Quantum enhanced classical imaging and metrology

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14.9.2017

1. Motivation
   - Quantum tomography
   - Quantum and beam optics correspondence
2. Results
   - Wavefront detector tomography
   - Two incoherent points resolution
3. Conclusion
   - Benefits from quantum reformulation of classical problems

Outline

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Quantum state tomography formalism

- The goal is to estimate the quantum state from the measurement data obtained from the ensemble of N identical copies the quantum system
- Measurements are described by a set of positive operators Π_j (POVM operators)
- Due to finite resources, quantum state can be only statistically estimated from probabilities $p_j = \mathrm{Tr}\, \rho \Pi_j$
- The probabilities are measured by the outcome frequencies f_j of the particular measurements $f_j = \frac{n_j}{N}$

Problems of quantum tomography

$$p_j = \mathrm{Tr}\, \rho \Pi_j$$

- Estimation algorithm (ML algorithm)
- Problem of tomography measurement completeness (MEML algorithm)
- Problem of measurement device calibration (data pattern tomography)

Fisher information and estimation

How good is my estimator of $\mathfrak{s}$ in the case of $\varrho(\mathfrak{s})$? Any ubiased estimator must follow Cramer-Rao bound:

$$(\Delta\hat{\mathfrak{s}})^2 \geq \frac{1}{\mathcal{F}}$$

Classical Fisher information: $\mathcal{F} = \int_{-\infty}^{\infty} \varrho_{\mathfrak{s}}(x) \left(\frac{\partial log \varrho_{\mathfrak{s}}(x)}{\partial \mathfrak{s}}\right)^2 \mathrm{d}x$.
Quantum Fisher information: $\mathcal{F}_{\mathcal{Q}} = \mathrm{Tr}[\varrho_{\mathfrak{s}} L_{\mathfrak{s}}^2]$
symmetric logarithmic derivative $L_{\mathfrak{s}}$ is the selfadjoint operator satisfying $\frac{1}{2}(L_{\mathfrak{s}}\varrho_{\mathfrak{s}} + \varrho_{\mathfrak{s}} L_{\mathfrak{s}}) = \partial\varrho_{\mathfrak{s}}/\partial\mathfrak{s}$

Fisher information notes

Quantum Fisher information is an upper bound for a classical Fisher information,
$\mathcal{F} >= \mathcal{F}_{\mathcal{Q}}$.
Quantum fisher information is independent of a measurement process. To test optimality of particular measurement, Fisher information for the measurement has to be computed:
$\mathcal{F}_{\mathcal{M}} >= \mathcal{F}_{\mathcal{Q}}$.

Outline

Correspondence classical and quantum description

There is a tight correspondence between beam optics and quantum mechanics.

- coherent waves (beam modes) $\rightarrow$ pure states
  $U(x) = \langle x|\psi\rangle$
- partially coherent fields $\rightarrow$ mixed states
  $G(x, x') = \langle x|\rho|x'\rangle$

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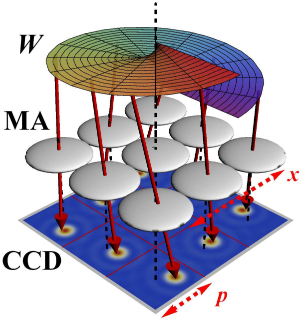
Wavefront detector tomography

Measurement of optical beams spatial coherence parametrized by a coherence matrix ϱ

$$I(\Delta x_i, \Delta p_j) = \mathrm{Tr}(\varrho|\Pi_{ij}\rangle\langle\Pi_{ij}|)$$

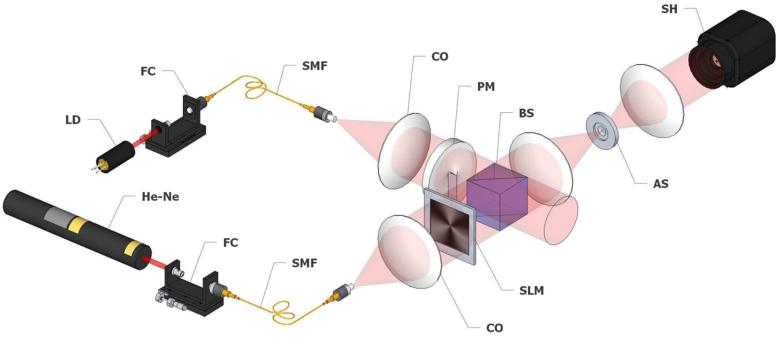
$$\left(\Pi_{ij}\right)_{mn} = \psi_{n,i}(\Delta p_j)\,\psi^*_{m,i}(\Delta p_j)$$



Problems of wavefront detection quantum description

- Suitable representation of ϱ has to be found (character of modes describing all relevant features of signal)
- Subspace establishing information complete measurement has to be formed (number of modes)
- If N is number of pixels of position detector (CCD), maximum reconstructed space dimension is $\sqrt{N}$

Experimental setup of wavefront tomography

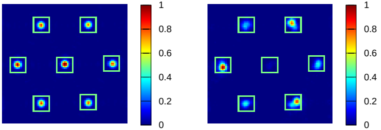


Vortex beam reconstruction

Vortex reconstruction space

$V_\ell = \langle r, \varphi | V_\ell \rangle \propto e^{i\ell\varphi}$

$\ell \in \{-9, -6, -3, 0, +3, +6, +9\}$



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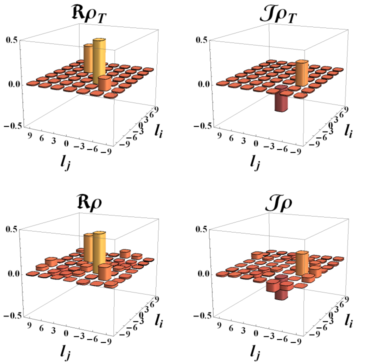
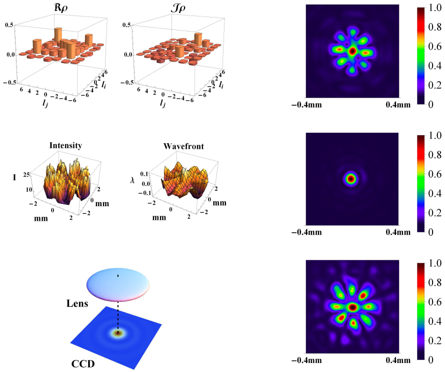


Partially coherent light intensity propagation

The propagation of transverse intensity distribution requires the explicit form of mutual coherence function at the input plane



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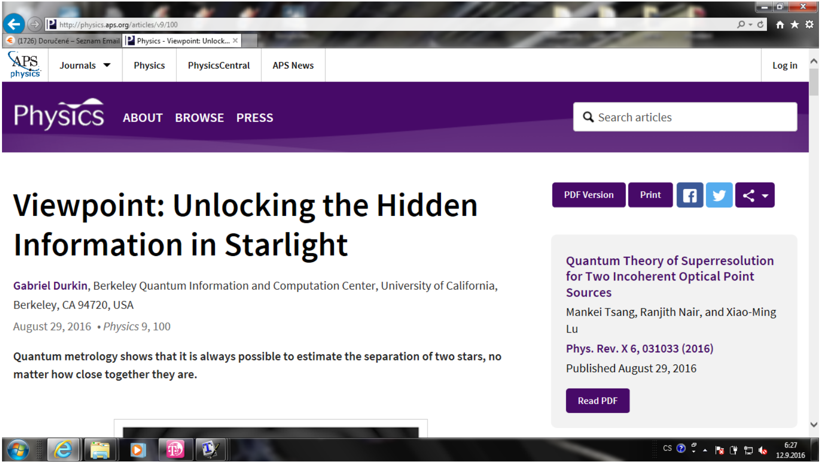
Hot topic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Viewpoint: Unlocking the Hidden Information in Starlight
Gabriel Durkin, Berkeley Quantum Information and Computation Center, University of California, Berkeley, CA 94720, USA
August 29, 2016 • Physics 9, 100
Quantum metrology shows that it is always possible to estimate the separation of two stars, no matter how close together they are.
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Quantum Theory of Superresolution for Two Incoherent Optical Point Sources
Mankei Tsang, Ranjith Nair, and Xiao-Ming Lu
Phys. Rev. X 6, 031033 (2016)
Published August 29, 2016
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12.9.2016

Tsangs theoretical work

Quantum theory of superresolution for two incoherent optical point sources

Mankei Tsang,[1,2,*] Ranjith Nair,[1] and Xiaoming Lu[1]
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(Dated: November 3, 2015)

Recent experimental work

Taking resolution to the limit: dispelling Rayleigh curse

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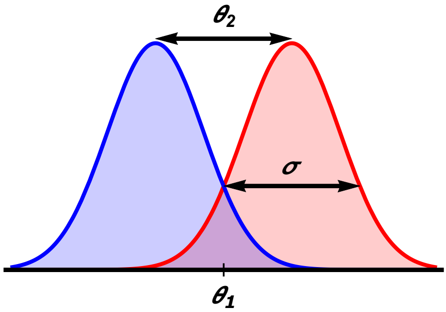
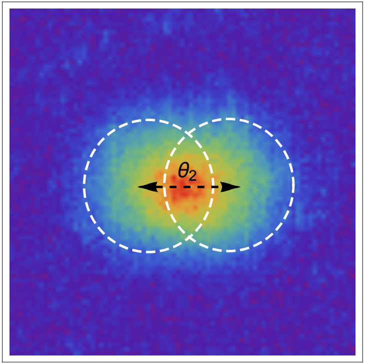
Incoherent image of two-points

PSF: $I(x) = |\langle x|\psi\rangle|^2 = |\psi(x)|^2$





$$|\psi_\pm\rangle = \exp(\pm iP\mathfrak{s}/2)|\psi\rangle,\ \varrho_\mathfrak{s} = \frac{1}{2}(|\psi_+\rangle\langle\psi_+| + |\psi_-\rangle\langle\psi_-|)$$

Classical Fisher information for position intensity measurement

Standard image plane intensity detection

$$\varrho_{\mathfrak{s}}(x) = \tfrac{1}{2}(|\psi(x - \mathfrak{s}/2)|^2 + |\psi(x + \mathfrak{s}/2)|^2)$$

$$\mathcal{F}_{\mathrm{std}} = \int_{-\infty}^{\infty} \frac{1}{\varrho_{\mathfrak{s}}(x)} \left(\frac{\partial \varrho_{\mathfrak{s}}(x)}{\partial \mathfrak{s}} \right)^2 \mathrm{d}x \, .$$

$$\mathcal{F}_{\mathrm{std}} \simeq \mathfrak{s}^2 \int_{-\infty}^{\infty} \frac{[I''(x)]^2}{I(x)} \mathrm{d}x \, .$$

Optimal measurement

$|\psi_\pm\rangle = \exp(\pm iP\mathfrak{s}/2)|\psi\rangle$, $\langle\psi_-|\psi_+\rangle \neq 0$

$$\begin{aligned} |\psi_{sm}\rangle &= C_{sm}(|\psi_+\rangle + |\psi_-\rangle) \simeq |\psi\rangle\,, \\ |\psi_a\rangle &= C_a(|\psi_+\rangle - |\psi_-\rangle) \simeq \frac{P|\psi\rangle}{\sqrt{\langle\psi|P^2|\psi\rangle}}\,, \end{aligned}$$

Once PSF is inversion symmetric, those modes are orthogonal and Quantum Fisher information is:

$$\mathcal{F}_\mathcal{Q} = 2\left[\frac{1}{p_a}\langle\psi_a|\frac{\partial\varrho_\mathfrak{s}}{\partial\mathfrak{s}}|\psi_a\rangle + \frac{1}{p_{sm}}\langle\psi_{sm}|\frac{\partial\varrho_\mathfrak{s}}{\partial\mathfrak{s}}|\psi_{sm}\rangle\right] \simeq \langle\psi|P^2|\psi\rangle\,,$$

Optimal measurement II

$\varrho_{\mathfrak{s}}$ is diagonal $\varrho_{\mathfrak{s}}|\psi_j\rangle = p_j|\psi_j\rangle$, with eigenvalues $p_a = \langle\psi|P^2|\psi\rangle\mathfrak{s}^2/4$ and $p_{sm} = 1 - p_a$.

$$\Pi_j = \psi_{\mathrm{opt}}(x) = \langle x|\psi_a\rangle = \frac{\psi'(x)}{\sqrt{\mathcal{F}}}$$

$$\mathcal{F} = \langle\psi|P^2|\psi\rangle = \int_{-\infty}^{\infty} [\psi'(x)]^2 \,\mathrm{d}x\,.$$

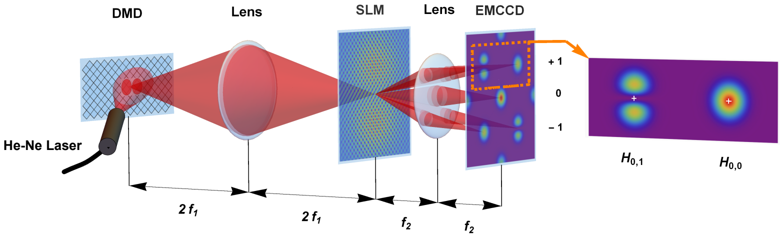
Specific PSF

$$\psi^G(x) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{4}}} \exp\left(-\frac{x^2}{4\sigma^2}\right), \quad \psi^S(x) = \frac{1}{\sqrt{w}} \frac{\sin(\pi x/w)}{\pi x/w},$$

The optimal measurements are then

$$\begin{aligned}
\psi^G_{\mathrm{opt}}(x) &= \frac{-1}{(2\pi)^{\frac{1}{4}}\sigma^{\frac{3}{2}}} x \exp\left(-\frac{x^2}{4\sigma^2}\right), \\
\psi^S_{\mathrm{opt}}(x) &= \sqrt{3}\left[\frac{w^{\frac{1}{2}}}{\pi x}\cos\left(\frac{\pi x}{w}\right) - \frac{w^{\frac{3}{2}}}{\pi^2 x^2}\sin\left(\frac{\pi x}{w}\right)\right].
\end{aligned}$$

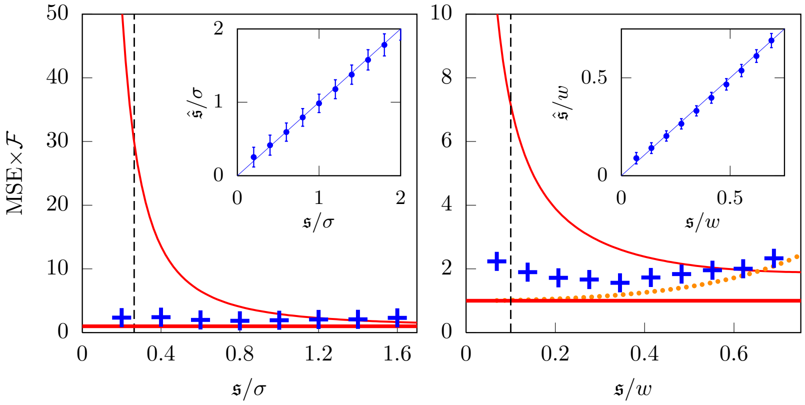
Experimental realization of mode projection



In the direction of the hologram reference wave, observed intensity is:

$$(|\int_{-\infty}^{\infty} dx\phi_q^*\phi_0(x+\frac{\theta_2}{2})|^2 + |\int_{-\infty}^{\infty} dx\phi_q^*\phi_0(x-\frac{\theta_2}{2})|^2)$$

Results


50
40
30
20
10
0
MSE×$\mathcal{F}$
0
0.4
0.8
1.2
1.6
$\mathfrak{s}/\sigma$
2
1
0
$\hat{\mathfrak{s}}/\sigma$
0
1
2
$\mathfrak{s}/\sigma$
10
8
6
4
2
0
0
0.2
0.4
0.6
$\mathfrak{s}/w$
0.5
0
$\hat{\mathfrak{s}}/w$
0
0.5
$\mathfrak{s}/w$

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Benefits of quantum description of measurement and imaging

- Recasting classical measurement scenario should provide a new point of view about a problem (Shack-Hartmann example).
- Proper treatment of Quantum Fisher Information provides real boundaries to measurement process and should lead to improvement in a measurement scheme (the image of two incoherent sources example).