Experimental Demonstration and Characterization of Multi-Qubit Linear Optical Quantum Gates

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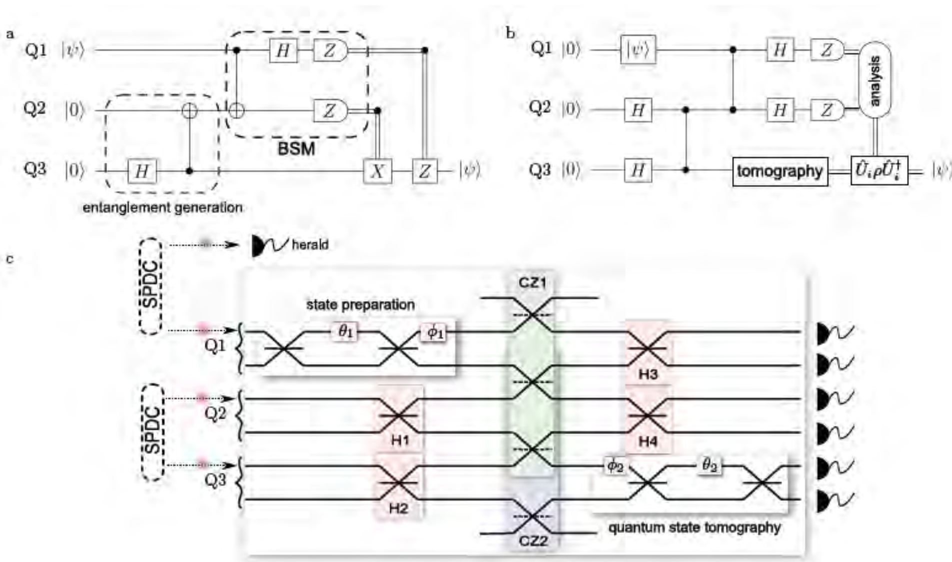
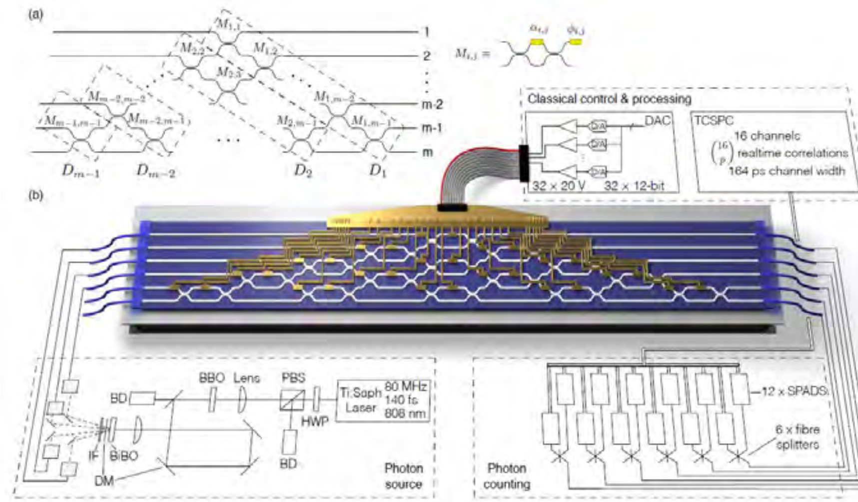
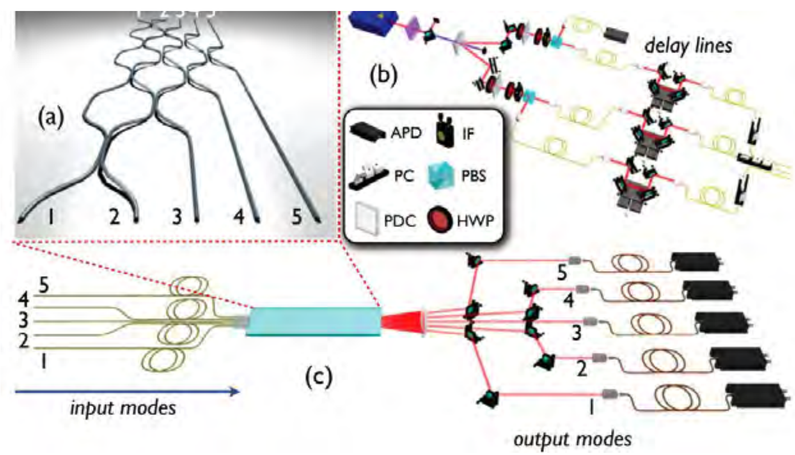
Outline of the talk

- Paths to increase the complexity of optical quantum logic circuits
- Four-qubit linear optical quantum processor
- Experimental characterization of a four-qubit linear optical quantum gate
- Generation of multiqubit entangled states by the optical quantum processor
- Monte Carlo sampling of quantum gate fidelity
- Comparison with three-qubit linear optical quantum Toffoli gate

Increasing the complexity of linear optics quantum logic circuits







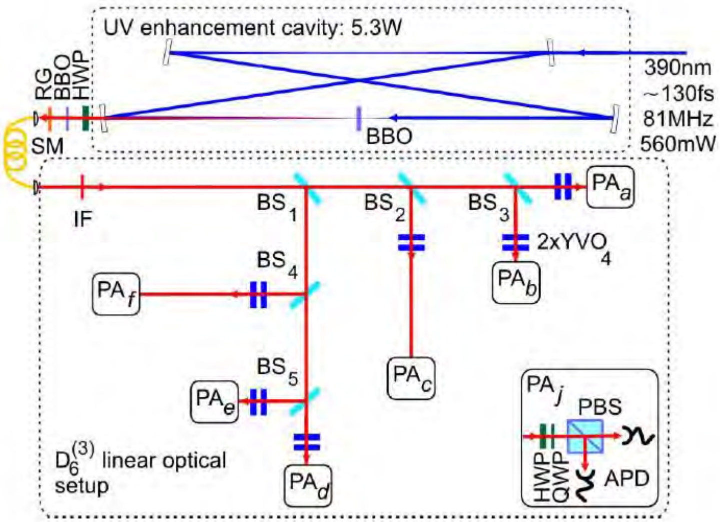
Integrated optics on a photonic chip

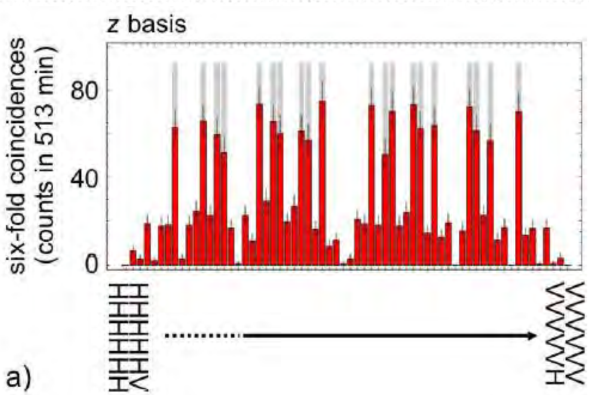
A. Crespi, R. Osellame, R. Ramponi, D. J. Brod, E. F. Galvao, N. Spagnolo, C. Vitelli, E. Maiorino, P. Mataloni, F. Sciarrino, Nature Photonics **7**, 545 (2013).

B.J. Metcalf, J.B. Spring, P.C. Humphreys, N. Thomas-Peter, M. Barbieri, W. S. Kolthammer, X.-M. Jin, N.K. Langford, D. Kundys, J.C. Gates, B.J. Smith, P.G.R. Smith, and I.A. Walmsley, Nature Photonics **8**, 770-774 (2014).

J. Carolan, C. Harrold, C. Sparrow, E. Martin-Lopez, N.J. Russell, J.W. Silverstone, P.J. Shadbolt, N. Matsuda, M. Oguma, M. Itoh, G.D. Marshall, M.G. Thompson, J.C.F. Matthews, T. Hashimoto, J.L. O’Brien, and A. Laing, Science **349**, 711 (2015).

Increasing the complexity of linear optics quantum logic circuits





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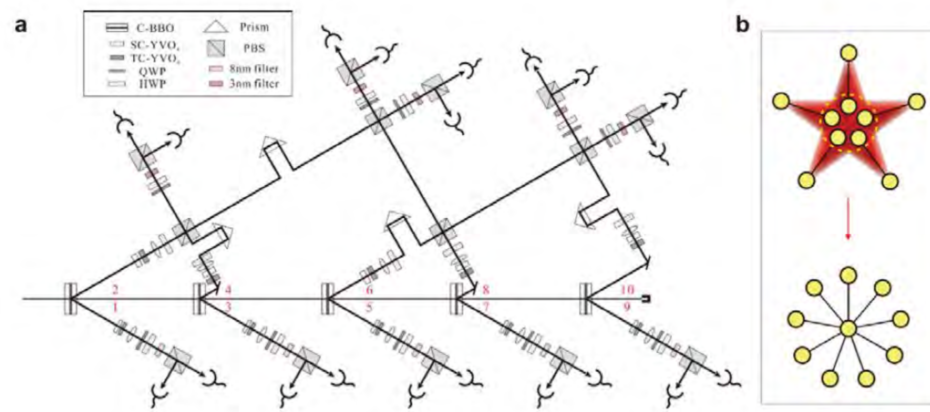
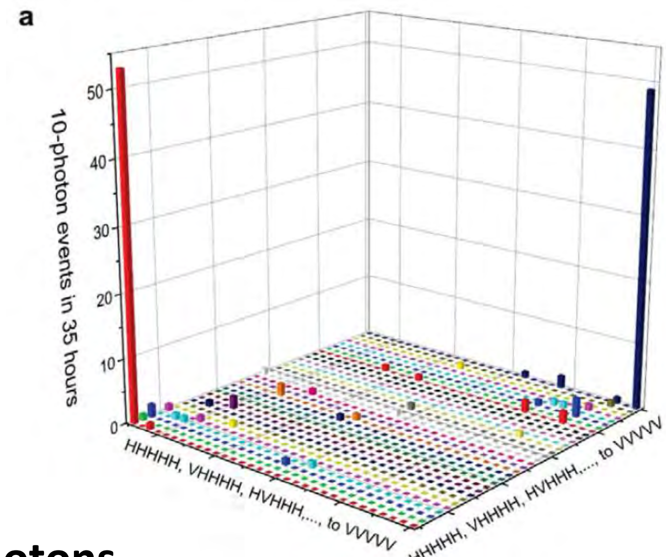


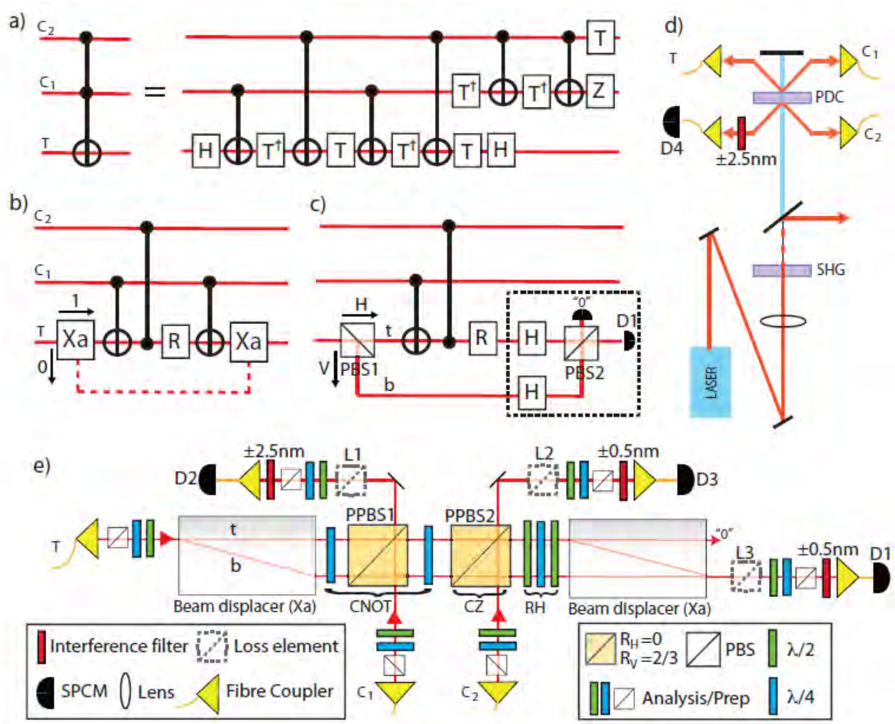


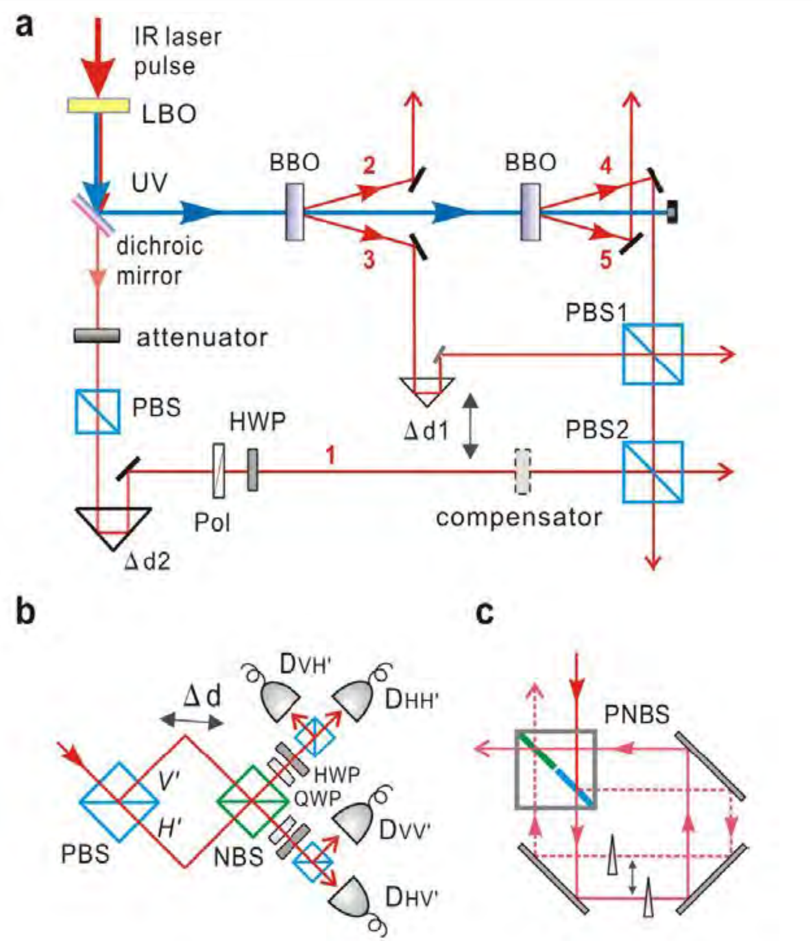
Increasing the number of photons

W. Wieczorek, R. Krischek, Nikolai Kiesel, P. Michelberger, G. Toth, H. Weinfurter Phys. Rev. Lett. **103**, 020504 (2009).

X.-L. Wang, L.-K. Chen, W. Li, H.-L. Huang, C. Liu, C. Chen, Y.-H. Luo, Z.-E. Su, D. Wu, Z.-D. Li, H. Lu, Y. Hu, X. Jiang, C.-Z. Peng, L. Li, N.-L. Liu, Y.-A. Chen, C.-Y. Lu, and J,-W. Pan, arXiv:1605.08547.

Increasing the complexity of linear optics quantum logic circuits



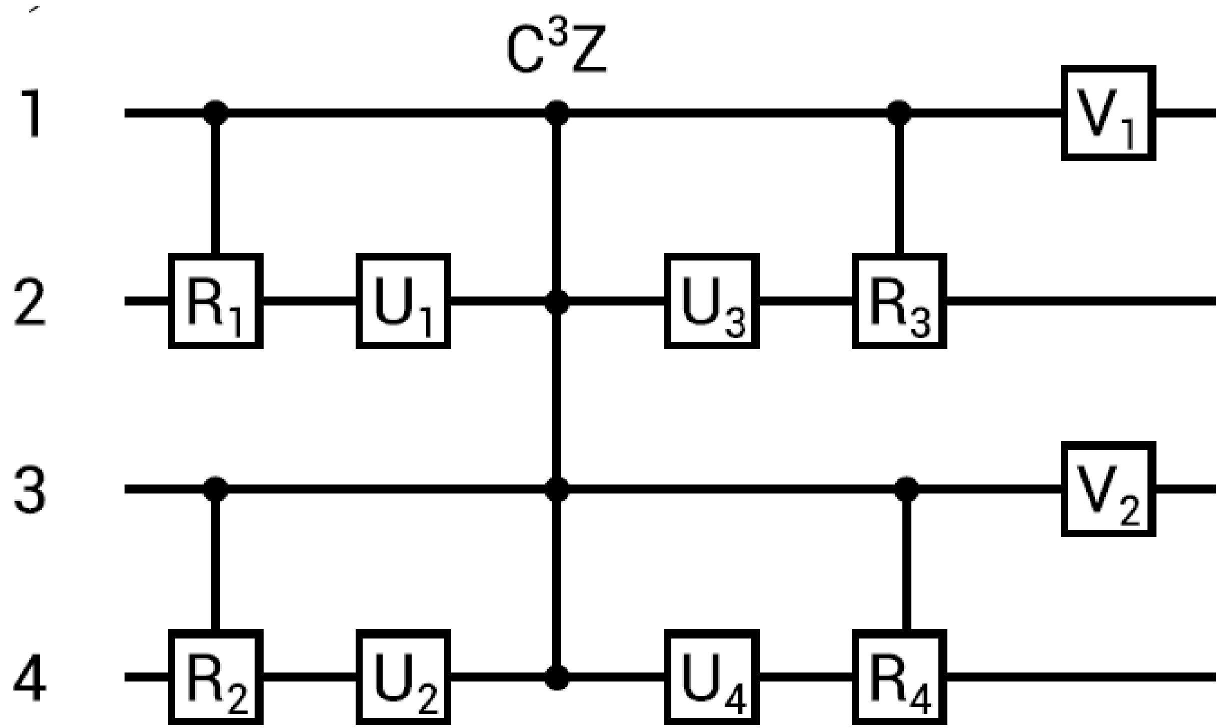


Utilization of inherently stable interferometers
and exploitation of several degrees of freedom of single photons

B. P. Lanyon, M. Barbieri, M. P. Almeida, T. Jennewein, T. C. Ralph, K. J. Resch, G. J. Pryde, J. L. O'Brien, A. Gilchrist, A. G. White, Nature Physics **5**, 134 (2009).

W.-B. Gao, C.-Y. Lu, X.-C. Yao, P. Xu, O. Gühne, A. Goebel, Y.-A. Chen, C.-Z. Peng, Z.-B. Chen, and J.-W. Pan, Nature Physics, **6**, 331 (2010).

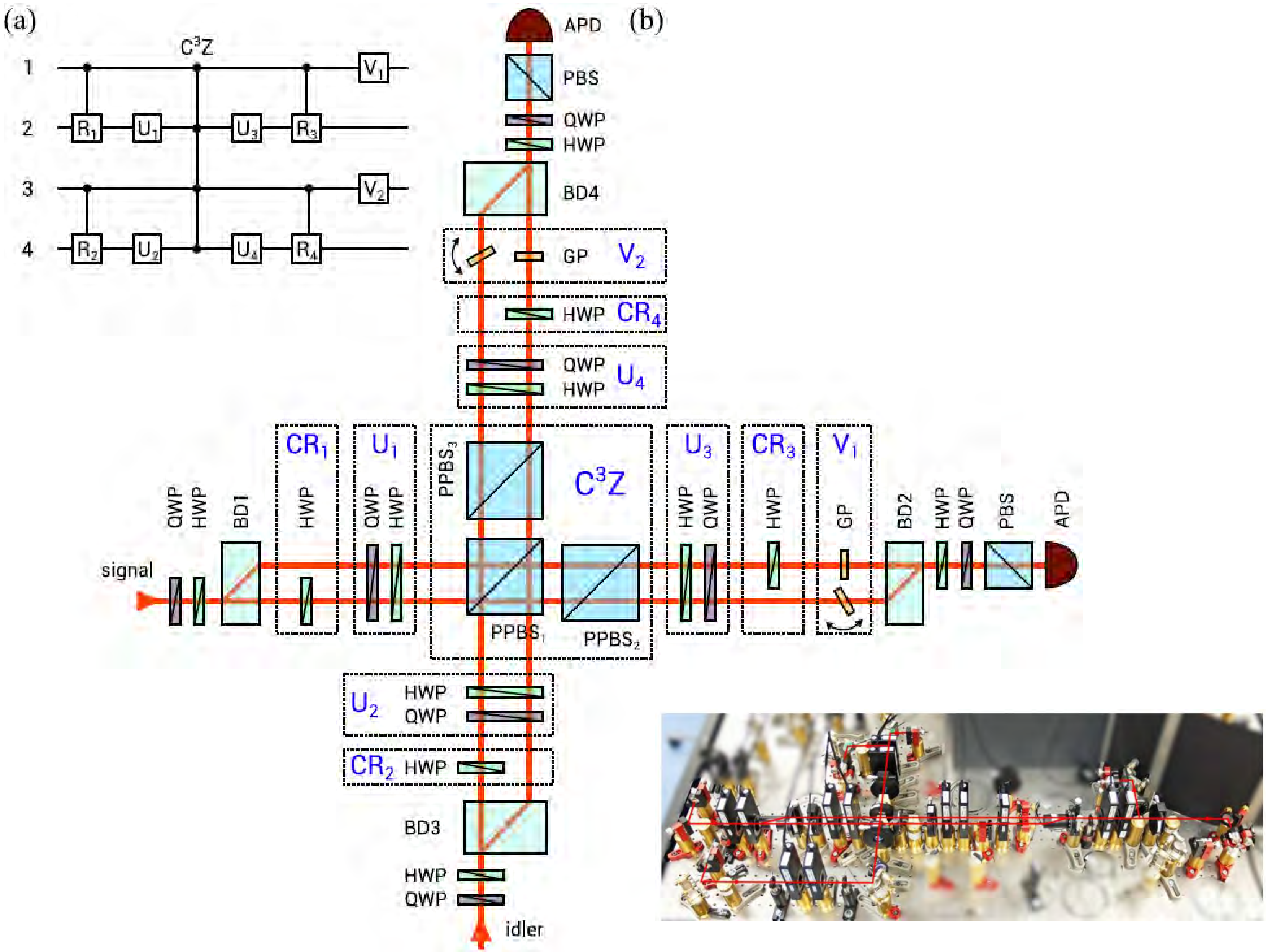
Quantum logic circuit of the four-qubit optical processor



- Four-qubit generalized controlled-Z gate C^3Z
- Two-qubit tunable controlled-rotation gates CR_k
- Single-qubit unitary gates U_k
- Single-qubit phase gates V_k

$$U_{C^3Z} = I - 2|1111\rangle\langle 1111|$$

Experimental setup



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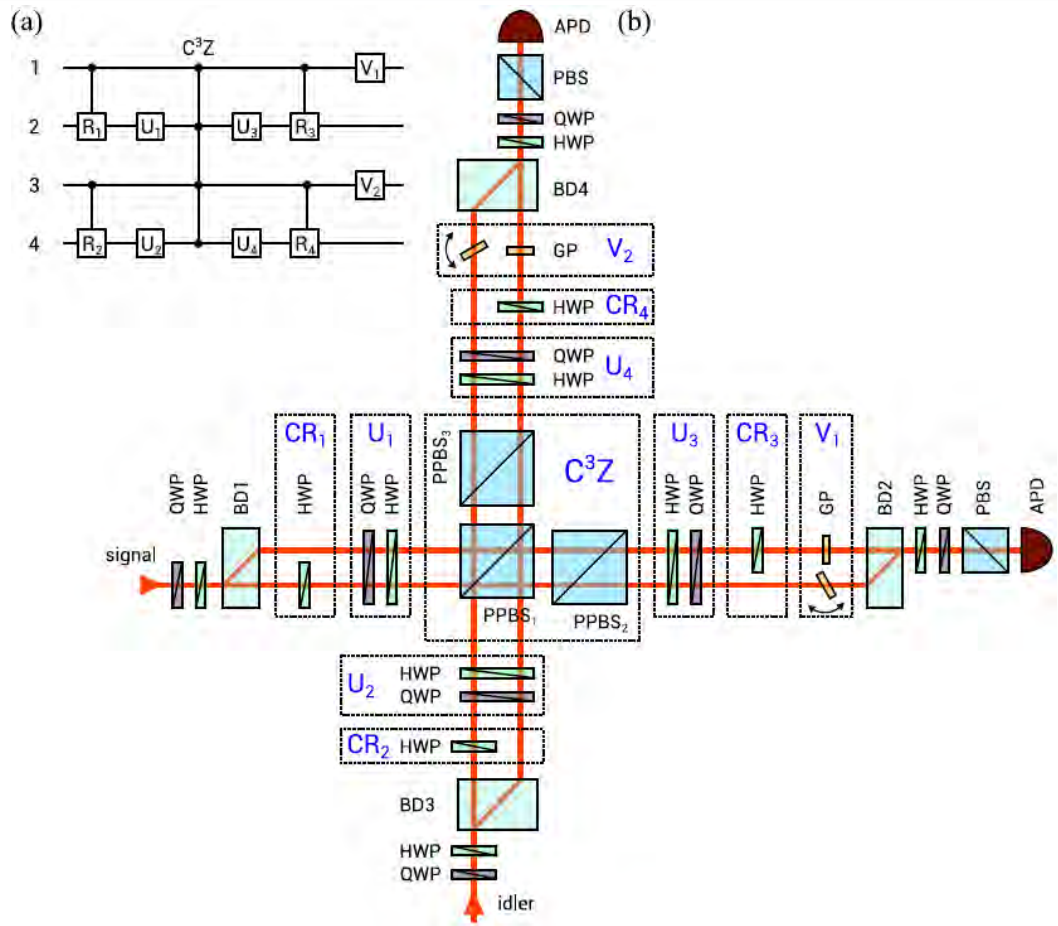


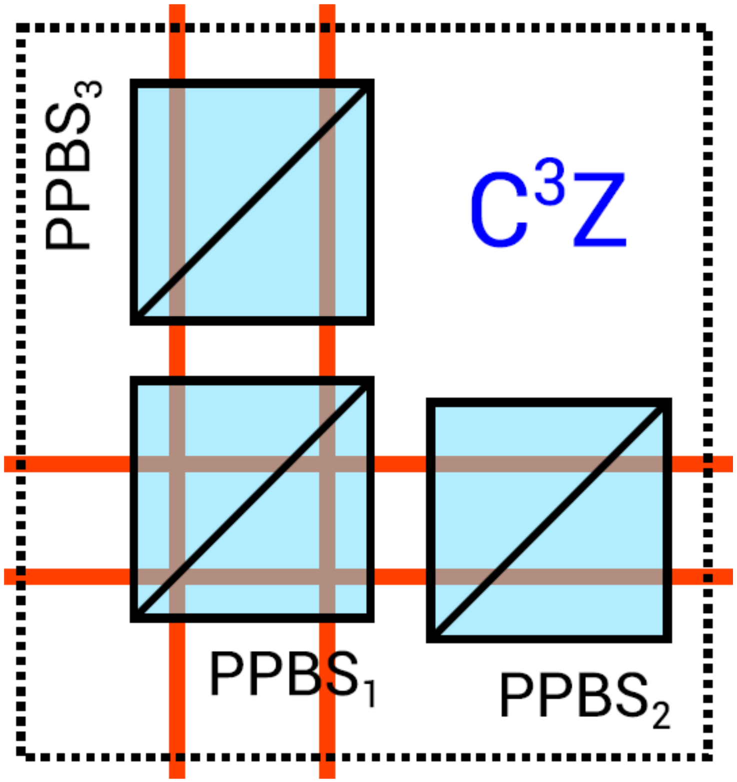
Correlated photon pairs are generated by Type-II SPDC in a BBO crystal

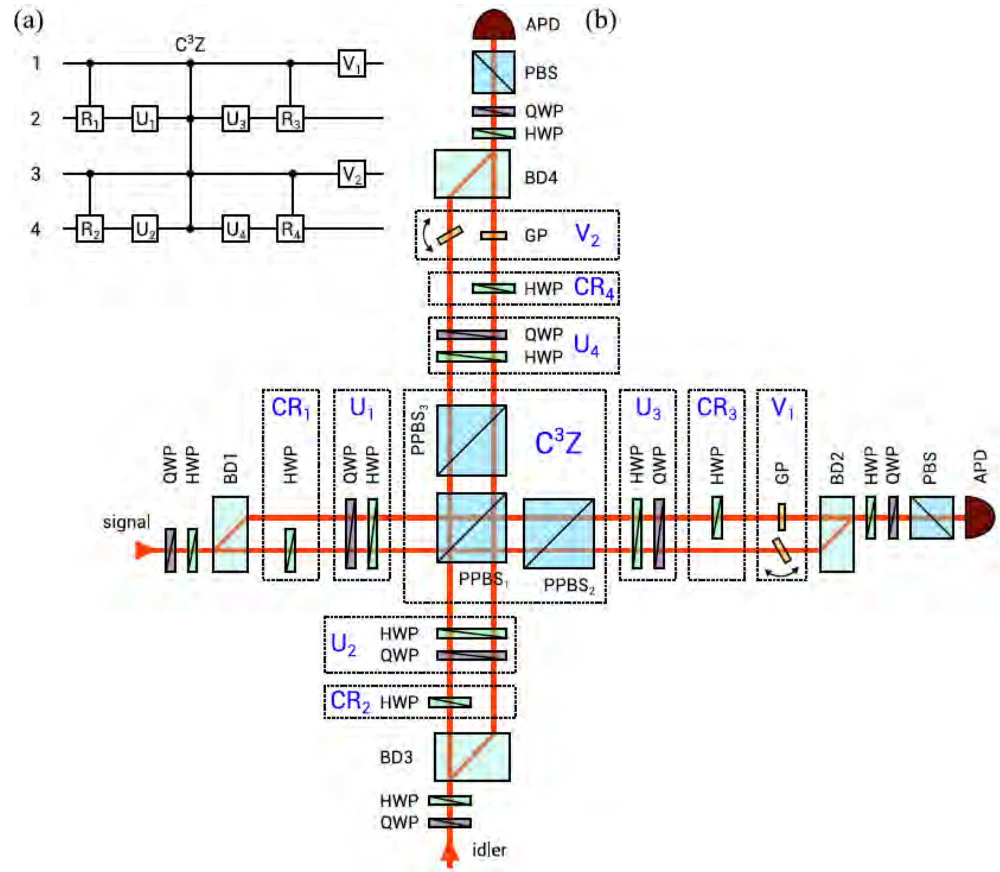
Qubits 1 and 2 encoded into polarization and path of signal photon

Qubits 3 and 4 encoded into polarization and path of idler photon

Two-photon coincidences are measured at the device output

Experimental setup

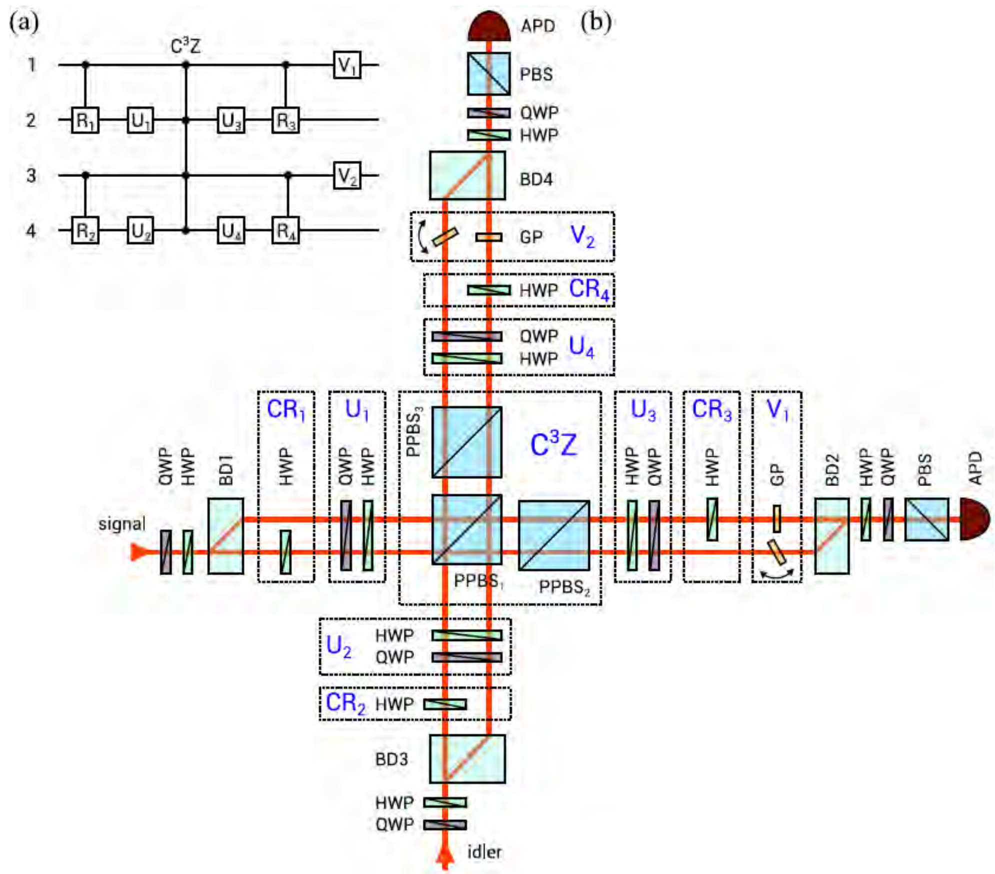




Four-qubit C^{3}Z gate implemented by two-photon interference on a PPBS

R. Okamoto, H.F. Hofmann, S. Takeuchi, and K. Sasaki, Phys. Rev. Lett. 95, 210506 (2005).
N. K. Langford, T.J. Weinhold, R. Prevedel, K. J. Resch, A. Gilchrist, J. L. OBrien, G. J. Pryde, and A. G. White, Phys. Rev. Lett. 95, 210504 (2005).
N. Kiesel, C. Schmid, U. Weber, R. Ursin, and H. Weinfurter, Phys. Rev. Lett. 95, 210505 (2005).

Experimental setup



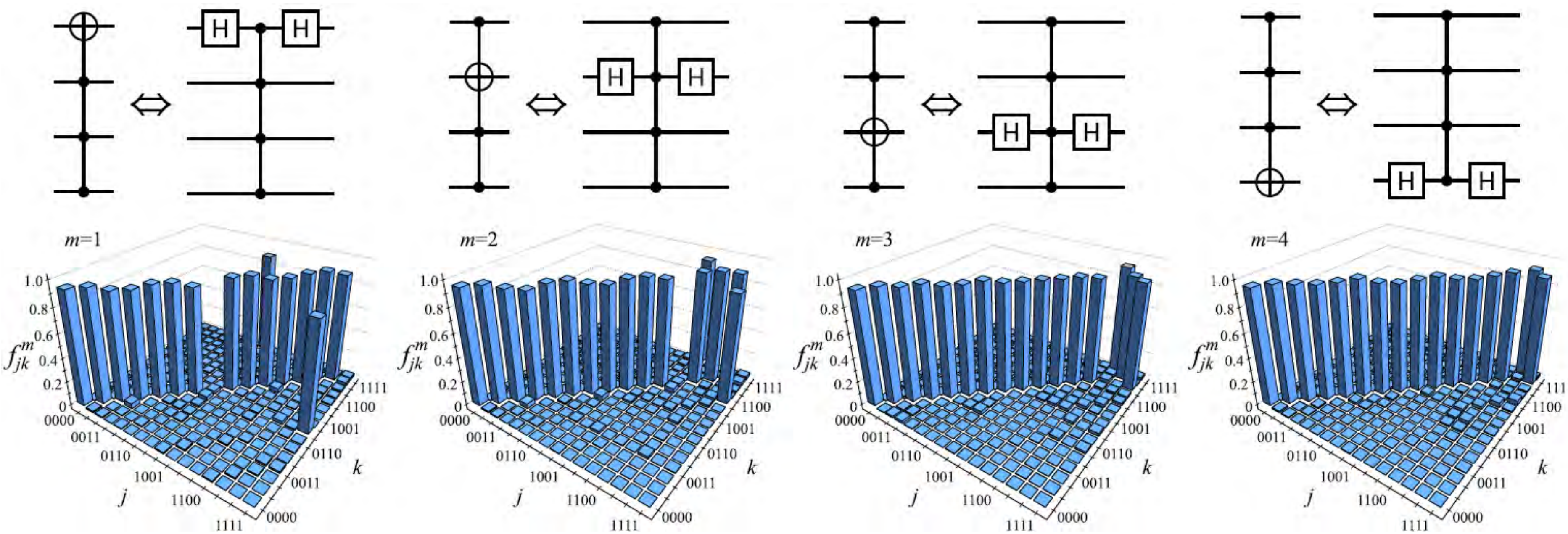
Four-qubit C^3Z gate implemented by two-photon interference on a PPBS

Two-qubit CR gates are realized by rotated HWPs inserted in one arm of the interferometer

Single-qubit unitary gates U_k are realized by combination of HWP and QWP addressing both interferometer arms

Single-qubit phase gates V_k are implemented by tilting glass plates GP

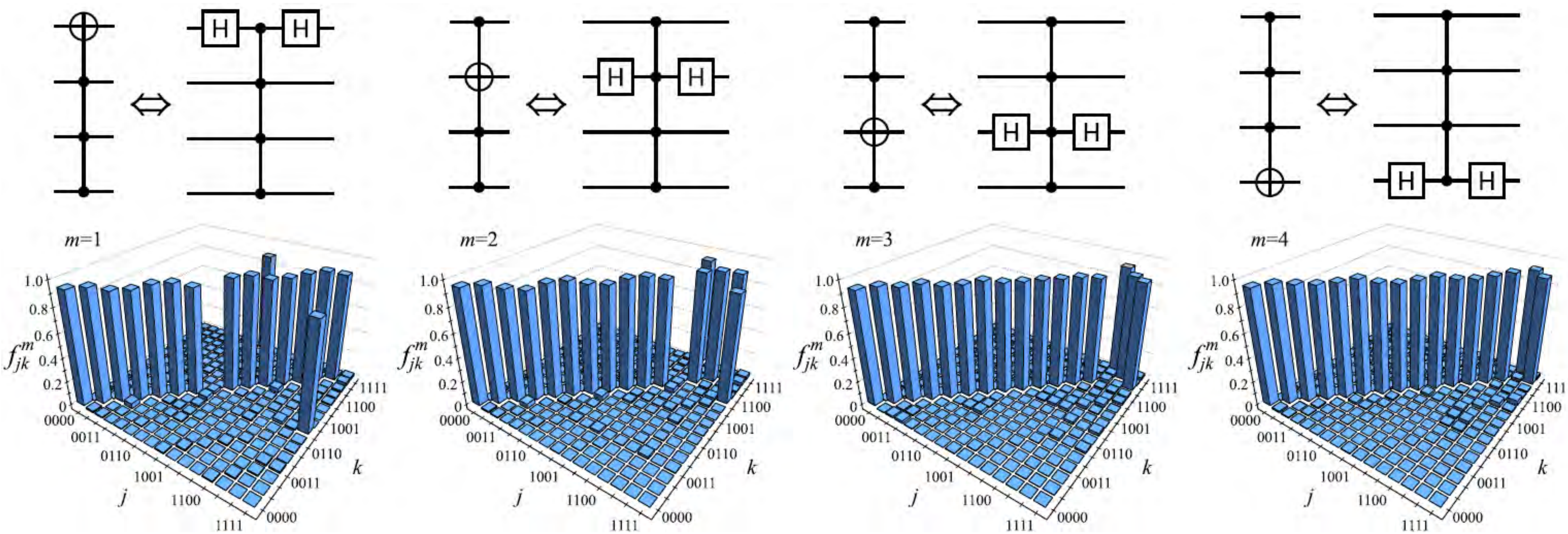
Truth tables of four-qubit Toffoli gates



The four-qubit C^3Z gate is equivalent to a four-qubit Toffoli gate up to single-qubit Hadamard transforms on the target qubit

The truth tables were measured for the four possible choices of the target qubit

Generalized Hofmann bound on C^3Z gate fidelity



$F_1 = 0.943(1)$ $F_2 = 0.952(1)$ $F_3 = 0.944(1)$ $F_4 = 0.955(1)$

$$F_1 + F_2 + F_3 + F_4 - 3 \leq F_{C^3Z} \leq \min_m F_m$$

$$0.794(2) \leq F_{C^3Z} \leq 0.943(1)$$

M. Mičuda, M. Sedlák, I. Straka, M. Miková, M. Dušek, M. Ježek, J. Fiurášek, Phys. Rev. Lett. **111**, 160407 (2013).
H.F. Hofmann, Phys. Rev. Lett. **94**,160504 (2005).

Generation of four-qubit entangled state by the C^3Z gate

Unitary C^3Z gate:

$$U_{\mathrm{C^3Z}} = I - 2|1111\rangle\langle 1111|$$

Input product four-qubit state:

$$|++++\rangle$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

Generation of four-qubit entangled state by the C^3Z gate

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Input product four-qubit state:

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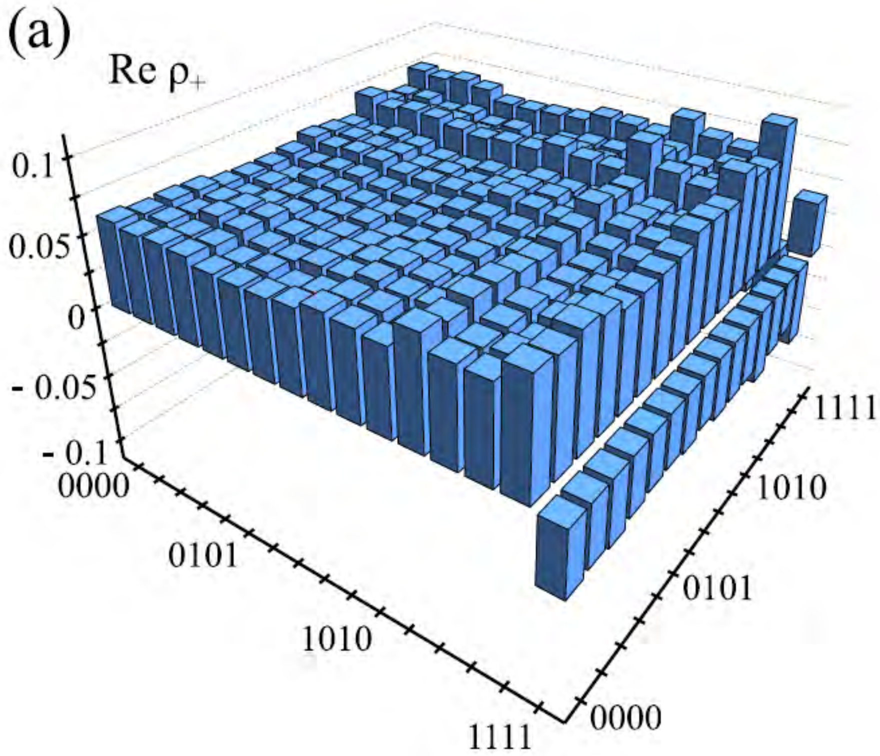
$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

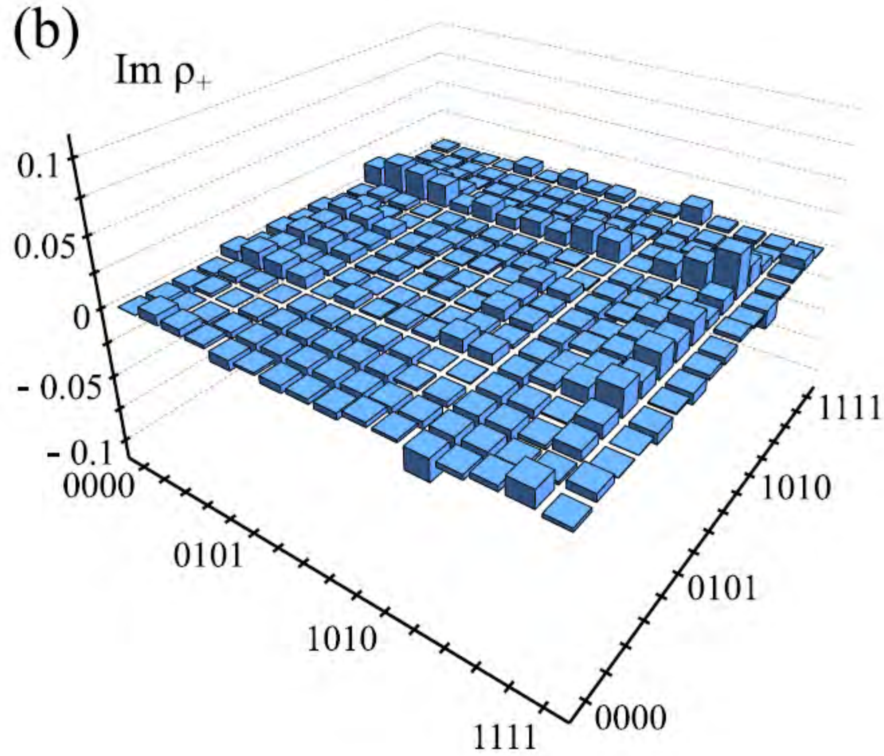
Output entangled four-qubit state:

$$|++++\rangle - \frac{1}{2}|1111\rangle$$

The output state belongs to the family of four-qubit GHZ states.

Reconstructed four-qubit state

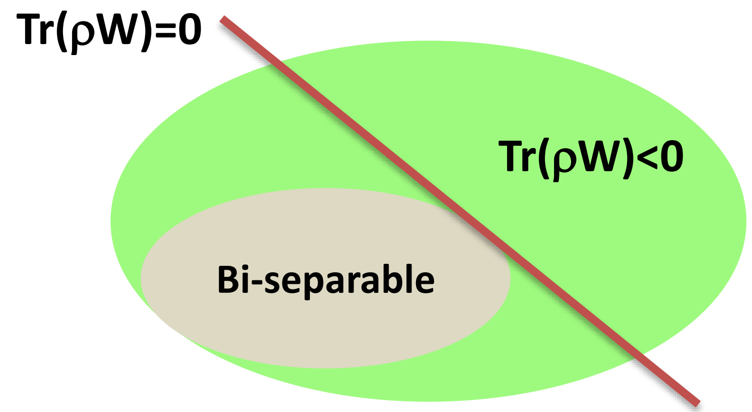




Quantum state fidelity F=0.942(2)

Quantum state purity P=0.931(2)

Certification of genuine four-partite entanglement



Application of entanglement witnesses

M. Lewenstein, B. Kraus, J. I. Cirac, and P. Horodecki, Phys. Rev. A **62**, 052310 (2000).

G. Toth and O. Guhne, Phys. Rev. Lett. **94**, 060501 (2005).

B. Jungnitsch, T. Moroder, and O. Guhne, Phys. Rev. A **84**, 032310 (2011).

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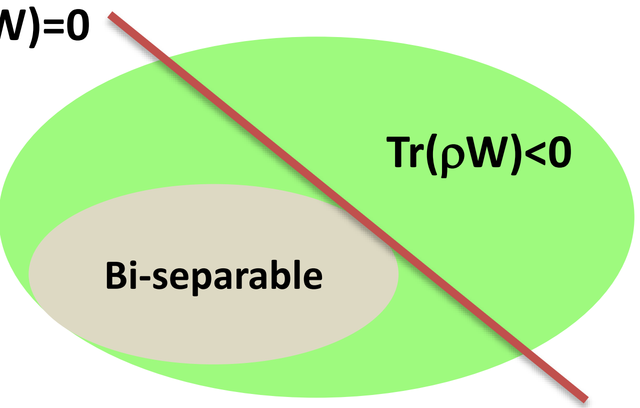

Standard optimal witness for a GHZ state

$$W_{\mathrm{GHZ}} = \frac{1}{2} I - |\mathrm{GHZ}_\phi\rangle\langle\mathrm{GHZ}_\phi| \qquad |\mathrm{GHZ}_\phi\rangle = \frac{1}{\sqrt{2}}(|\phi\phi\phi\phi\rangle - |\phi_\perp\phi_\perp\phi_\perp\phi_\perp\rangle)$$

Optimal projector witness for the generated state

$$W_{\mathrm{proj}} = \frac{7}{8} I - |\Psi_{4+}\rangle\langle\Psi_{4+}|$$

Witness based on transformation of the generated state onto a canonical GHZ state

$$W_{\mathrm{filter}} = \frac{1}{2} G^\dagger G - G^\dagger |\mathrm{GHZ}_0\rangle\langle\mathrm{GHZ}_0| G \qquad G = g^{\otimes 4} \qquad g = \frac{1}{2^{1/4}} |0\rangle\langle 0| + |1\rangle\langle -|$$

Performance of the entanglement witnesses

	W_{GHZ}	W_{filter}	W_{proj}
$\langle W \rangle$	$-0.112(2)$	$-0.0146(3)$	$-0.067(2)$
$\mathscr{S}$	53	49	44
$p_{\max}$	$\frac{2}{9} \approx 0.222$	$\frac{1}{(2^{1/4}+1)^4-2} \approx 0.048$	$\frac{2}{15} \approx 0.133$

Significance of the entanglement test

$$\mathscr{S} = -\langle W \rangle / \Delta W$$

Maximum tolerable fraction of white noise p_{max} - maximum value of p such that the witness still detects multipartite entanglement of the following mixed state

$$(1-p)|\Psi_{4+}\rangle\langle\Psi_{4+}| + \frac{p}{16} I$$

B. Jungnitsch, S. Niekamp, M. Kleinmann, O. Guhne, H. Lu, W.-B. Gao, Y.-A. Chen, Z.-B. Chen, and J.-W. Pan, Phys. Rev. Lett. **104**, 210401 (2010).

Monte Carlo sampling of quantum gate fidelity

Normalized overlap of Choi matrices:

$$F_{\mathrm{C^3Z}} = \frac{\mathrm{Tr}(\chi\chi_{\mathrm{C^3Z}})}{\mathrm{Tr}(\chi)\mathrm{Tr}(\chi_{\mathrm{C^3Z}})}$$

Expansion of Choi matrix of $\mathrm{C^3Z}$ gate into superposition of tensor products of Pauli operators:

$$\chi_{C^3Z} = \sum_{j=1}^{1936} a_j \Sigma_j \qquad \Sigma_j = \sigma_{j_1} \otimes \sigma_{j_2} \otimes \sigma_{j_3} \otimes \sigma_{j_4} \otimes \sigma_{j_5} \otimes \sigma_{j_6} \otimes \sigma_{j_7} \otimes \sigma_{j_8}$$

S. T. Flammia and Y.-K. Liu, Phys. Rev. Lett. **106,** 230501 (2011);

M. P. da Silva, O. Landon-Cardinal, and D. Poulin, Phys. Rev. Lett. **107,** 210404 (2011).

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Expansion of Choi matrix into linear combination of tensor products of single-qubit projectors:

$$\chi_{\mathrm{C^3Z}} = \sum_{k=1}^{N_+} b_k^+ \Pi_k^+ - \sum_{k=1}^{N_-} b_k^- \Pi_k^- \qquad \Pi_k^\pm = \pi_{k_1}^\pm \otimes \pi_{k_2}^\pm \otimes \pi_{k_3}^\pm \otimes \pi_{k_4}^\pm \otimes \pi_{k_5}^\pm \otimes \pi_{k_6}^\pm \otimes \pi_{k_7}^\pm \otimes \pi_{k_8}^\pm$$

S. T. Flammia and Y.-K. Liu, Phys. Rev. Lett. **106,** 230501 (2011);
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$N_+=22416$ $N_-=22400$

Monte Carlo sampling of quantum gate fidelity

Expansion of Choi matrix into linear combination of tensor products of single-qubit projectors:

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$N_+=22416$ $N_-=22400$

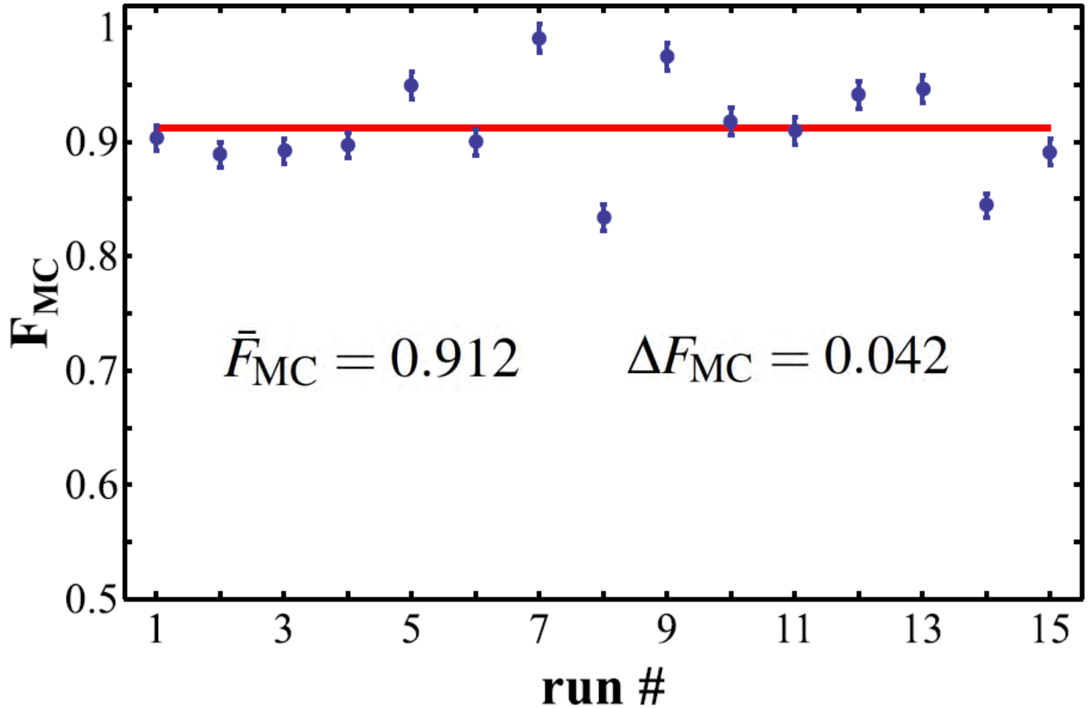
Random sampling o the positive and negative terms according to probability distributions:

$$p_k^+ = \frac{b_k^+}{B_+}, \qquad p_k^- = \frac{b_k^-}{B_-}, \qquad B_+ = \sum_{k=1}^{N_+} b_k^+, \qquad B_- = \sum_{k=1}^{N_-} b_k^-$$

Monte Carlo estimate of gate fidelity:

$$F_{\mathrm{MC}} = \frac{1}{\mathrm{Tr}[\chi]\mathrm{Tr}[\chi_{\mathrm{C^3Z}}]} \left(\frac{B_+}{M_+} \sum_{m=1}^{M_+} \langle \Pi_{c_m}^+ \rangle - \frac{B_-}{M_-} \sum_{m=1}^{M_-} \langle \Pi_{d_m}^- \rangle \right)$$

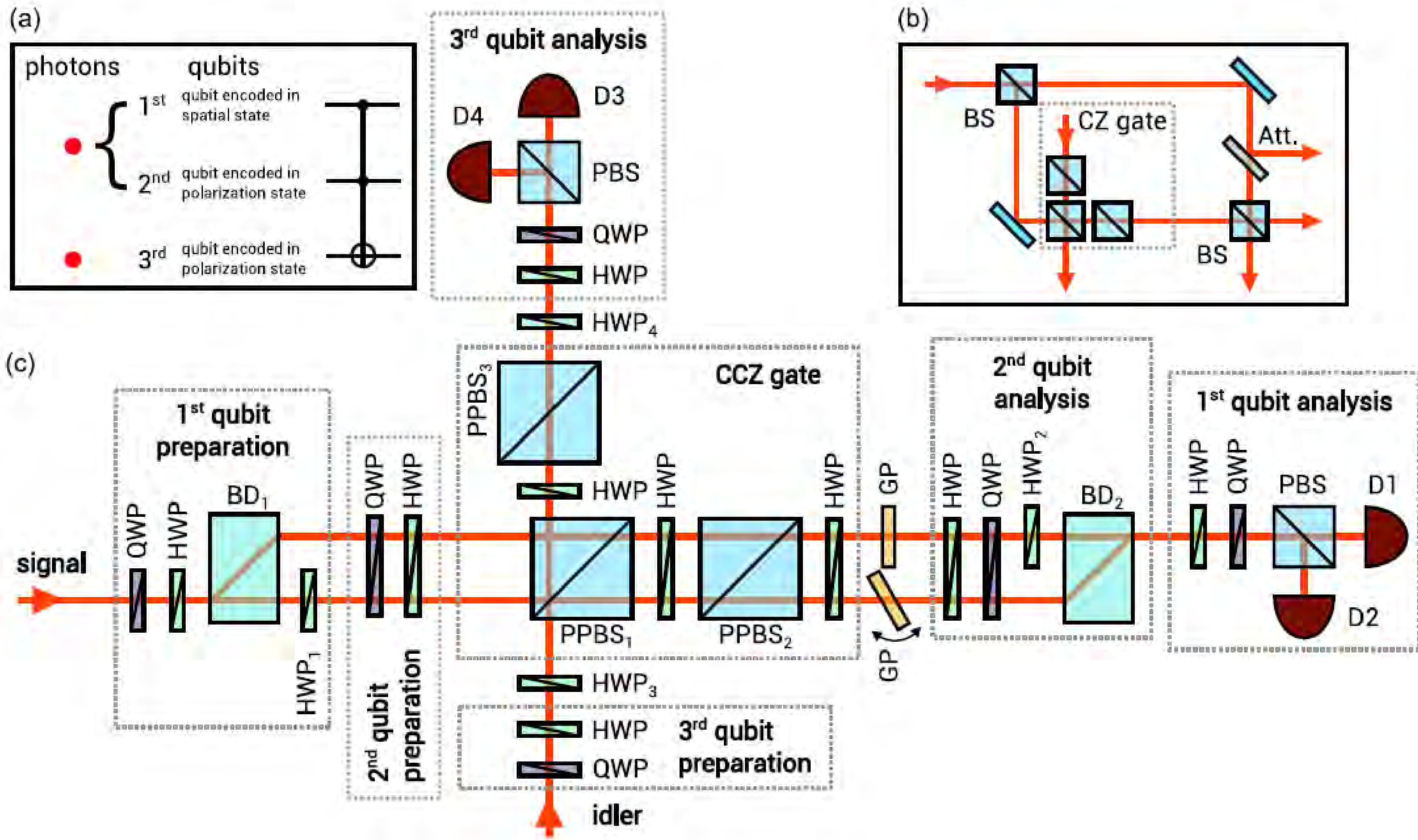
Experimental Monte Carlo Sampling



Optimal sampling strategy is asymmetric: $M_+/M_-=10$, we choose $M_+=1000$, $M_-=100$

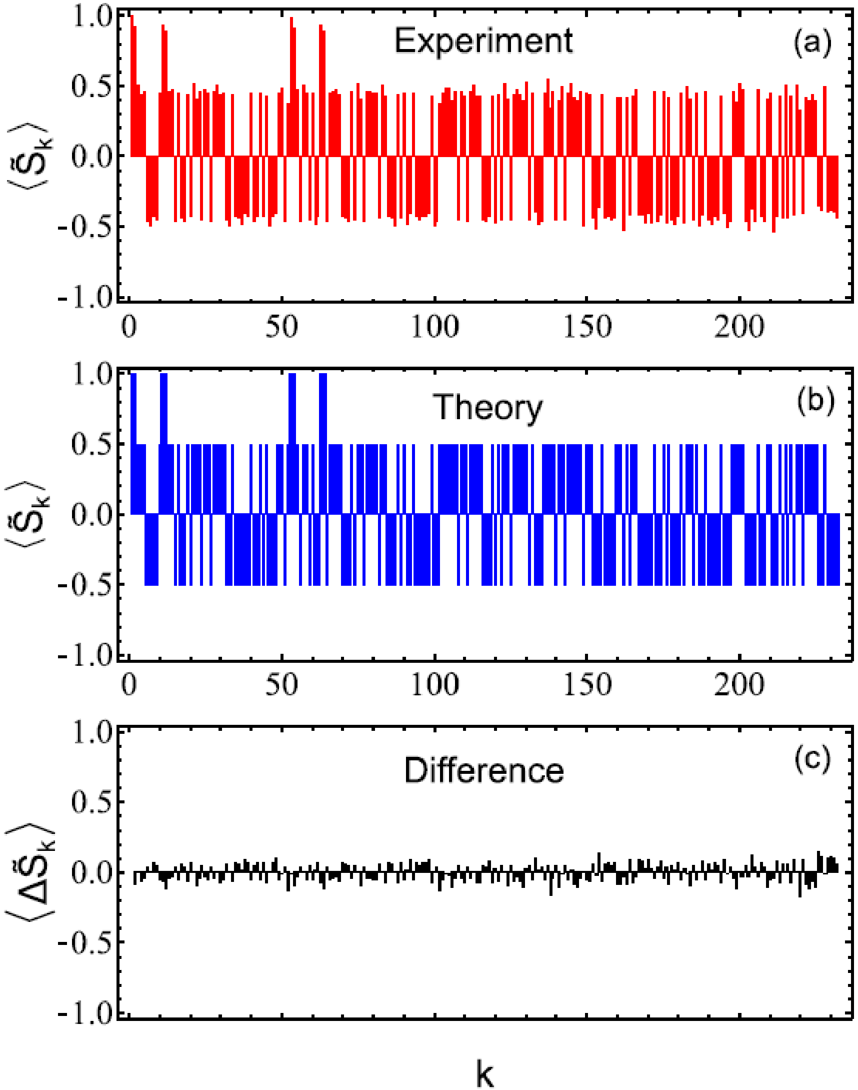
Systematic error due to finite number of samples: $\langle(\Delta F_{\mathrm{MC}})^2\rangle_{\min} \approx \frac{2.496}{M_T}$

Comparison with three-qubit quantum CCZ/Toffoli gate



M. Mičuda, M. Sedlák, I. Straka, M. Miková, M. Dušek, M. Ježek, and J. Fiurášek, Phys. Rev. Lett. **111**, 160407 (2013).

Exact estimation of fidelity of three-qubit CCZ gate



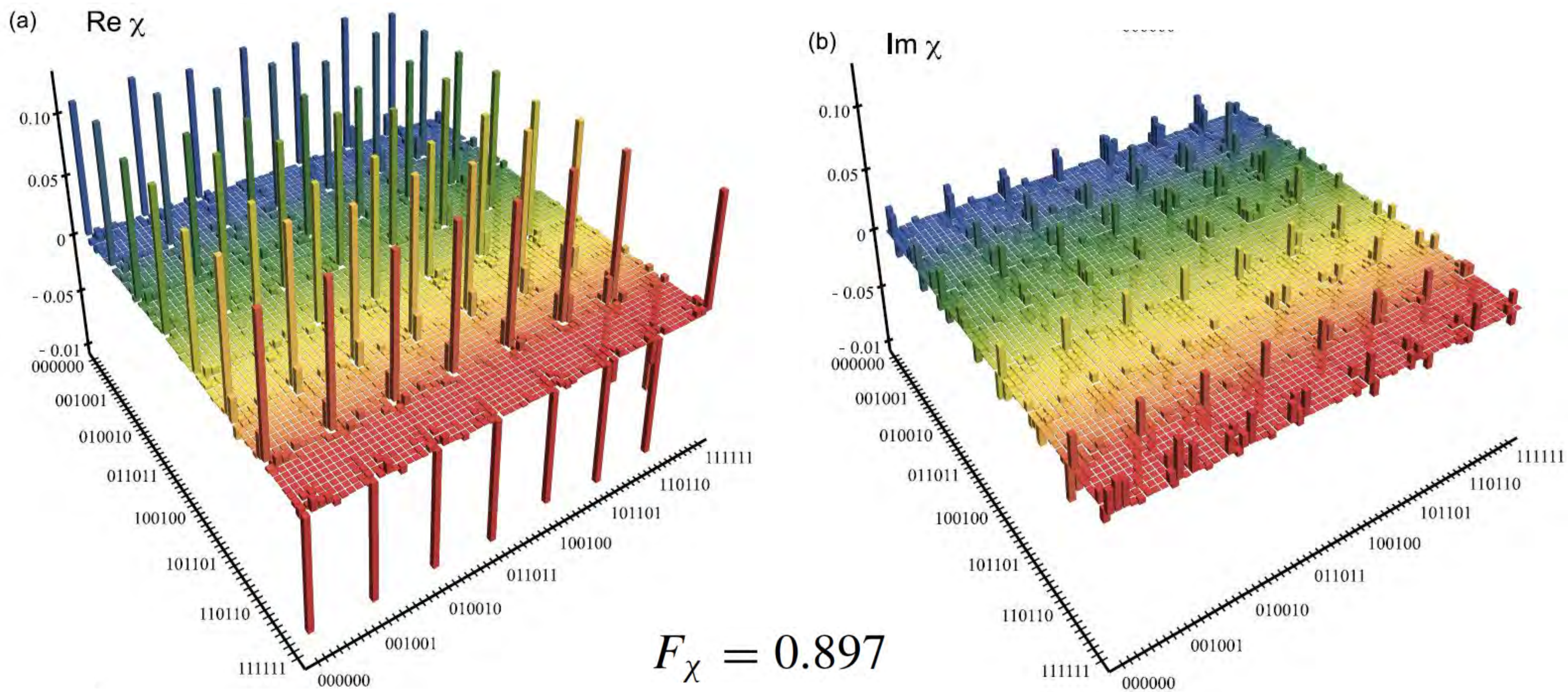
$$F_\chi = \frac{1}{64}\sum_{k=1}^{232} A_k \langle \tilde{S}_k \rangle$$

Experimental estimation of all 232 mean values of 6-fold tensor products of single-qubit Pauli operators.

$$F_\chi = 0.894(2)$$

M. Mičuda, M. Miková, I. Straka, M. Sedlák, M. Dušek, M. Ježek, and J. Fiurášek, Phys. Rev. A **92**, 032312 (2015).

Full reconstruction of Choi matrix of three-qubit CCZ gate

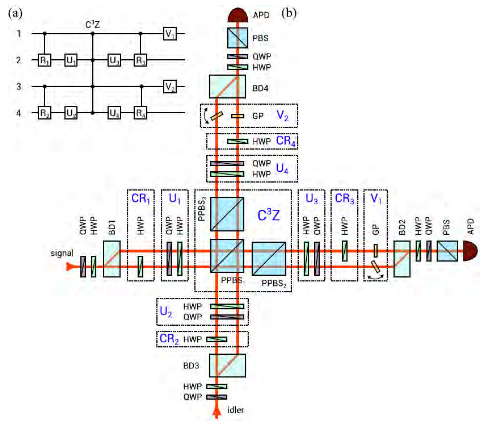


Reconstruction from incomplete data: 1570 linearly independent data vs. 4096 parameters characterizing the three-qubit quantum operation.

Maximum Likelihood-Maximum entropy reconstruction algorithm was utilized

M. Mičuda, M. Miková, I. Straka, M. Sedlák, M. Dušek, M. Ježek, and J. Fiurášek, Phys. Rev. A **92**, 032312 (2015).

Conclusions



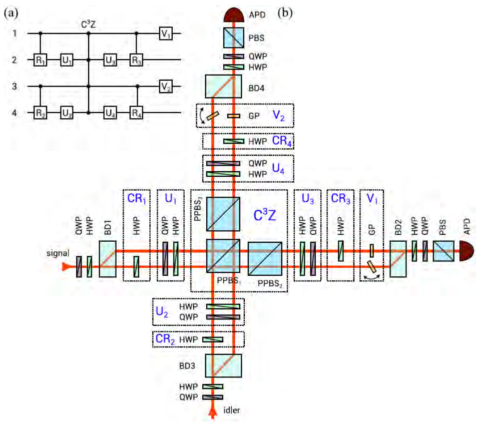
Four-qubit quantum C^3Z gate was experimentally demonstrated and characterized

Multipartite entanglement of four-qubit state was certified using suitable entanglement witnesses

	W_{GHZ}	W_{filter}	W_{proj}
$\langle W \rangle$	$-0.112(2)$	$-0.0146(3)$	$-0.067(2)$
$\mathscr{S}$	53	49	44
$p_{\max}$	$\frac{2}{9} \approx 0.222$	$\frac{1}{(2^{1/4}+1)^4-2} \approx 0.048$	$\frac{2}{15} \approx 0.133$

R. Stárek, M. Mičuda, M. Miková, I. Straka, M. Dušek, M. Ježek, and J. Fiurášek, submitted.

Conclusions



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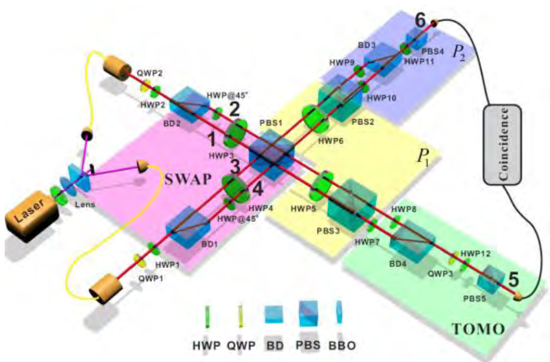
Possible modifications and extensions of the scheme:

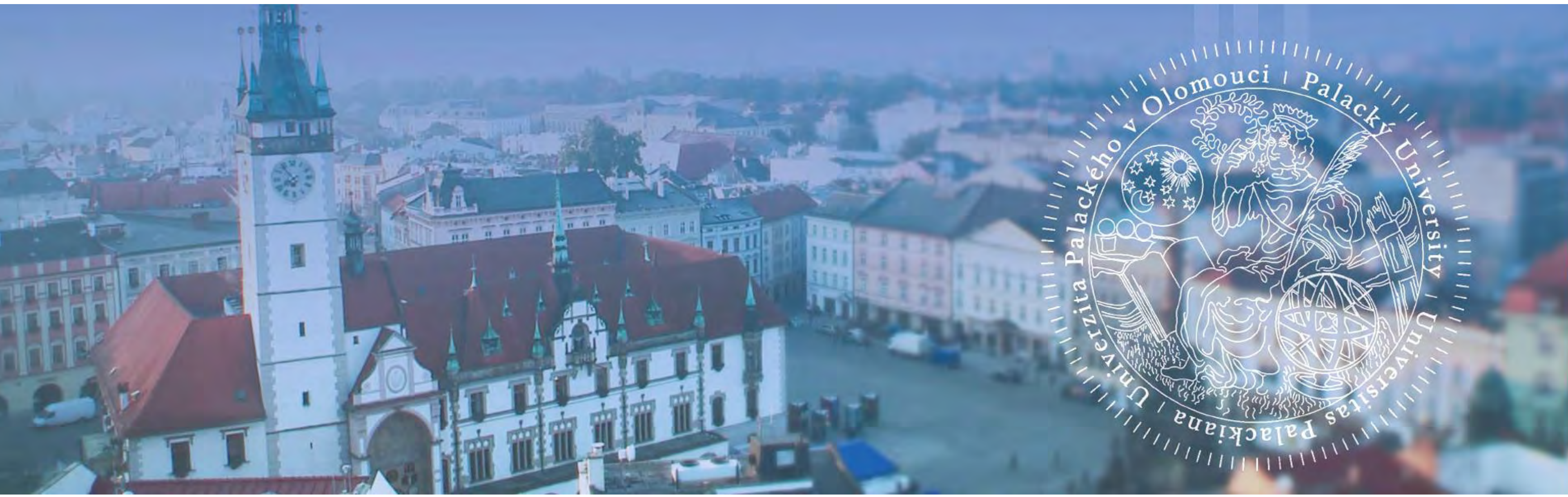
- creation of superpositions of unknown quantum states
- implementation of quantum Fredkin gate
- experiments with qudits (multiple path interferometers)



R. Stárek, M. Mičuda, M. Miková, I. Straka, M. Dušek, M. Ježek, and J. Fiurášek, submitted.

X.-M. Hu, M.-J. Hu, J.-S. Chen, B.-H. Liu, Y.-F. Huang, C.-F. Li, G.-C. Guo, and Y.-S. Zhang, arXiv:1605.02339.

Thank you for your attention!



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