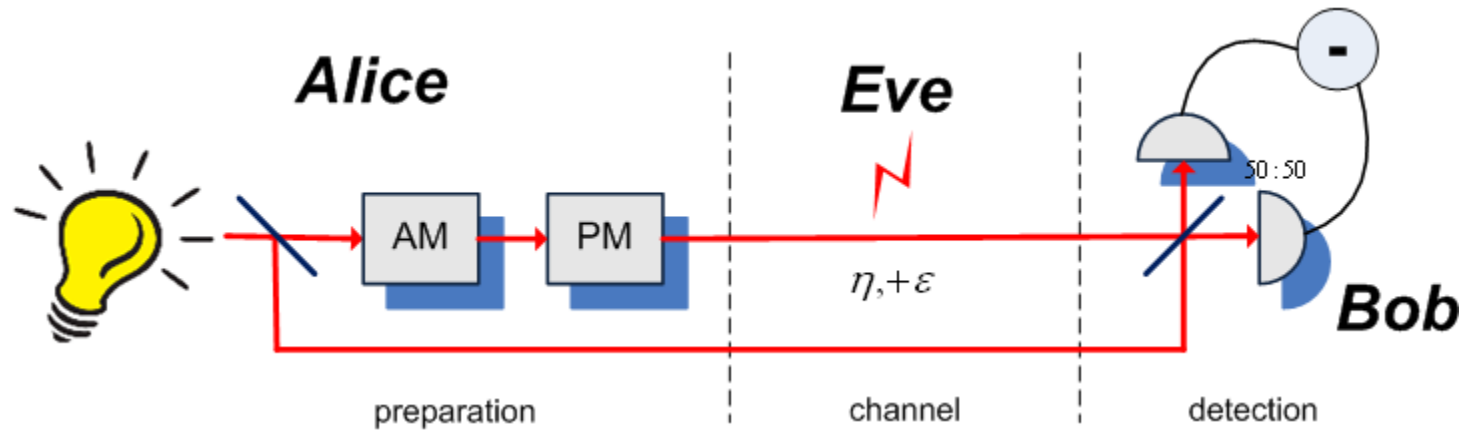


Mixture

Coherent states-based protocol

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CV Quantum Key Distribution



Coherent states-based protocol: achievements

25 km, 2 kbps [J. Lodewyck et al., *PRA* 76, 042305 (2007)]

80 km, ~150 bps [P. Jouguet et al., *Nature Photonics* 7, 378-381 (2013)]

CV Quantum Key Distribution

SECURITY

- Collective attacks: **asymptotic**
[Navascués et al. PRL 97, 190503;
Garcia-Patron & Cerf, PRL 97, 190503]
- finite-size**
[Leverrier & Grangier, PRA 81, 062314;
Leverrier et al. PRA 81, 062343;
Ruppert et al., PRA 90 062310]
- General attacks: **asymptotic / finite-size**
[Leverrier et. al. PRL 110 030502]
- Composable security: **asymptotic**
[Leverrier PRL 114, 070501]

CV Quantum Key Distribution

ADVANTAGES

- Large alphabet, no empty pulses, no need for decoy states
- Efficient mode-selective homodyne detection
- Relative simplicity (possible to do on a chip)

CV Quantum Key Distribution

ISSUES

- Continuous influence of channel imperfections (subject to analysis)
- Possible attacks on local oscillator (can be monitored/self-referenced/classically locked)
- Side channels and other loopholes ([M]DI CV QKD possible)
- Post-processing (mainly solved)

Environment for CV QKD

- Attenuating channels (fiber-optical links)
- Channels with excess noise (fiber links+noise)
- Fluctuating channels (atmospheric links)

Environment for CV QKD

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Environment for CV QKD

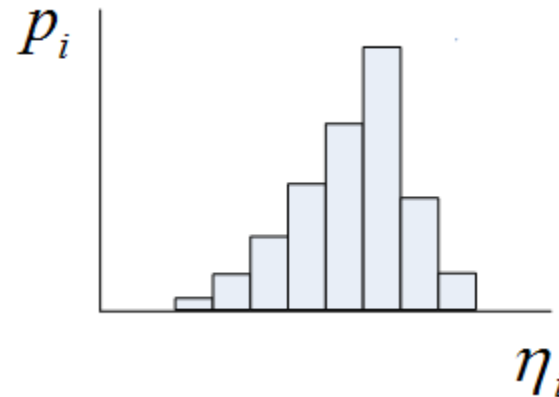
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Our study

Analysis of CV QKD in free-space channels, proof-of-principle test.

Fading channels

Described by the distributions of transmittance values $\{\eta_i\}$ and respective probabilities $\{p_i\}$:



Fading is typically observed in atmospheric channels, where it is caused by the turbulence effects.

Security of CV QKD

Individual attacks: $I_i = I_{AB} - I_{BE}$

$$I_{AB} = \frac{1}{2} \log_2 \frac{V_B}{V_{B|A}} \quad I_{BE} = \frac{1}{2} \log_2 \frac{V_B}{V_{B|E}}$$

Collective attacks: $I = I_{AB} - \chi_{BE}$

Holevo quantity: $\chi_{BE} = S_E - \int P(B) S_{E|B} dB$ $\chi_{BE} = S(\rho_E) - S(\rho_{E|B})$

In case of channel noise Eve is assumed to be able to hold purification:

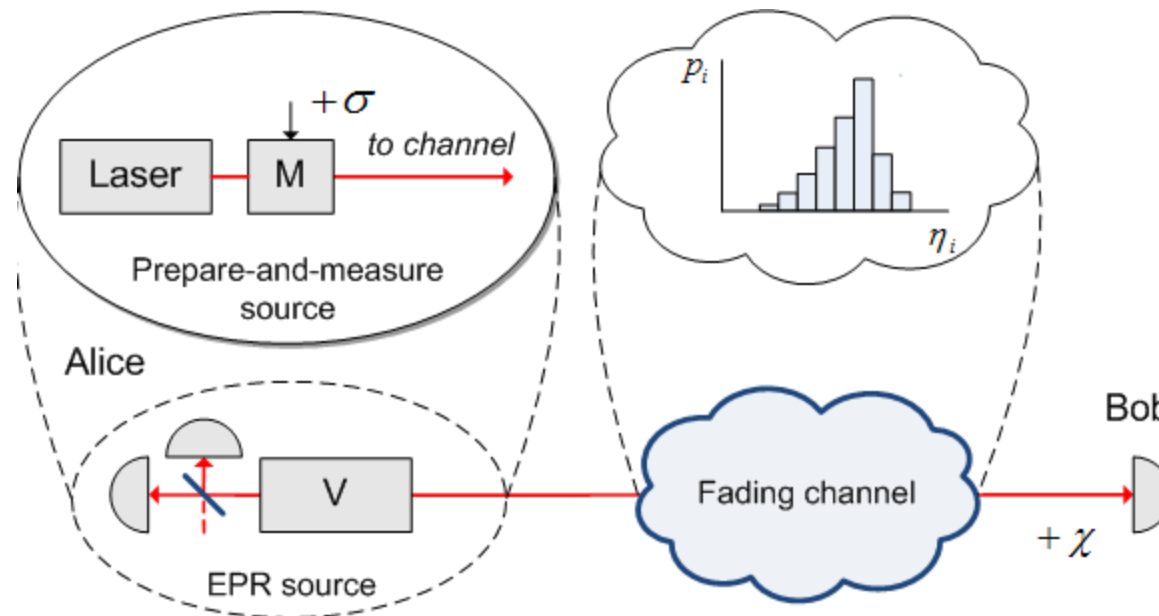
$$S(\rho_E) = S(\rho_{AB}) \quad S(\rho_{E|B}) = S(\rho_{A|B})$$

von Neumann entropy is given by:

$$S_\gamma = \sum_i G\left(\frac{\lambda_i - 1}{2}\right), \quad G(x) = (x + 1) \log_2 (x + 1) - x \log_2 x$$

Fading channels: effect on CV QKD

Equivalent entanglement-based scheme:



Effect of a fading channel upon individual attacks:

$$\text{Var}(\sqrt{\eta})_{\max, \text{ind}} = \frac{\langle \sqrt{\eta} \rangle^2 \sigma - 2(\sigma + 1)(\chi + 1) + \sqrt{\langle \sqrt{\eta} \rangle^4 \sigma^2 + 4(\sigma + 1)^2}}{2\sigma(\sigma + 1)}$$

Where $\sigma = V - 1$ - modulation variance

Fading channels: effect on CV QKD

Initial two-mode covariance matrix: $\gamma_{AB}^0 = \begin{pmatrix} \gamma_A & \sigma_{AB} \\ \sigma_{AB} & \gamma_B \end{pmatrix}$

Effect of an i -th channel:

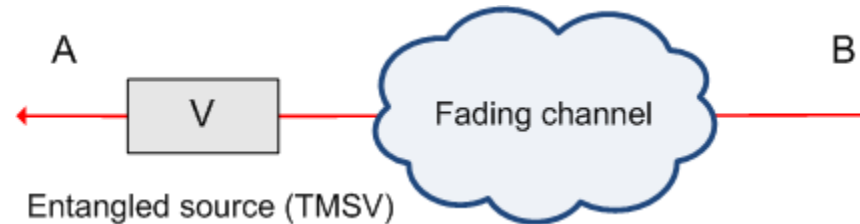
$$\gamma_{AB}^i = \begin{pmatrix} \gamma_A & \sqrt{\eta_i} \sigma_{AB} \\ \sqrt{\eta_i} \sigma_{AB} & \eta_i \gamma_B + [1 - \eta_i] \mathbb{I} \end{pmatrix}$$

Effect of the fading channel:

$$\gamma_{AB} = \begin{pmatrix} \gamma_A & \langle \sqrt{\eta} \rangle \sigma_{AB} \\ \langle \sqrt{\eta} \rangle \sigma_{AB} & \langle \eta \rangle \gamma_B + [1 - \langle \eta \rangle] \mathbb{I} \end{pmatrix}$$

[Dong et al. PRA 82 012312 (2010) / arXiv:1002.0280]

Fading channels: effect on CV QKD



Initial two-mode squeezed-vacuum state:

$$\gamma_{AB} = \begin{pmatrix} V\mathbb{I} & \sqrt{V^2 - 1}\sigma_z \\ \sqrt{V^2 - 1}\sigma_z & V\mathbb{I} \end{pmatrix}$$

After a fading channel:

$$\gamma'_{AB} = \begin{pmatrix} V\mathbb{I} & \langle\sqrt{\eta}\rangle\sqrt{V^2 - 1}\sigma_z \\ \langle\sqrt{\eta}\rangle\sqrt{V^2 - 1}\sigma_z & (V\langle\eta\rangle + 1 - \langle\eta\rangle + \chi)\mathbb{I} \end{pmatrix}$$

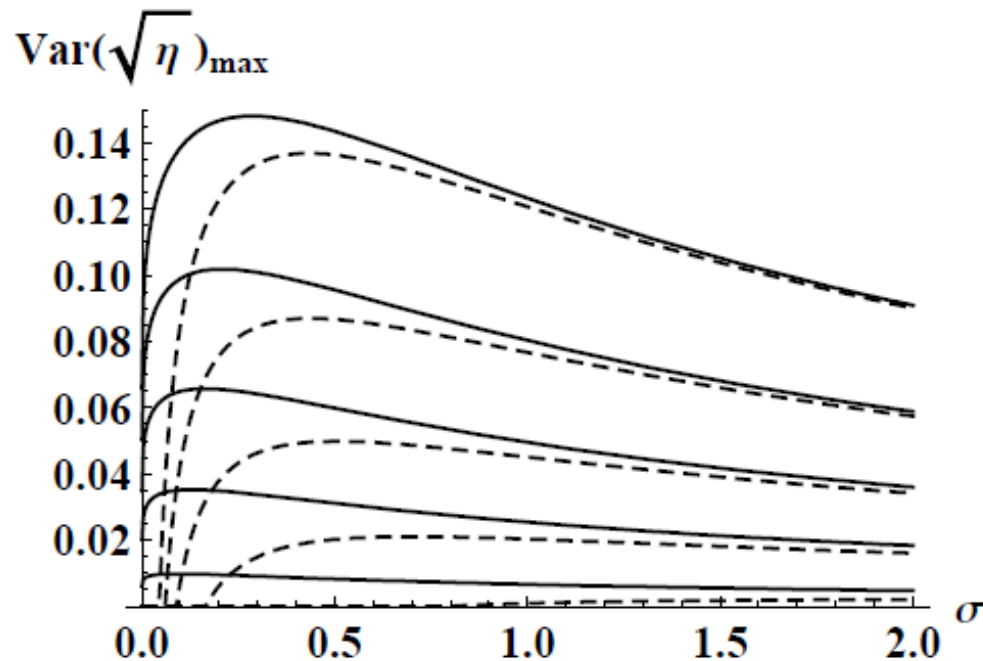
Is equivalent to a fixed channel with variance-dependent excess noise:

$$\gamma'_{AB} = \begin{pmatrix} V\mathbb{I} & \langle\sqrt{\eta}\rangle\sqrt{V^2 - 1}\sigma_z \\ \langle\sqrt{\eta}\rangle\sqrt{V^2 - 1}\sigma_z & \langle\sqrt{\eta}\rangle^2(V - 1) + \epsilon_f + \chi + 1)\mathbb{I} \end{pmatrix}$$

where $\epsilon_f = \text{Var}(\sqrt{\eta})(V - 1)$ and $\text{Var}(\sqrt{\eta}) = \langle\eta\rangle - \langle\sqrt{\eta}\rangle^2$

Fading channels: effect on CV QKD

Security against collective attacks:

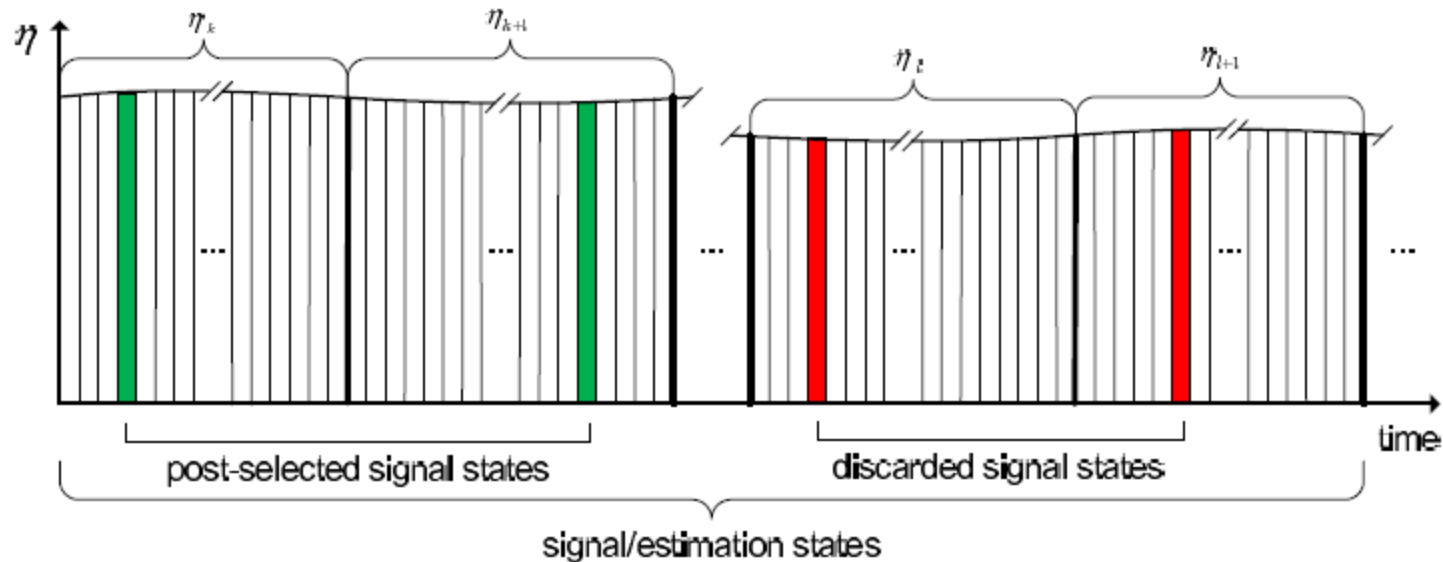


solid lines: no excess noise

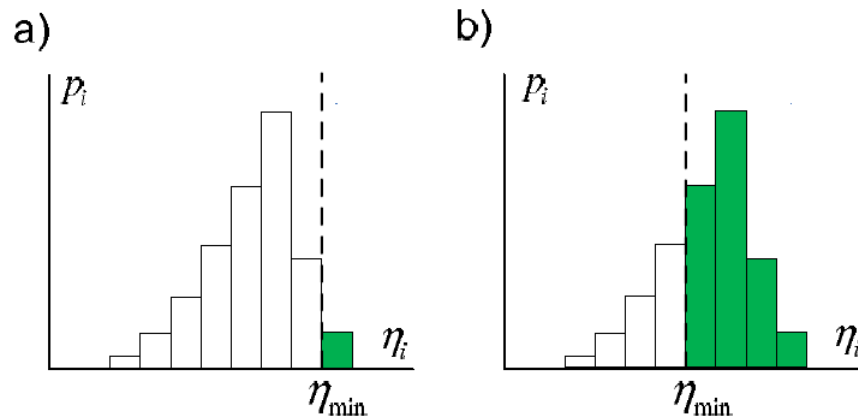
dashed lines: excess noise $\chi = 1.2 \cdot 10^{-2}$

Post-selection of sub-channels

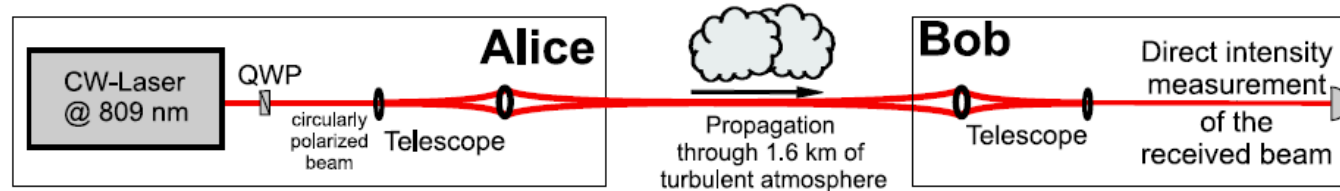
Post-selection time-flow:



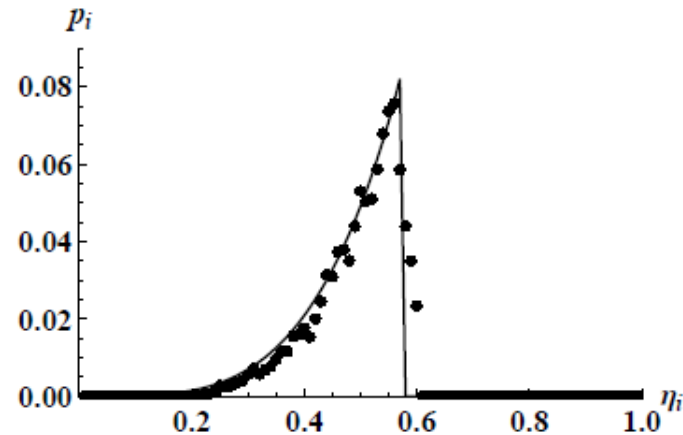
Post-selection of a single / multiple subchannels:



Real fading channel



Transmittance distribution obtained from a 1.6 km atmospheric link in Erlangen

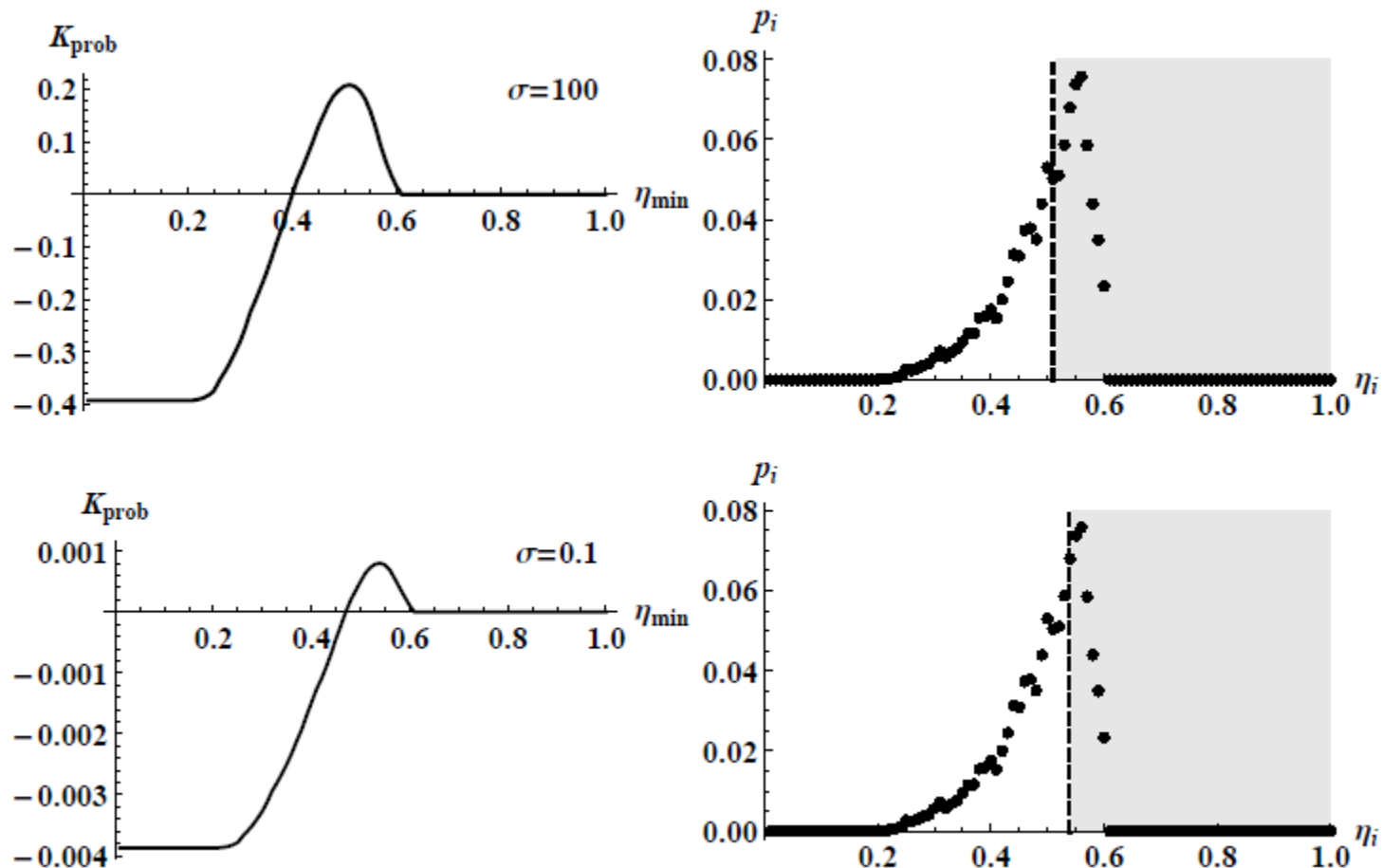


Sampling rate 150 kHz, bin size $\Delta\eta = 0.01$

Experimental distribution is well fitted by the log-normal.

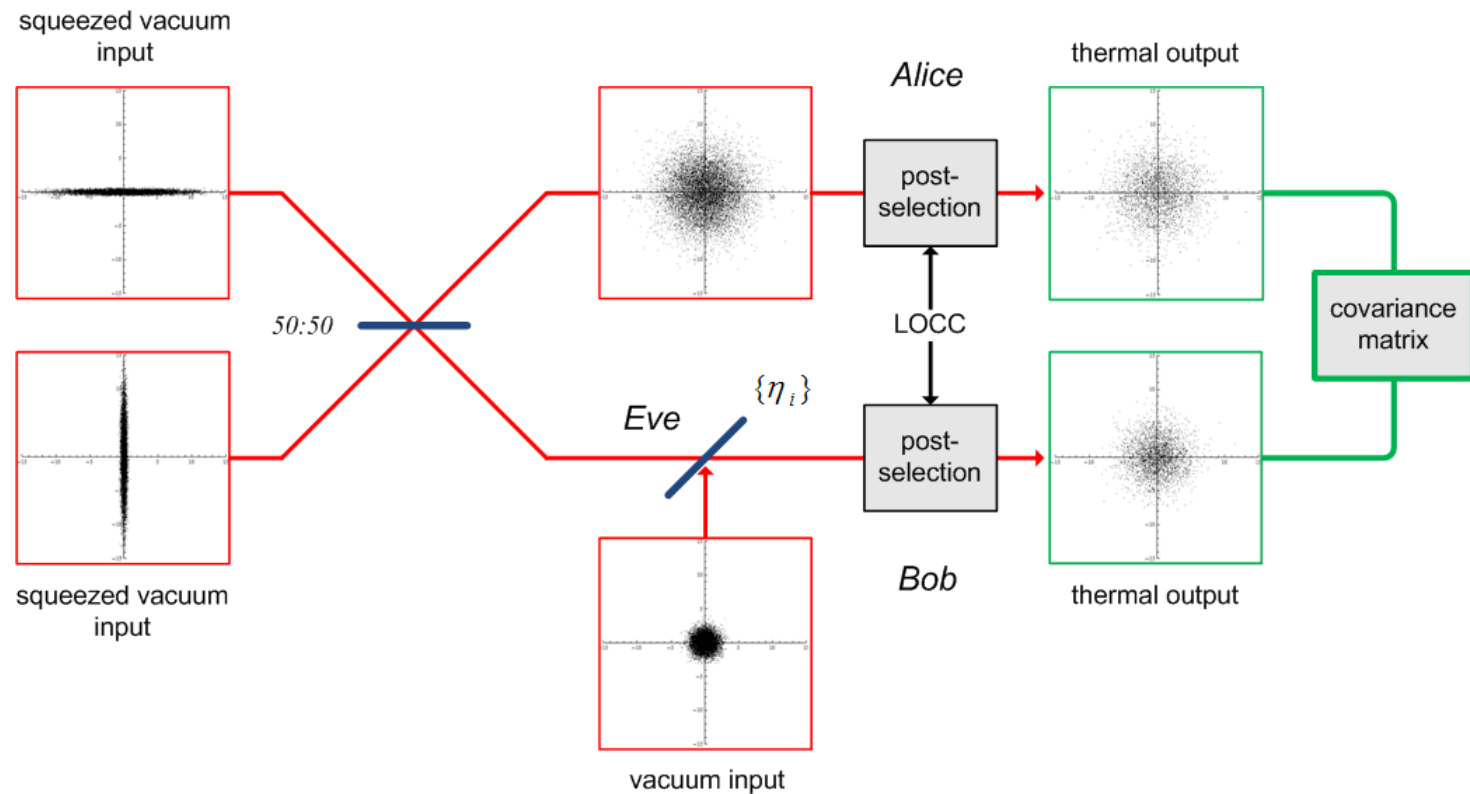
Channel is characterized by $\langle\sqrt{\eta}\rangle^2 \approx 0.492$ and $Var(\sqrt{\eta}) \approx 3 \cdot 10^{-3}$

Real fading channel



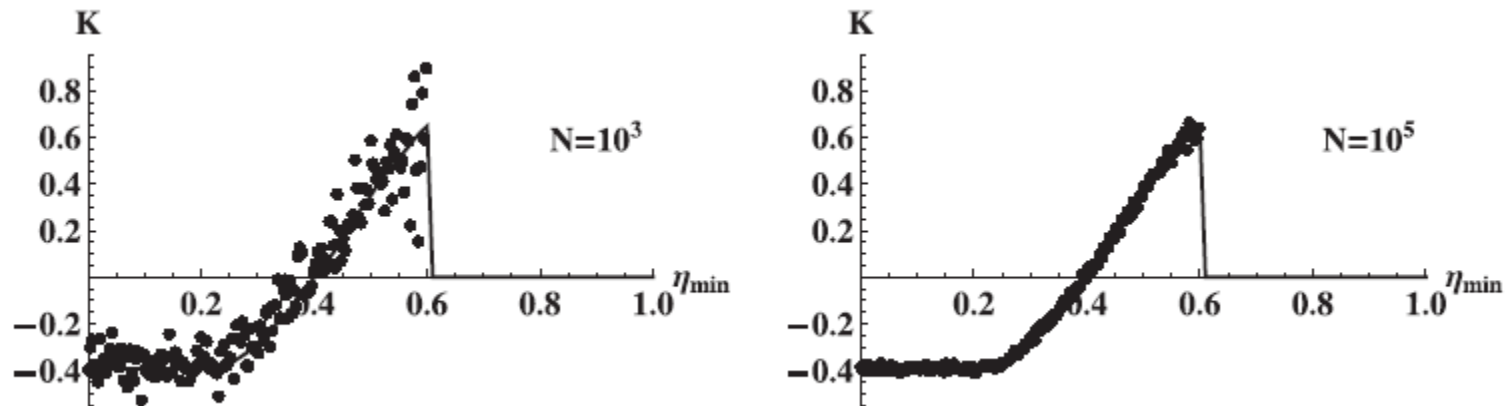
Effect of post-selection after the real fading channel on the security of the coherent-state protocol in terms of the weighted key rate (left). Corresponding optimal PS region is given at the right. Noise $\chi = 3.2 \cdot 10^{-2}$

Finite-size effects



Scheme for numerical modeling of the fading and post-selection effects.

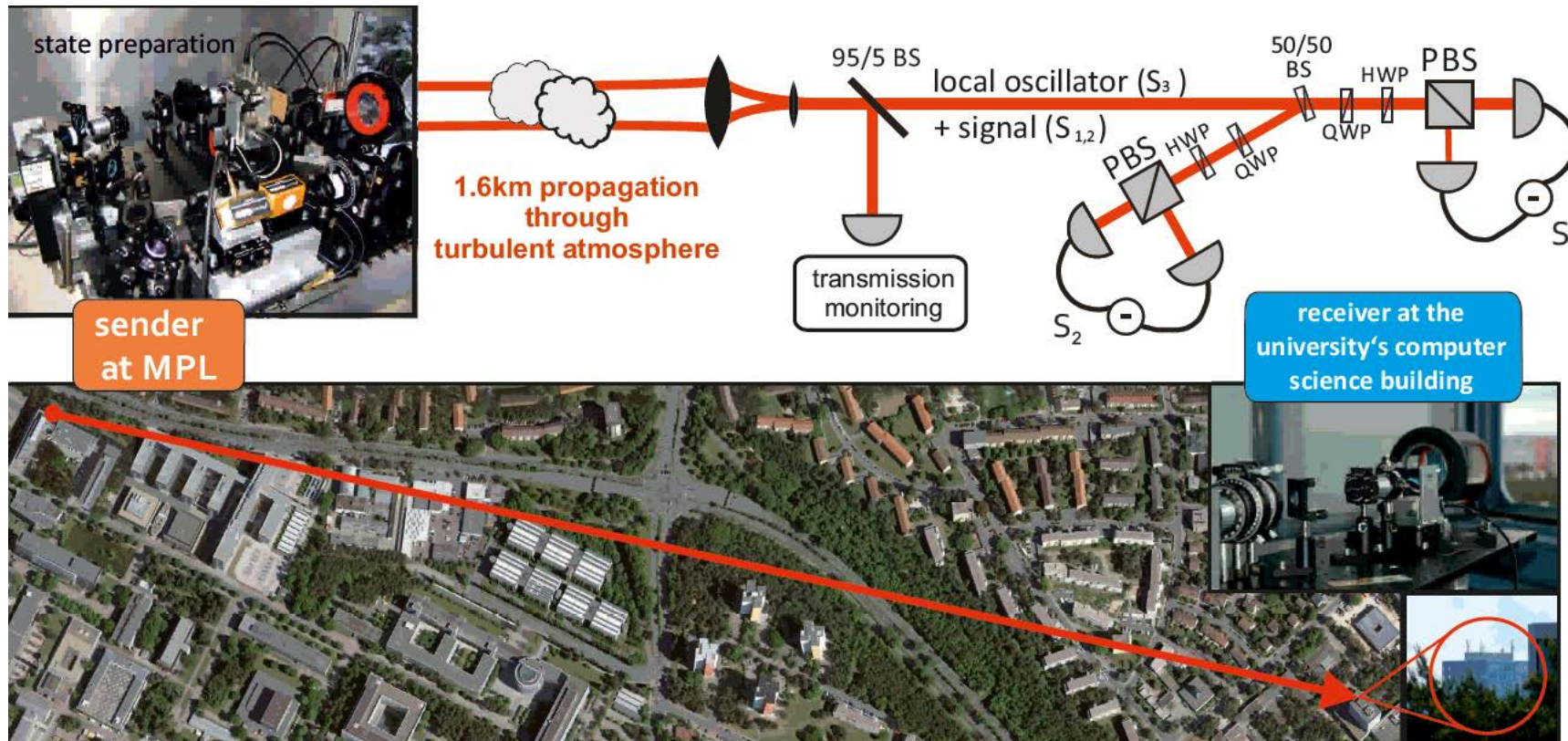
Finite-size effects



Effect of the finite ensemble size on the key rate upon post-selection.

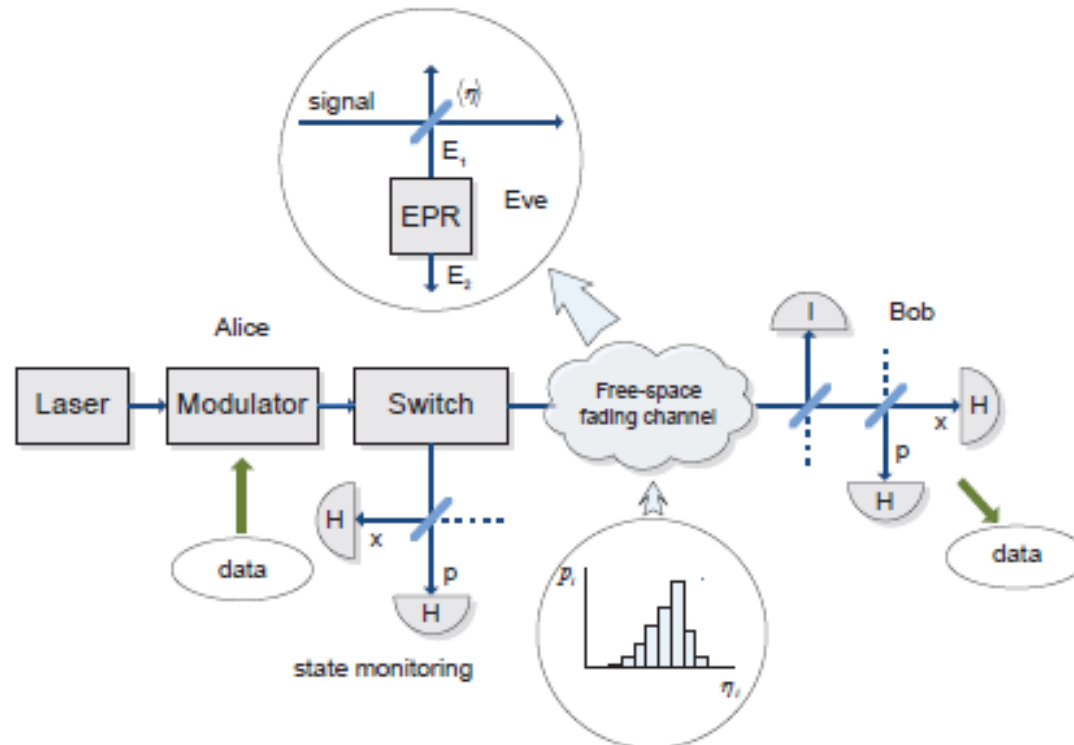
[VU, Heim, Peuntinger, Wittmann, Marquardt, Leuchs, Filip, New J. Phys., 14 093048 (2012)]

Proof-of-principle test



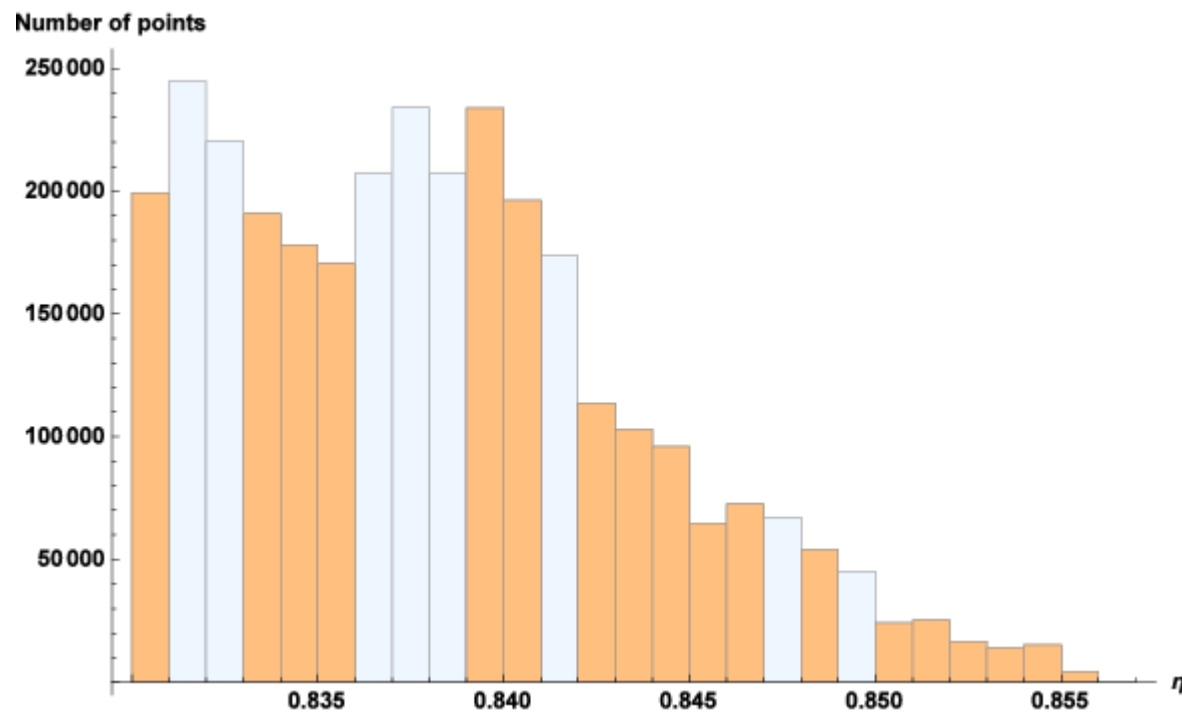
We illustrate the effect of sub-channel post-selection by testing Gaussian-modulated coherent-state CV QKD in the free-space link in Erlangen.

Proof-of-principle test



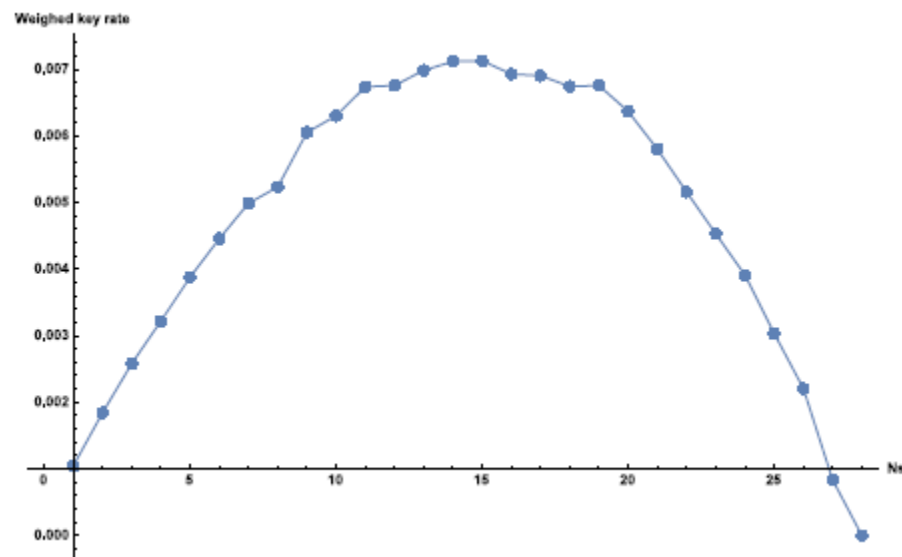
Sketch of the experiment with the Gaussian-modulated coherent states, free-space link and double-homodyne detection.

Proof-of-principle test



Sub-channels with the positive key rate (orange).

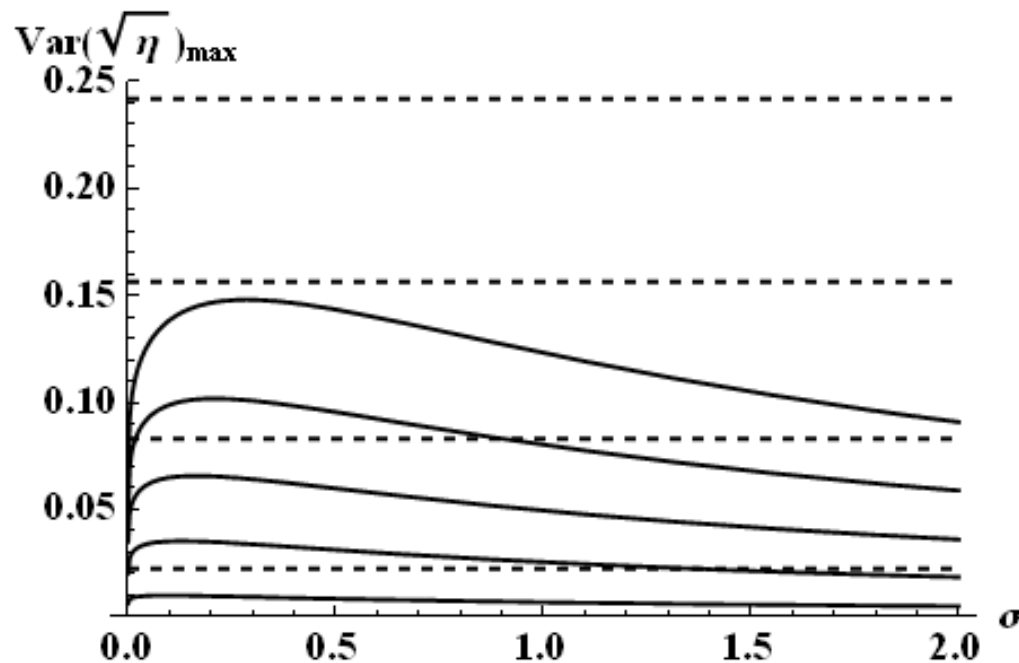
Proof-of-principle test



Lower bound of the key rate secure against collective attacks versus number of post-selected sub-channels.

Coming next

Realization of the squeezed-state protocol (following successful distribution of squeezing [Peuntinger et al. PRL 113, 060502 (2014)])



Coherent-state protocol (solid lines) and squeezed-state protocol (dashed lines) with -0.8 dB squeezing ($V=1.2$).

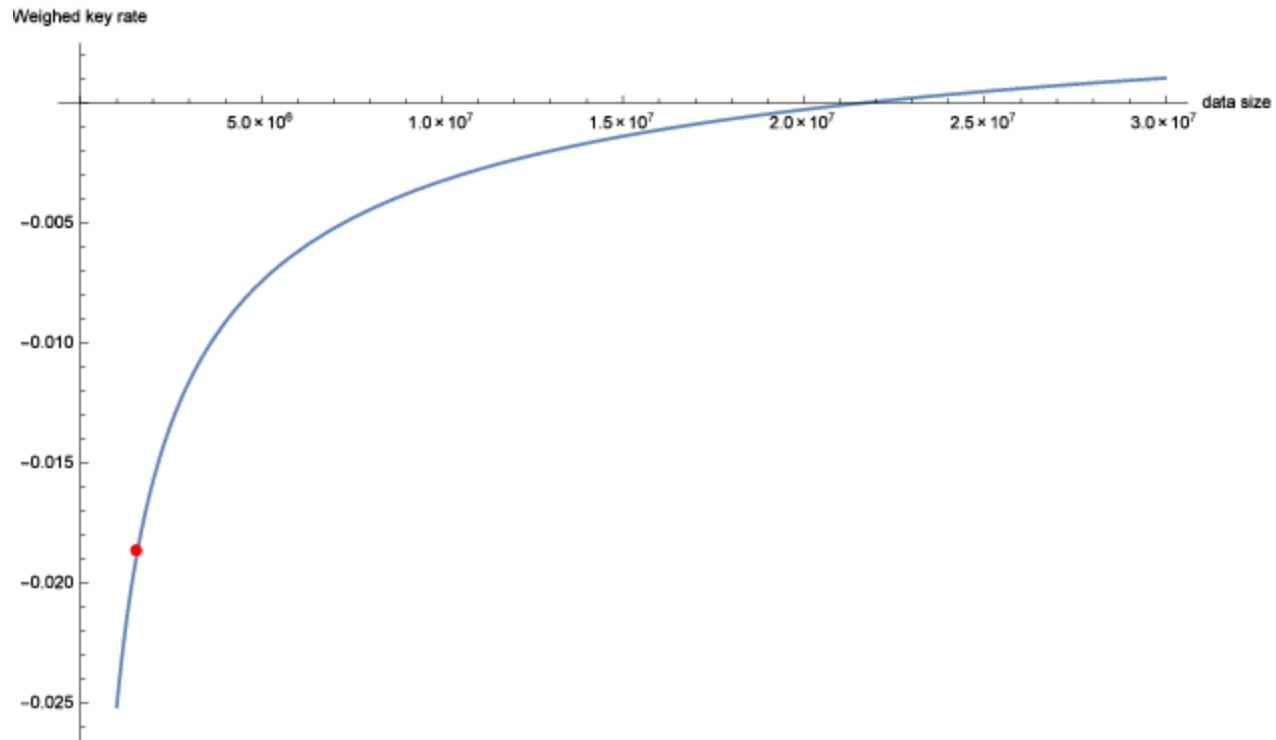
Summary

- Fading channels affect security of QKD protocols on the basis of the Gaussian states.
- States with the higher variance are more sensitive to fading, while presence of even small excess noise combined with fading strongly affects the states with lower variance.
- Security can be restored by the use of post-selection.
- Alternatively, squeezed states can be used and are more robust to fading (even without sub-channel post-selection).

Thank you for the attention!

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Finite-size effects



Weighted key rate versus number of data points.