Quantum nonlinear gates with continuous variables

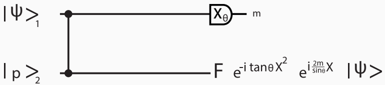
A measurement based approach to nonlinear interaction

Seckin Sefi

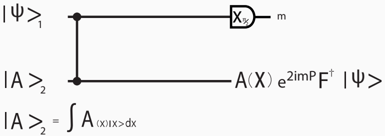
Teleportation-based implementations of quantum optical continuous-variable gates

- Can we generate arbitrary quantum optical evolution using available experimental tools without resorting to highly nonlinear medium such as Kerr medium.
- Driving a quantum system to a desired state by coupling the input to a ancilla and then making a measurement on to the input

Quadratic gate teleportation



Nonlinear gate teleportation



Toolbox

Gaussian operators

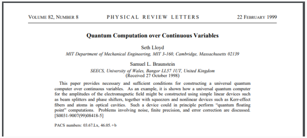
- Beam splitters $\equiv e^{\theta(\hat{a}_2^\dagger \hat{a}_1 - \hat{a}_1^\dagger \hat{a}_2)}$
- Phase shifters $\equiv e^{i\theta \hat{a}^\dagger \hat{a}}$
- Displacement operators $\equiv e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}}$
- Squeezing operation $\equiv e^{\frac{r(\hat{a}^2 - \hat{a}^{\dagger 2})}{2}}$
- Homodyne detection

Non-Gaussian operators

- Photon detectors
- Photon subtraction
- Marek, Filip, Furusawa method $\Rightarrow \int e^{itx^3} dx$ or even $\int e^{itx^4} dx$ $\Rightarrow e^{itX^3}$ or $\Rightarrow e^{itX^4}$
- Adaptive measurements

Corresponding mathematical problem

$$e^{it\mathcal{H}(a,a^\dagger)} \stackrel{?}{=} \{e^{\theta(\hat{a}_2^\dagger \hat{a}_1 - \hat{a}_1^\dagger \hat{a}_2)}, e^{i\theta \hat{a}^\dagger \hat{a}}, e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}}, e^{\frac{r(\hat{a}^2 - \hat{a}^{\dagger 2})}{2}}, e^{itX^3}\}$$


VOLUME 82, NUMBER 8 PHYSICAL REVIEW LETTERS 22 FEBRUARY 1999

Quantum Computation over Continuous Variables

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(Received 27 October 1998)

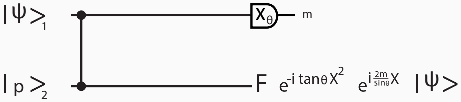
This paper provides necessary and sufficient conditions for constructing a universal quantum computer over continuous variables. As an example, it is shown how a universal quantum computer for the amplitudes of the electromagnetic field might be constructed using simple linear devices such as beam splitters and phase shifters, together with squeezers and nonlinear devices such as Kerr-effect fibers and atoms in optical cavities. Such a device could in principle perform "quantum floating point" computations. Problems involving noise, finite precision, and error correction are discussed. [S0031-9007(99)08418-5]

PACS numbers: 03.67.Lx, 46.05.+b

- Aims to prove the principle. Are all operators accessible?
- Lacking a systematic recipe for converting the optical tools that are presented into an arbitrary unitary operator using the continuous variable gate teleportation model.
- How to avoid redundancy?

Gaussian decompositions

- $$e^{it\mathcal{H}_G(a,a^\dagger)} \stackrel{?}{=} \{e^{\theta(\hat{a}_2^\dagger \hat{a}_1 - \hat{a}_1^\dagger \hat{a}_2)}, e^{i\theta \hat{a}^\dagger \hat{a}}, e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}}, e^{\frac{r(\hat{a}^2 - \hat{a}^{\dagger 2})}{2}}\}$$
- Gaussian operators corresponds to linear mode transformations, thus matrix representation and matrix decomposition techniques can be used
- Two level matrix decomposition - Reck, Zeilinger decomposition
- Singular value decomposition - Bloch, Messiah decomposition



- Decomposition to quadratic operator and the (*Universal linear Bogoliubov transformations through one-way quantum computation, PRA, 2010*)
- $$e^{it\mathcal{H}_G(a,a^\dagger)} = \{F, e^{itX^2}, e^{itX}\}$$

Decomposing non-Gaussian operator

- $e^{it\mathcal{H}(a,a^\dagger)} \stackrel{?}{=} \{F, e^{itX}, e^{itX^2}, e^{itX^3}\}$
- $e^{it\mathcal{H}(a,a^\dagger)} = e^{\sum_i c_i X^m P^n + c_i^* P^n X^m}$
- $e^{itn^2} = e^{it\left(X^4 + P^4 + X^2 P^2 + P^2 X^2\right)}$

1. Rewrite the Hamiltonian in terms of monomial Hamiltonians and rewrite each monomial Hamiltonian as a linear combination of commutations of the elementary operators you employ
2. Split the Hamiltonian into separate commutation and nested commutation operators and realize each separate element through the tools we posses in such a way that it will fit into our gate teleportation setup.

Operator relations

$$X^m P^n + P^n X^m = -\frac{4i}{(n+1)(m+1)}[X^{m+1}, P^{n+1}] - \frac{1}{n+1}\sum_{k=1}^{n-1}[P^{n-k}, [X^m, P^k]]$$

$$X^m = -\frac{2}{3(m-1)}[X^{m-1}, [X^3, P^2]]$$

$$P^m = -\frac{2}{3(m-1)}[P^{m-1}, [P^3, X^2]]$$

For example Kerr gate is:

$$e^{it\left(X^4+P^4+X^2P^2+P^2X^2\right)} = e^{-it\frac{2}{9}[X^3,[X^3,P^2]]-it\frac{2}{9}[P^3,[P^3,X^2]]-\frac{4}{9}t[X^3,P^3]}e^{-\frac{it}{6}}$$

- $e^{\frac{3}{2}it_1^2t_2X_1^2X_2} = e^{it_1X_1P_2}e^{it_2X_2^3}e^{-2it_1X_1P_2}e^{it_2X_2^3}e^{it_1X_1P_2}e^{-2it_2X_2^3}$
- $e^{it^2X_1^4} = e^{itX_1^2P_2}e^{itX_1^2X_2}e^{-itX_1^2P_2}e^{-itX_1^2X_2}$
- $e^{3it_1^2t_2X_1^2X_2^2} = e^{it_1X_1P_2}e^{it_2X_2^4}e^{-2it_1X_1P_2}e^{it_2X_2^4}e^{it_1X_1P_2}e^{-2it_2X_2^4}e^{-\frac{1}{8}it_1^4t_2X_1^4}$
- $e^{it_1X^3+it_2(XP+PX)} = e^{i\left(-\frac{t_1}{3t_2}(1-e^{-3t_2})\right)X^3}e^{it_2(XP+PX)}$

S.S., van Loock, PRL, 2011

Operator approximations

- Splitting approximations: (Trotter-Suzuki-Yoshida approximations)
  $e^{t(A+B)} = e^{tA}e^{tB} + f(t^2, A, B) + \dots$
  $e^{t(A+B)} = e^{\frac{t}{2}A}e^{tB}e^{\frac{t}{2}A} + f(t^3, A, B) + \dots$
  $e^{t(A+B)} = Q_2(st)Q_2(s't)Q_2(st) + f(t^5, A, B) + \dots$
- Commutation approximation

$$e^{t^2(AB-BA)} = e^{itB}e^{itA}e^{-itB}e^{-itA} + f(t^3) + \dots$$

- $$e^{it^3[B,[B,A]]} = e^{itB}e^{t^2[B,A]}e^{-itB}e^{-t^2[B,A]} + f'(t^4) + \dots$$

  and for the nested commutation operator with reasonable interaction and error, millions of operators are needed
- It is possible to use Suzuki's idea on commutation approximations as well.

Method for approximations

$$\log(e^{c_1 itA} e^{c_2 itB} e^{c_3 itA} ... e^{c_{10} itB}) = \quad f_1 tA + f_2 tB + f_3 t^2[A,B] + f_4 t^3 [A[A,B]] \\ + f_5 t^3 [B,[A,B]] + f_6 t^4 [A,[A,[A,B]]] \\ + f_7 t^4 [B,[A,[A,B]]] + f_8 t^4 [B,[B,[A,B]]] + ...$$

Can be calculated by repeatedly applying *Baker Campbell Hausdorff* formulas or any equivalent method: **M.Reinsch**, *A simple expression for the terms in the Baker-Campbell-Hausdorff series* 2000.

Solve the polynomials: $f_1, f_2, ...,$ obtain the values of $c_1, c_2, ...$ which is another technical difficulty. *Homotopy continuation method* is recommended.

$$e^{t^2[A,B]} = e^{1.2itA} e^{-itB} e^{-2.121itA} e^{-1.680itB} e^{-0.019itA} e^{1.602itB} e^{2.025itA} e^{2.743itB} e^{0.061itA} \\ e^{-1.170itB} e^{-3.235itA} e^{0.018itB} e^{1.519itA} e^{-0.989itB} e^{0.571itA} e^{0.476itB} + f(t^6, A, B) + \ldots$$

$$e^{it^3[A,[A,B]]} = e^{0.5itB}e^{-itA}e^{-itB}e^{itA}e^{itB}e^{itA}e^{-itB}e^{-itA}e^{0.5itB} + f(t^5, A, B) + \dots$$

$$e^{it^3[A,[A,B]]} = e^{0.5itB}e^{-0.912itA}e^{-itB}e^{2.439itA}e^{-0.531itB}e^{0.184itA}e^{2itB}e^{-0.477itA}$$
$$e^{-1.659itB}e^{-0.761itA}e^{1.465itB}e^{1.181itA}e^{-1.312itB}e^{-1.654itA}e^{0.537itB} + f(t^6, A, B) + \dots$$

Use the concatenations of first-step approximations. For example a fifth order nested commutation:

$Q_5(p_i t) : e^{i(p_i t)^3[A,[A,B]] + (p_i t)^6 F + (p_i t)^7 G + \dots}$

$Q_9(t) = \prod_i Q_5(p_i t)$

$$e^{it^3[A,[A,B]]} = Q_5(t)Q_5(1.2t)Q_5(-0.948048t)Q_5(-0.830428t)Q_5(1.049237t)$$
$$Q_5(-0.88766t)Q_5(1.35820t)Q_5(1.32998t) + f(t^{10}, A, B)$$

For interaction time 0.1 and the dominant error term 10^{-3}, number of operators for the commutation operator reduced from 4000 to 45 and for nested commutation 10^7 to 120

Kerr gate

Expressing Kerr interaction $e^{it(X^2+P^2)^2}$ through the elementary gate set

$$e^{it(X^4+X^2P^2+P^2X^2+P^4)} \stackrel{?}{=} \{F, e^{it_1X}, e^{it_2X^2}, e^{it_3X^3}\}$$

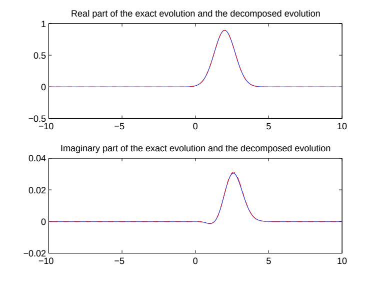
$\Rightarrow e^{-it\frac{2}{9}[X^3,[X^3,P^2]]-it\frac{2}{9}[P^3,[P^3,X^2]]-\frac{4}{9}t[X^3,P^3]}$

$$e^{i0.5t'P^2}e^{-it'X^3}e^{-it'P^2}e^{it'X^3}e^{it'P^2}e^{it'X^3}e^{-it'P^2}e^{-it'X^3}e^{i0.5t'P^2}$$
$$e^{i0.5t'X^2}e^{-it'P^3}e^{-it'X^2}e^{it'P^3}e^{it'X^2}e^{it'P^3}e^{-it'X^2}e^{-it'P^3}e^{i0.5t'X^2}$$
$$e^{it''P^3}e^{it''X^3}e^{-it''P^3}e^{-it''X^3}$$

$t' = -0.605706t^{1/3}$

Numerical simulation of the decomposition

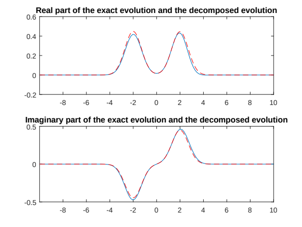
Exact evolution and the decomposed evolution is applied to the same coherent state



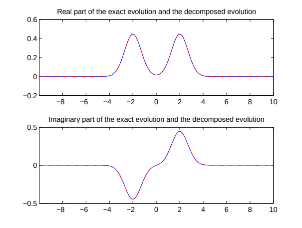
- Interaction time is 10^{-3}
- $(1 - |\langle \psi_{exact} | \psi_{approximate} \rangle|) < 10^{-6}$

- Errors depends on the interaction time. 10^{-2} Err.$< 10^{-2}$
- Depends on the input state, av. 25 photon Err.$< 10^{-2}$.
- Depends on the quality of the operators, a first order approximate cubic phase gate increase errors. Err.$< 10^{-2}$
- Higher order interactions can be generated by multiplying decompositions for weak interactions.

Initial: 10^{-3} Total: $\pi/2$, Err. $< 10^{-2}$



Initial: 0.6×10^{-3} Total: $\pi/2$, higher order approximation. Err. $< 10^{-3}$



Kerr decomposition with minimal teleportations

- Avoid any specific operator approximations and instead, utilize a more brute force approach especially tailored for the Kerr interaction.
- Decomposing the Kerr interaction using the concatenations of the following set of interactions:

$$\left\{e^{itX^3+itX^4}, e^{itP^3+itP^4}\right\}. \tag{1}$$

-

$$e^{ip_1 t^{1/2}P^3+ip_2 tP^4}e^{ip_3 t^{1/2}X^3+ip_4 tX^4}e^{ip_5 t^{1/2}P^3+ip_6 tP^4}e^{ip_7 t^{1/2}X^3+ip_8 tX^4}. \tag{2}$$

-

$$e^{it\left(X^2+P^2-\frac{1}{2}\right)^2} \propto e^{t\left(iX^4+iP^4+\frac{4}{9}[X^3,P^3]\right)}$$

$$e^{in^2} \approx e^{it^{1/2}P^3}e^{\frac{4}{9}it^{1/2}X^3}e^{-it^{1/2}P^3+itP^4}e^{-i\frac{4}{9}t^{1/2}X^3+itX^4}. \tag{3}$$

We found similar errors

S.S., Vaibhav, van Loock, PRA, 2013

Two mode operators

- An entangling gate: beam splitter or equivalently $e^{itX_1 \otimes X_2}$ required
-
$$P_1^n \otimes P_2^s = -\frac{1}{(n+1)(s+1)}[P_2^{s+1}, [P_1^{n+1}, X_1 \otimes X_2]].$$
- Single mode operator relations also required to tailor the interaction for each mode.
- Are these operators accesible?
$O_1(X_1, P_1) \otimes O_2(X_2, P_2) \otimes ... + O_1^\dagger(X_1, P_1) \otimes O_2^\dagger(X_2, P_2) \otimes ...$
For example:
$X_1 P_1 \otimes X_2 P_2 + P_1 X_1 \otimes P_2 X_2$

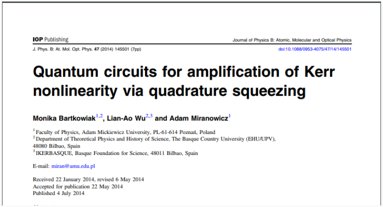
- Cross Kerr gate

$$e^{it(X^2+P^2)_1\otimes(X^2+P^2)_2} \stackrel{?}{=} \left\{e^{i\theta(X^2+P^2)}, e^{it_1X}, e^{it_2X^2}, e^{it_3X^3}, e^{itX_1\otimes X_2}\right\}$$

$$e^{it([P_2^3,[P_1^3,X_1\otimes X_2]]-[X_2^3,[P_1^3,X_1\otimes P_2]]-[P_2^3,[X_1^3,P_1\otimes X_2]]+[X_2^3,[X_1^3,P_1\otimes P_2]])}$$

- $e^{it(X^2+P^2)_1\otimes(X^2+P^2)_2} \approx e^{itX_1^2\otimes X_2^2}e^{itX_1^2\otimes P_2^2}e^{itP_1^2\otimes X_2^2}e^{itP_1^2\otimes P_2^2}$
  $e^{3it_1^2t_2X_1^2X_2^2} = e^{it_1X_1P_2}e^{it_2X_2^4}e^{-2it_1X_1P_2}e^{it_2X_2^4}e^{it_1X_1P_2}e^{-2it_2X_2^4}e^{-\frac{1}{8}it_1^4t_2X_1^4}$
- When applied to Fock states leads to entanglement and errors are smaller since the input is weak energy. We can use a decomposition for 0.15. Applied 20 times to get an amplitude of π. Errors are smaller than 10^{-3}.

Amplifying weak interactions through Gaussian operators.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Quantum circuits for amplification of Kerr nonlinearity via quadrature squeezing

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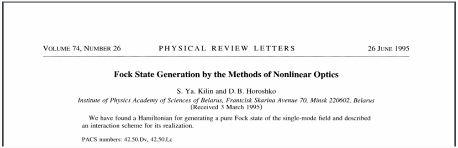
- One can use squeezing to amplify the weak cross Kerr interaction with the condition that the second mode is limited to single photon level
- $\Gamma_3 = \frac{1}{2}(2n_1+1)(2n_2-1) \quad \Gamma_2 = \frac{i}{2}(a_1^2 - a_1^{\dagger 2}) \quad \Gamma_1 = \frac{1}{2}(a_1^2 + a_1^{\dagger 2})(2n_2-1)$
- $\Gamma_1, \Gamma_2, \Gamma_3$ satisfy the commutation relations of the generators of the SU(1,1) group
- $e^{i\gamma\Gamma_3} = e^{i\theta_1\Gamma_2} e^{i\sigma\Gamma_3} e^{i\theta_2\Gamma_2} e^{i\sigma\Gamma_3} e^{i\theta_1\Gamma_2}$
- How does it fit into our gate teleportation scheme?
- State dependent decomposition and amplifying non-Gaussian states through Gaussian operators. Might be applied to many operators.

Decomposing Artificial Interactions

Hamiltonians of Fock space

Transformation $\rightarrow$ Hamiltonians $\rightarrow$ Decompositions



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Fock State Generation by the Methods of Nonlinear Optics

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We have found a Hamiltonian for generating a pure Fock state of the single-mode field and described an interaction scheme for its realization.

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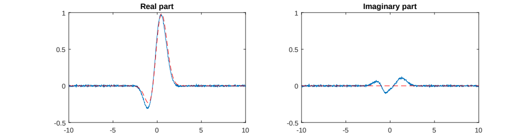
Is it possible to express arbitrary unitary transformations acting on finite dimensional Fock space in terms of interactions;

- $$H_{ansatz} = f_0(a^\dagger a) + \sum_{n=1}^{\infty} f_n(a^\dagger a)a^n + (a^\dagger)^n f_n^*(a^\dagger a)$$
- $H^2 = I \quad \Rightarrow \quad e^{i\theta H_{ansatz}} = \cos(\theta)I + i\sin(\theta)H$
- $e^{i\theta H_{ansatz}} |0\rangle = cos(\theta)|0\rangle + isin(\theta)(f_0(0)|0\rangle + \sum_{n=1}^{\infty} \sqrt{n!} f_n^*(0)|n\rangle)$
- $e^{i\theta H_{ansatz}} |1\rangle = cos(\theta)|1\rangle + isin(\theta)(f_0(1)|1\rangle + f_1(0)|0\rangle + \sum_{n=1}^{\infty} \sqrt{(n+1)!} f_n^*(1)|n+1\rangle)$

Hadamard interaction

- $$e^{i\frac{\pi}{2}H}|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
$$e^{i\frac{\pi}{2}H}|1\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$
- $\exp(i\theta(\sqrt{2} + 2\sqrt{2}X - \sqrt{2}X^2 - \sqrt{2}P^2 - \sqrt{2}X^3 + i\frac{\sqrt{2}}{3}[P^3, X^2]))$
- $e^{-i\theta\sqrt{2}P^2} e^{i\sqrt{\theta\frac{\sqrt{2}}{3}}P^3} e^{i\sqrt{\theta\frac{\sqrt{2}}{3}}X^2} e^{-i\sqrt{\theta\frac{\sqrt{2}}{3}}P^3} e^{i\theta 2\sqrt{2}X} e^{-i\theta\sqrt{2}X^2} e^{-i\sqrt{\theta\frac{\sqrt{2}}{3}}X^2} e^{-i\theta\sqrt{2}X^3}$
- Initial decomposition for 0.15, applied for 10 times. The fidelity is around $1 - 10^{-2}$



- Solution set is not unique.
- Arbitrary transformation can be reduced to Hamiltonians using two level matrix decomposition techniques in arbitrary dimension.

Non-Gaussianity vs Clifford

C.V. picture

- $|0\rangle \rightarrow |0\rangle + |1\rangle$
  non-Gaussian
- Entangling is cross Kerr
  non-Gaussian
- No algorithm known for simulation

D.V. picture

- $|0\rangle \rightarrow |0\rangle + |1\rangle$ Clifford operation
- Entangling is Clifford operation
- Can be simulated classically using Gottesmann-Knill theorem

A photon counting Hamiltonian

- Transformation → Hamiltonian
-

$$e^{i\theta(X_1^2+P_1^2)P_2}\left(\sum_n c_n |n\rangle_1\right)|x\approx 0\rangle_2 = \sum_n c_n |n\rangle_1 \left|x\approx -\frac{\theta}{2}(n+\frac{1}{2})\right\rangle_2$$

- Shift in the second mode depends on the photon number
- $e^{i\theta(X_1^2+P_1^2)P_2} \approx e^{i\theta X_1^2 P_2} e^{i\theta P_1^2 P_2} + O(\theta^2)$
- $e^{it_1 t_2 X_1^2 X_2} = e^{it_1 P_1 X_2} e^{i\frac{t_2}{3}X_1^3} e^{-2it_1 P_1 X_2} e^{-i\frac{t_2}{3}X_1^3} e^{it_1 P_1 X_2}$
- Depends on initial squeezing or repeated implementation
- Kerr interaction can be exactly realized through this interaction

$$e^{i\theta(X_1^2+P_1^2)^2} = e^{i\theta(X_1^2+P_1^2)P_2} e^{i\theta(X_1^2+P_1^2)X_2} e^{-i\theta(X_1^2+P_1^2)P_2} e^{-i\theta(X_1^2+P_1^2)X_2}$$

Decomposing Measurements

Decomposing X^2 measurement

Cat states are one of the eigenfunctions of this operator thus it has been argued that this measurement high amplitude superposition states and applied in optomechanics.

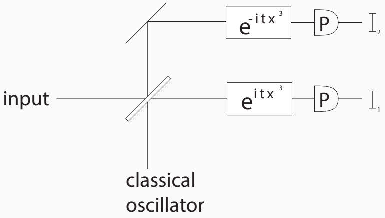
$X^2(|-x\rangle + e^{i\theta}|x\rangle) = x^2(|-x\rangle + e^{i\theta}|x\rangle)$

Not clear
Quadrature eigenstates are also eigenstate of this operator and the superposition states are eigenstate for different phases

Cubic phase gate is almost doing the job

$$e^{itX^3} P e^{-itX^3} \rightarrow P - t\frac{3}{2}X^2$$

Circuit for X^2 measurement



$$
\begin{aligned}
P_1 - \tfrac{3}{2} t X_1^2 \quad &\rightarrow \quad \tfrac{1}{\sqrt{2}} P_1 - \tfrac{1}{\sqrt{2}} P_2 - \tfrac{3}{2} t\left(\tfrac{1}{2} X_1^2 - X_1 X_2 + \tfrac{1}{2} X_2^2\right) \\
P_2 - \tfrac{3}{2}(-t) X_2^2 \quad &\rightarrow \quad \tfrac{1}{\sqrt{2}} P_1 + \tfrac{1}{\sqrt{2}} P_2 - \tfrac{3}{2}(-t)\left(\tfrac{1}{2} X_1^2 + X_1 X_2 + \tfrac{1}{2} X_2^2\right)
\end{aligned}
$$

using classical oscillator and the subtracting two signals lead to the following operator:

$I_2 - I_1$

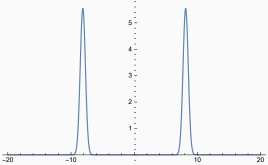
$$\frac{3t}{\sqrt{2}} X_1^2 + \left(-\frac{2}{\sqrt{2}} \alpha_{im} + \frac{3t}{2} \alpha_{re}^2\right) I$$

Projection operator

Projection operator $\int_{-\infty}^{\infty} e^{2i(m_1+m_2)x} Airy(\sigma x^2 - c) e^{x^2} |x\rangle \, dx$

$$\sigma x^2 - c = \frac{3tx^2}{(6t)^{1/3}} - \frac{2(m_2-m_1)}{(6t)^{1/3}} + \frac{2}{3t(6t)^{1/3}} + \frac{-\sqrt{2}\alpha_{im} + i2\sqrt{2}\alpha_{re}}{(6t)^{1/3}}$$

$t = 0.1,\ m_2 - m_1 = 10$



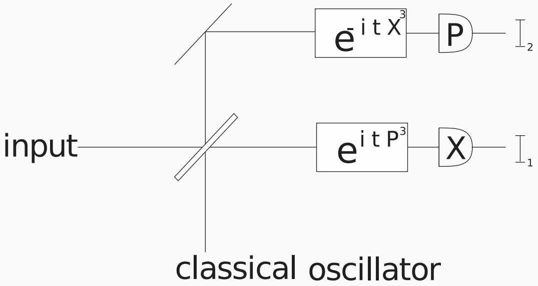
- Interaction strength of the cubic phase gate squeezes the peaks.
- Difference between measurement results increase the distance.
- Doesn't depend on the coherent state amplitude!
- Approximate cubic phase gate?
- Might be generated by injecting a non Gaussian state instead of applying two cubic phase gates.

Computation with superposition states

- Phase information has to be clear.
- Entangling gate
  $e^{i\phi X_1 X_2}(|0\rangle + |x\rangle) \otimes (|0\rangle + |y\rangle) \rightarrow |0\rangle |0\rangle + |0\rangle |y\rangle + |x\rangle |0\rangle + e^{ixy\phi} |x\rangle |y\rangle$
- Phase operation
  $e^{i\phi X}(|0\rangle + |x\rangle) \rightarrow (|0\rangle + e^{i\phi x} |x\rangle)$
- Z measurement is a Homodyne detection.
- Measurement induced Hadamard gate:
  $(\langle 0| + \langle x|)(\alpha |0\rangle |0\rangle + \alpha |0\rangle |y\rangle + \beta |x\rangle |0\rangle - \beta |x\rangle |y\rangle) \rightarrow$
  $(\alpha + \beta) |0\rangle + (\alpha - \beta) |y\rangle$

Photon counting circuit

How to implement $X^2 + P^2$ operator?



Summary

- It is possible to use measurement based nonlinearities in a more constructive way to implement i.e. self-Kerr, cross-Kerr gates.
- Gaussian operations to amplify weak interactions for certain operators. Existing results still not satisfactory.
- Possible to express transformations in terms of interactions and then decompose them such as Fock state transformations, photon counting operator...
- Allows superposition basis measurements.
- Using superposition states for encoding information.