Quantum enhanced classical imaging and metrology

Bohumil Stoklasa[1,3], Jaroslav Rehacek[1], Zdenek Hradil[1], Martin Paur[1], Libor Motka[1,3], L. L. Sanchez-Soto[2]

Department of Optics, Palacky University, Czech Republic [1]
Departamento de Optica, Universidad Complutense, Madrid, Spain [2]
Meopta-optika, Czech Republic [3]

15.11.2017

University-company collaboration

Palacky University- Optics department

Meopta-Optika,
2500 employees
150 people R&D

Quantum state tomography formalism

- The goal is to estimate the quantum state from the measurement data obtained from the ensemble of N identical copies the quantum system
- Measurements are described by a set of positive operators Π_j (POVM operators)
- Due to finite resources, quantum state can be only statistically estimated from probabilities $p_j = \mathrm{Tr}\,\rho\Pi_j$
- The probabilities are measured by the outcome frequencies f_j of the particular measurements $f_j = \frac{n_j}{N}$

Fisher information and estimation

How good is my estimator of $\mathfrak{s}$ in the case of $\varrho(\mathfrak{s})$? Any ubiased estimator must follow Cramer-Rao bound:

$$(\Delta\hat{\mathfrak{s}})^2 \geq \frac{1}{\mathcal{F}}$$

Classical Fisher information: $\mathcal{F} = \int_{-\infty}^{\infty} \varrho_{\mathfrak{s}}(x) \left(\frac{\partial log \varrho_{\mathfrak{s}}(x)}{\partial \mathfrak{s}} \right)^2 \mathrm{d}x$.
Quantum Fisher information: $\mathcal{F}_{\mathcal{Q}} = \mathrm{Tr}[\varrho_{\mathfrak{s}} L_{\mathfrak{s}}^2]$
symmetric logarithmic derivative $L_{\mathfrak{s}}$ is the selfadjoint operator satisfying $\frac{1}{2}(L_{\mathfrak{s}}\varrho_{\mathfrak{s}} + \varrho_{\mathfrak{s}} L_{\mathfrak{s}}) = \partial\varrho_{\mathfrak{s}}/\partial\mathfrak{s}$

Fisher information notes

Quantum Fisher information is an upper bound for a classical Fisher information,
$\mathcal{F} >= \mathcal{F}_{\mathcal{Q}}$.
Quantum fisher information is independent of a measurement process. To test optimality of particular measurement, Fisher information for the measurement has to be computed:
$\mathcal{F}_{\mathcal{M}} >= \mathcal{F}_{\mathcal{Q}}$.

Correspondence classical and quantum description

There is a tight correspondence between beam optics and quantum mechanics.

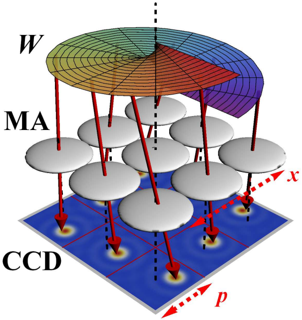
- coherent waves (beam modes) $\rightarrow$ pure states
  $U(x) = \langle x|\psi\rangle$
- partially coherent fields $\rightarrow$ mixed states
  $G(x, x') = \langle x|\rho|x'\rangle$

Wavefront detector tomography

Measurement of optical beams spatial coherence parametrized by a coherence matrix ϱ

$$I(\Delta x_i, \Delta p_j) = \mathrm{Tr}(\varrho |\Pi_{ij}\rangle\langle\Pi_{ij}|)$$

$$\left(\Pi_{ij}\right)_{mn} = \psi_{n,i}(\Delta p_j)\, \psi^*_{m,i}(\Delta p_j)$$

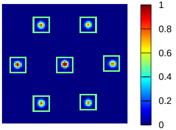


Vortex beam reconstruction

Vortex reconstruction space

$V_\ell = \langle r, \varphi | V_\ell \rangle \propto e^{i\ell\varphi}$

$\ell \in \{-9, -6, -3, 0, +3, +6, +9\}$



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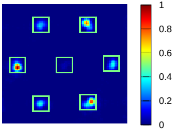

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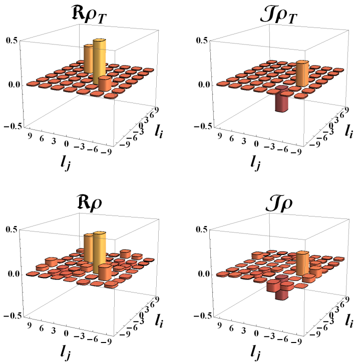
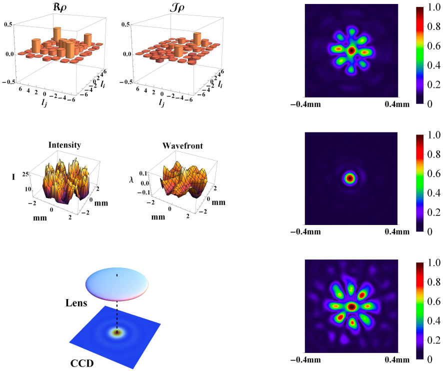


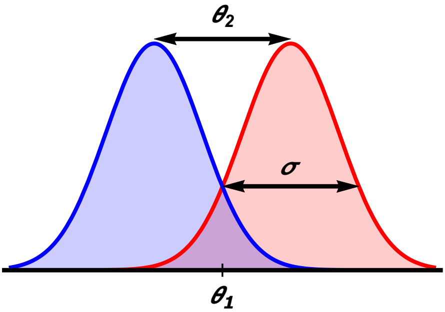
Partially coherent light intensity propagation

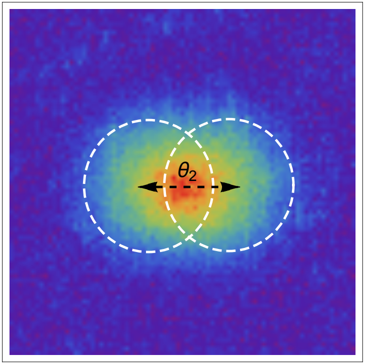
The propagation of transverse intensity distribution requires the explicit form of mutual coherence function at the input plane



Incoherent image of two-points

PSF: $I(x) = |\langle x|\psi\rangle|^2 = |\psi(x)|^2$





$$|\psi_\pm\rangle = \exp(\pm iP\mathfrak{s}/2)|\psi\rangle,\ \varrho_\mathfrak{s} = \frac{1}{2}(|\psi_+\rangle\langle\psi_+| + |\psi_-\rangle\langle\psi_-|)$$

Recent experimental work

Taking resolution to the limit: dispelling Rayleigh curse

MARTIN PAÚR[1], BOHUMIL STOKLASA[1], ZDENEK HRADIL[1], LUIS L. SÁNCHEZ-SOTO[2,3,*], AND JAROSLAV REHACEK[1]

[1] *Department of Optics, Palacky University, 17. listopadu 12, 771 46 Olomouc, Czech Republic*
[2] *Departamento de Óptica, Facultad de Física, Universidad Complutense, 28040 Madrid, Spain*
[3] *Max-Planck-Institut für die Physik des Lichts, Günther-Scharowsky-Straße 1, Bau 24, 91058 Erlangen, Germany*
[*] *Corresponding author: lsanchez@fis.ucm.es*

Optimal measurement

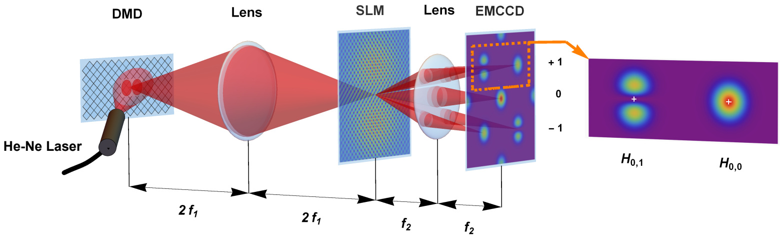
$|\psi_\pm\rangle = \exp(\pm i P \mathfrak{s}/2)|\psi\rangle$, $\langle\psi_-|\psi_+\rangle \neq 0$

$$\begin{aligned} |\psi_{sm}\rangle &= C_{sm}(|\psi_+\rangle + |\psi_-\rangle) \simeq |\psi\rangle\,, \\ |\psi_a\rangle &= C_a(|\psi_+\rangle - |\psi_-\rangle) \simeq \frac{P|\psi\rangle}{\sqrt{\langle\psi|P^2|\psi\rangle}}\,, \end{aligned}$$

Once PSF is inversion symmetric, those modes are orthogonal and Quantum Fisher information is:

$$\mathcal{F}_\mathcal{Q} = 2\left[\frac{1}{p_a}\langle\psi_a|\frac{\partial\varrho_\mathfrak{s}}{\partial\mathfrak{s}}|\psi_a\rangle + \frac{1}{p_{sm}}\langle\psi_{sm}|\frac{\partial\varrho_\mathfrak{s}}{\partial\mathfrak{s}}|\psi_{sm}\rangle\right] \simeq \langle\psi|P^2|\psi\rangle\,,$$

Experimental realization of mode projection



In the direction of the hologram reference wave, observed intensity is:

$$(|\int_{-\infty}^{\infty} dx \phi_q^* \phi_0(x + \frac{\theta_2}{2})|^2 + |\int_{-\infty}^{\infty} dx \phi_q^* \phi_0(x - \frac{\theta_2}{2})|^2)$$

Benefits of quantum description of measurement and imaging

- Recasting classical measurement scenario should provide a new point of view about a problem (Shack-Hartmann example).
- Proper treatment of Quantum Fisher Information provides real boundaries to measurement process and should lead to improvement in a measurement scheme (the image of two incoherent sources example).