Experimental demonstration of superresolution of two incoherent point sources using SPADE method

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Abstract

We present an experimental realization of a simple method to assess the quantum Fisher information for resolving two incoherent point sources under standard Rayleigh limit. The final quantum Cramér-Rao bound (qCRB) shows that limit can be surpassed by eligible coherent measurements. We determine optimal procedure which incorporates a projection onto optimal bases what is achieved by digital holography.

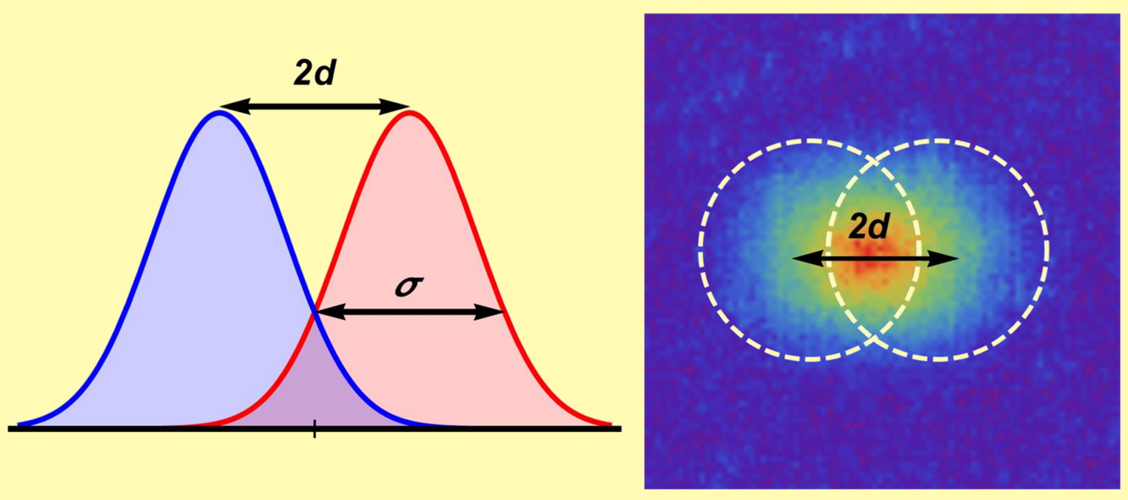
Theoretical background

We assume spatially invariant unit-magnification imaging system characterized by its point spread function (PSF) as

$$I(x) = |\langle x|\psi\rangle|^2 = |\psi(x)|^2.$$

Two imaged incoherent point sources with unknown separation $X_{\pm} = \pm d$ which are described by density matrix:

$$\varrho_d(x) = \frac{1}{2}\left(|\psi(x-d)|^2 + |\psi(x+d)|^2\right).$$



Particularly for close points we get:

$$|\psi_{\pm}\rangle = \mathcal{N}(1 \pm idP),$$

where $\mathcal{N} = \left[1 + d^2\langle\psi|P^2|\psi\rangle\right]$ is normalization constant. Problem is that the spatial modes are not orthogonal ($\langle\psi_-|\psi_+\rangle \neq 0$).

Due to, we define symetric and antisymetric states:

$$|\psi_s\rangle = C_s(|\psi_+\rangle + |\psi_-\rangle) \simeq |\psi\rangle,$$

$$|\psi_a\rangle = C_a(|\psi_+\rangle - |\psi_-\rangle) \simeq \frac{P|\psi\rangle}{\sqrt{\langle\psi|P^2|\psi\rangle}},$$

where modes are orthogonal when $\langle\psi|P|\psi\rangle = 0$.

The Fisher information is then defined as

$$\mathcal{F} \simeq \langle\psi|P^2|\psi\rangle = \int_{-\infty}^{\infty}[\psi'(x)]^2\,dx,$$

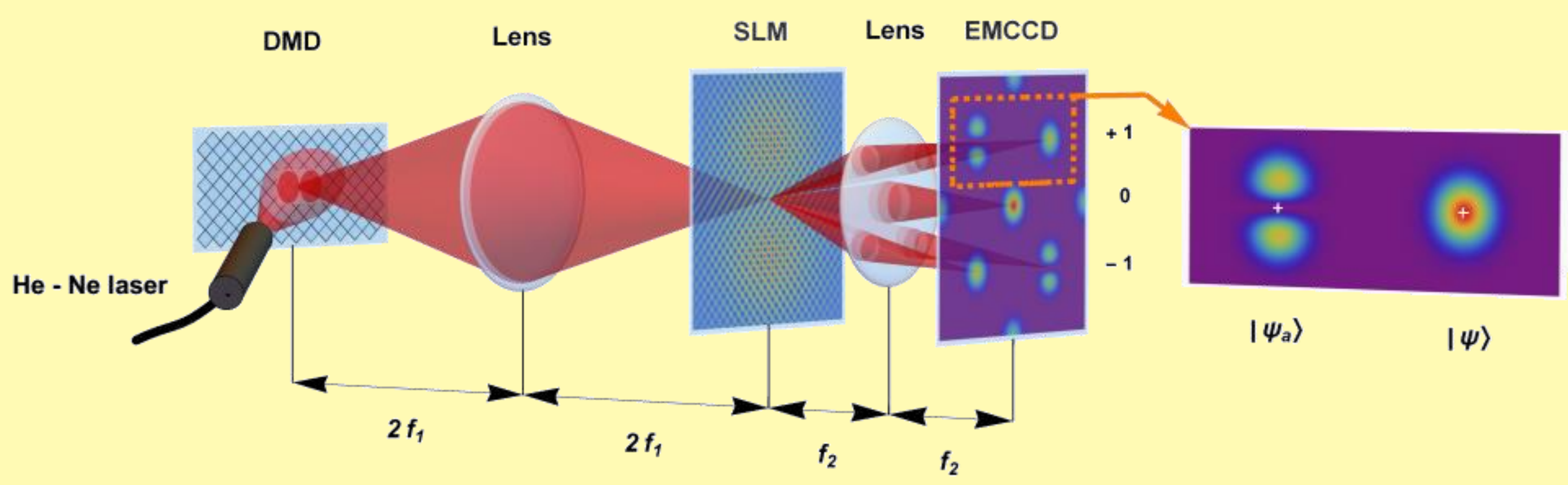
which is crucial for separation estimation and independent on d.

Conclusion

We have developed and demostrated simple technique that surpasses traditional imaging in its abilitiy to resolve two closely separated point sources in sub-Reyleigh region. The experimental realization implies simple technique how to provide the projection of PSFs into optimal spatial modes. Finally we present results for two particular PSFs, which stress that the diffraction resolution limit is not a fundamental constraint.

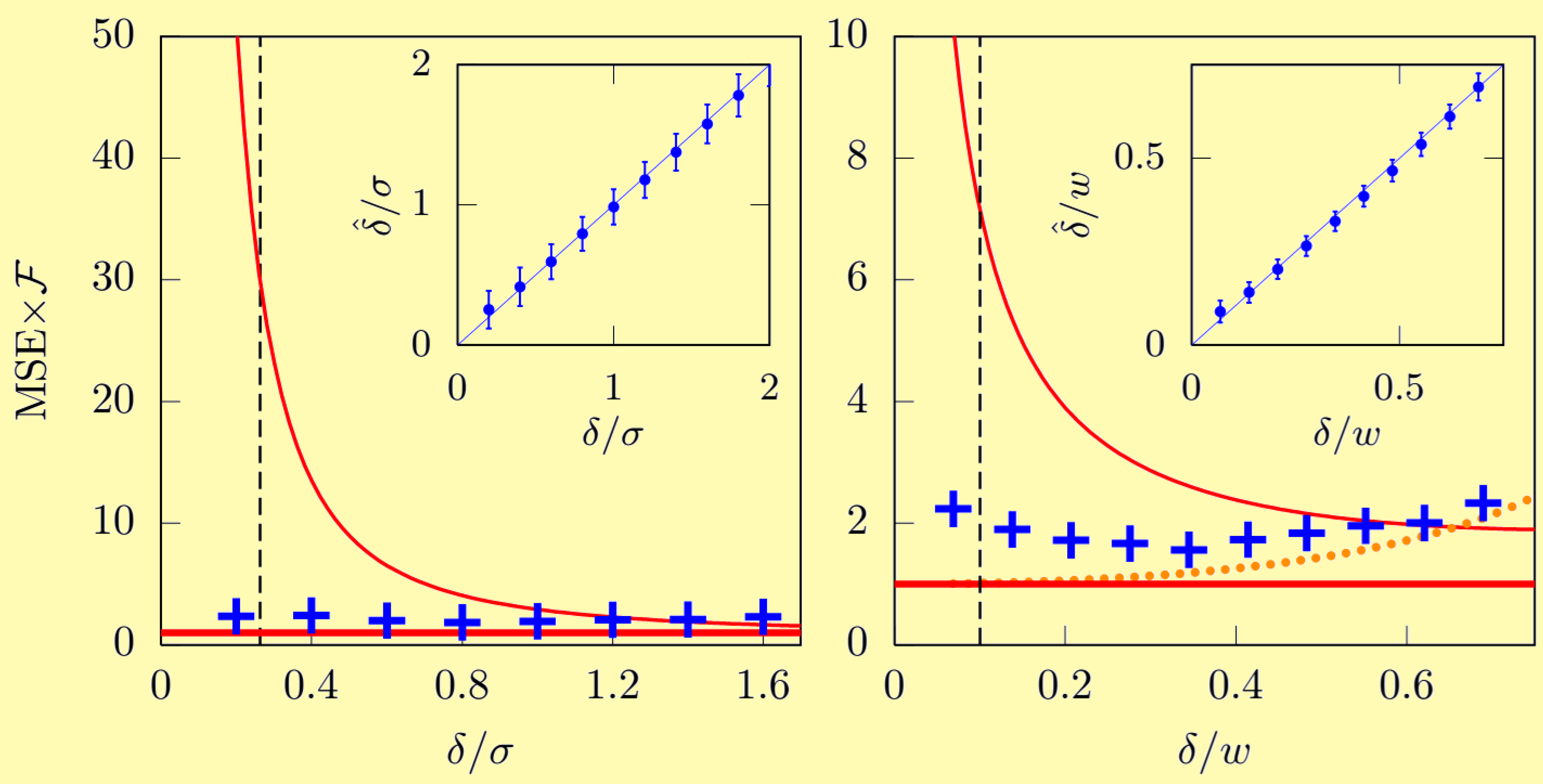
Experimental setup

- Generation of two incoherent point-like sources by the **Digital Micro-mirror Device** (**DMD**).
- Imaging point sources by a low-aperture lens.
- Two-point image projection onto the different modes by the digital hologram and the **amplitude spatial light modulator** (**SLM**).
- Detection of the first-order diffraction spectrum by cooled **EMCCD** camera.



Results

- Measurement of two relevant PSFs: **Gaussian** (left) and **Sinc** (Right).
- Comparison of our method (**Blue crosses**), theoretical lower bound for classical CCD measurement (**thin red curve**), ultimate quantum limit (**thick red curve**) and classical CRLB for antisymetric projection (**orange dots**).
- **Vertical dashed lines** delimit the 10% of the Rayleigh limit.
- Inset graphs show the statistics of the experimental estimates (**Blue dots with error bars**).



References

[1] M. Tsang, R. Nair and X.-M. Lu, arXiv:1511.00552

[2] M. Paur, B. Stoklasa, J. Rehacek, Z. Hradil, L. L. Sanchez-Soto, arXiv: 1606.08332

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