Storage and manipulation of a frequency light comb for quantum cluster state generation

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St. Petersburg State University -2017

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b. Quantum memory
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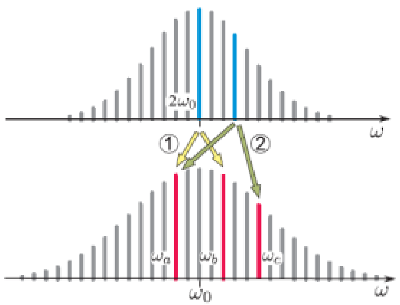
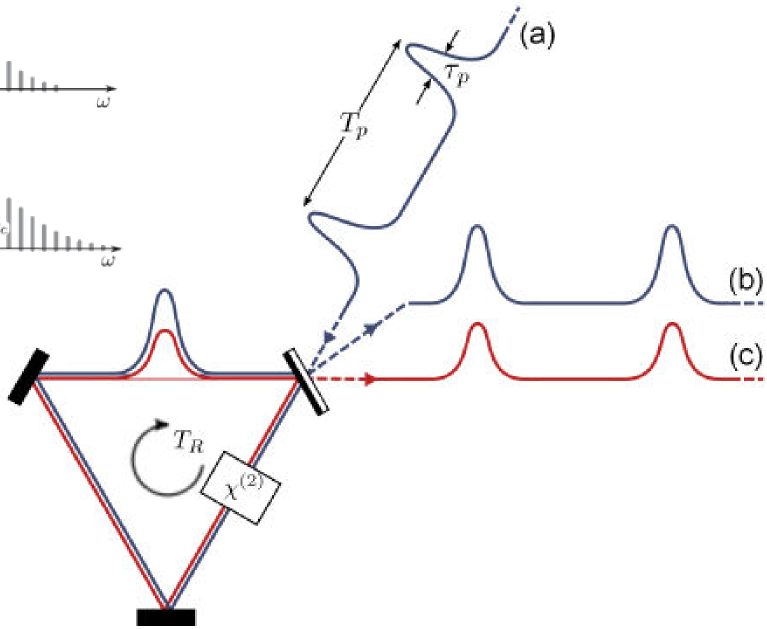
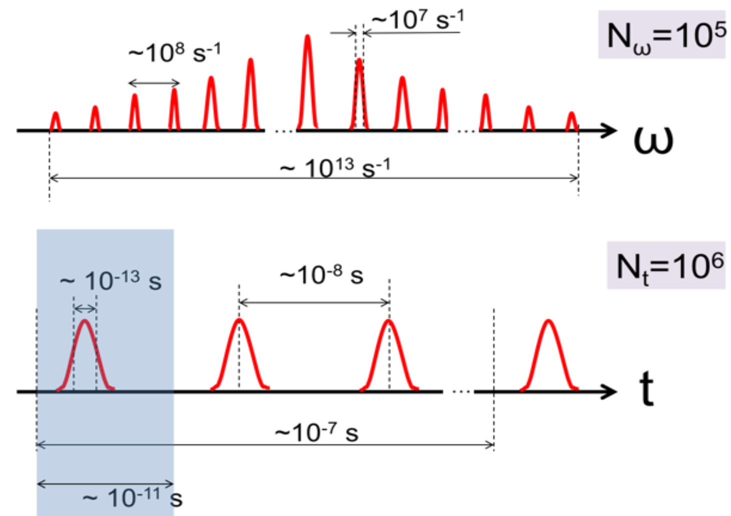
IV. Cluster state generation

Synchronously pumped optical parametric oscillator (SPOPO) as a source of a signal field

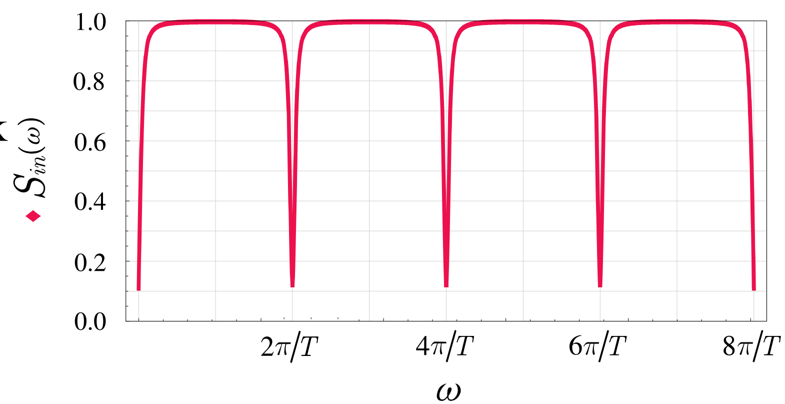






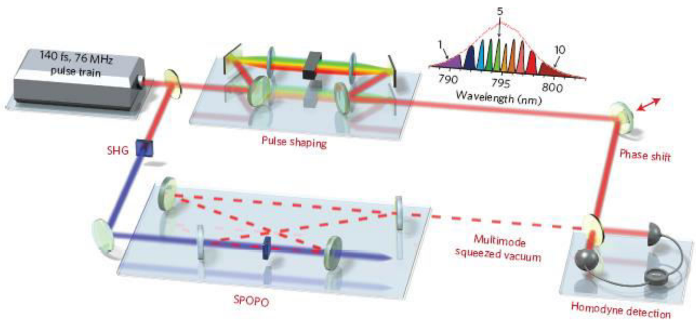
$\tau \neq T$ **Non-synchronous pumping** All correlation are weak

$\tau = T$ **Synchronous pumping** Strong correlations at times of the order of T



1. V. A Averchenko, Yu. M. Golubev, C. Fabre, N. Treps. // Eur. Phys. J. D. – 2011. – Vol. 61(1). – P. 207–214.
2. S. Gerke, J. Sperling, W. Vogel, Y. Cai, J. Roslund, N. Treps, and C. Fabre // Phys. Rev. Lett. – 2016. – Vol. 117. – P. 110502.

Synchronously pumped optical parametric oscillator (SPOPO) as a source of a signal field



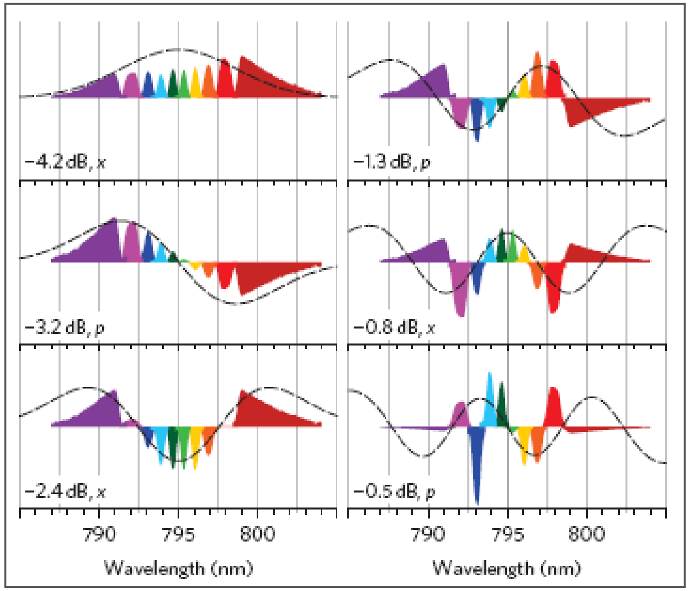

nature photonics
LETTERS
PUBLISHED ONLINE: 15 DECEMBER 2013 | DOI: 10.1038/NPHOTON.2013.340

Wavelength-multiplexed quantum networks with ultrafast frequency combs

Jonathan Roslund, Renné Medeiros de Araújo, Shifeng Jiang, Claude Fabre and Nicolas Treps*

the output radiation of the SPOPO can be represented as a **Hermite-Gaussian** polynomial expansion:

$$\hat{a}_{in}(t) = \sum_k L_k(t)\hat{e}_k$$



Supermodes – eigen functions of an effective Hamiltonian of the parametric downconversion (*smooth* Hermite-Gaussian polynomial functions)

Structure characteristics of the SPOPO radiation

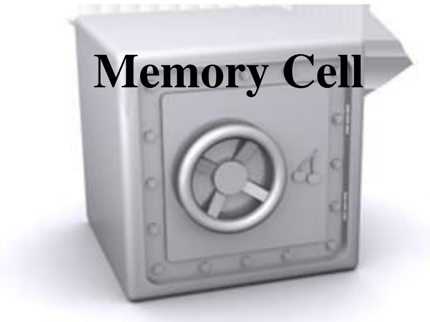
- correlations in the SPOPO radiation at times of the order of the several cavity round-trip times
- the genuine quantum degrees of freedom of such a complex system

Quantum memory

the writing process:

signal →

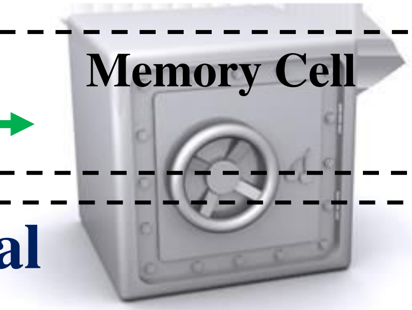
driving →

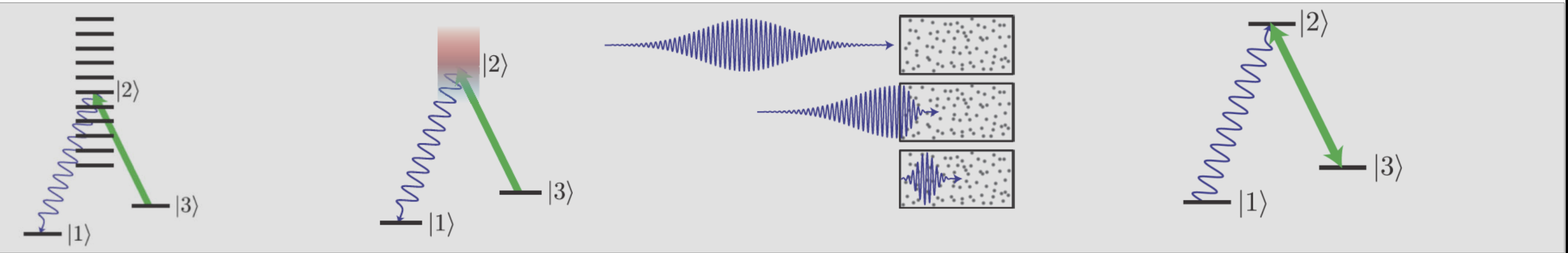


the readout process:

Forward reading: driving → signal →

Backward reading: ← signal ← driving

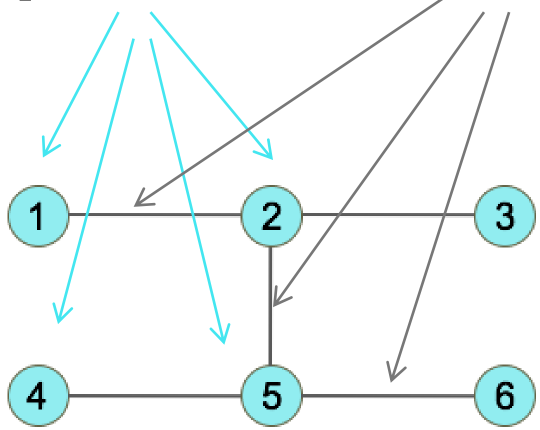




Cluster states. One-way quantum computation

nodes
(quantum oscillators)

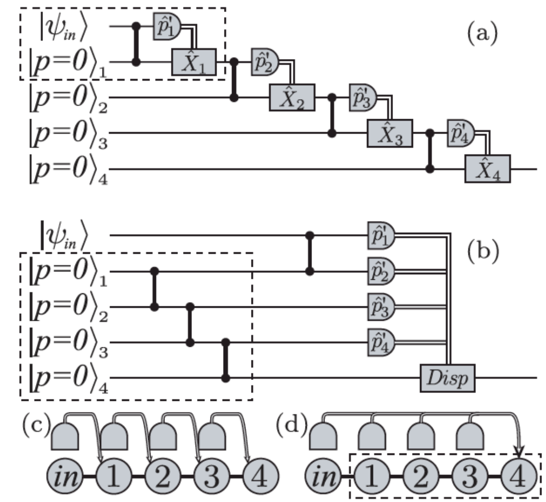
edges (entangling
between oscillators)



6-mode cluster graph

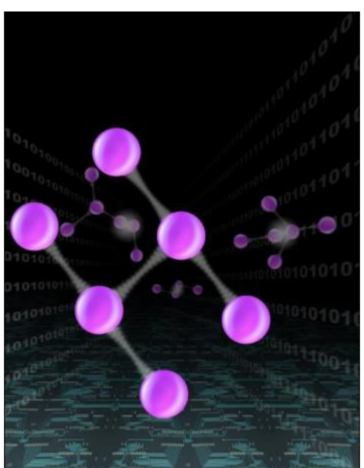
$$C = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Adjacency matrix



One-way computation
(quantum teleportation)

Cluster states – definition by nullifiers

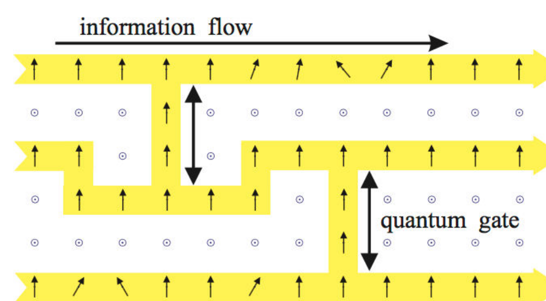


Nullifiers

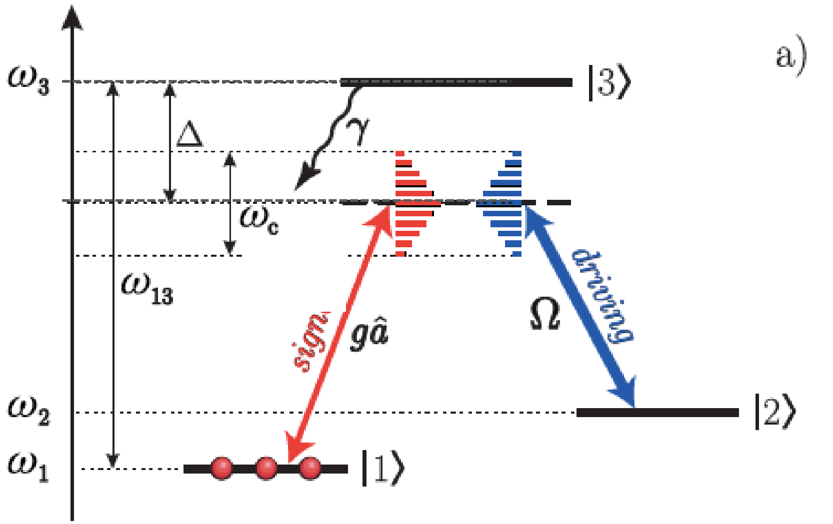
$$\hat{N}_i = \hat{P}_i - \sum_m C_{mi}\hat{Q}_m$$

$$i, m = 1, \ldots, n$$

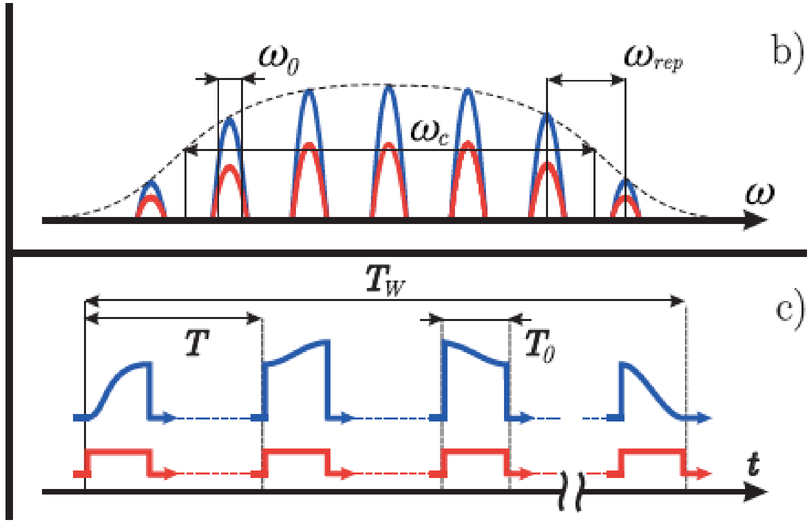
$$\hat{N}_i|\psi_{cluster}\rangle \longrightarrow 0$$



II. Quantum memory scheme for SPOPO radiation



Energy-level scheme of an atomic ensemble, interacting with combs of the signal and driving in the Raman memory protocol.



Frequency and time profiles of the signal and driving fields

<u>commutation relations:</u>

$$[\hat{a}(t,z),\hat{a}^\dagger(t',z)] = \delta(t-t')$$

$$[\hat{a}(t,z),\hat{a}^\dagger(t,z')] = c\left(1-\frac{i}{k_s}\frac{\partial}{\partial z}\right)\delta(z-z')$$

<u>time profile of the driving:</u>

$$\Omega(t,z) = \Omega_0 f(t-z/c)$$

$$f(t) = \sum_{n=1}^{N} F(t)\Theta(t-(n-1)T)$$

NB: the rotating-wave and dipole approximations

$$\hat{V} = \int_0^L dz\ \left(i\hbar g\hat{a}(t,z)\hat{\sigma}_{31}(t,z)e^{-i\Delta t+ik_s z} + i\hbar\Omega(t,z)\hat{\sigma}_{32}(t,z)e^{-i\Delta t+ik_d z} + H.c.\right).$$

Math framework

Heisenberg-Langevin equations:

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z} + ic\frac{g^2 N_{at}/L}{\Delta}\right)\hat{a}(t,z) = -ic\frac{g\sqrt{N_{at}/L}\,\Omega_0}{\Delta} f(t-z/c)\hat{b}(t,z) + \hat{f}_a(t,z),$$

$$\left(\frac{\partial}{\partial t} + i\frac{\Omega_0^2}{\Delta} f^2(t-z/c)\right)\hat{b}(t,z) = -i\frac{g\sqrt{N_{at}/L}\,\Omega_0}{\Delta} f(t-z/c)\hat{a}(t,z) + \hat{f}_b(t,z),$$

Math framework

Heisenberg-Langevin equations:

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z} + ic\frac{g^2 N_{at}/L}{\Delta}\right)\hat{a}(t,z) = -ic\frac{g\sqrt{N_{at}/L}\,\Omega_0}{\Delta}f(t-z/c)\hat{b}(t,z) + \hat{f}_a(t,z),$$

$$\left(\frac{\partial}{\partial t} + i\frac{\Omega_0^2}{\Delta}f^2(t-z/c)\right)\hat{b}(t,z) = -i\frac{g\sqrt{N_{at}/L}\,\Omega_0}{\Delta}f(t-z/c)\hat{a}(t,z) + \hat{f}_b(t,z),$$

Dimensionless variables:

$$\frac{g^2(N_{at}/L)}{\Delta}z \to z$$

$$\frac{\Omega_0^2}{\Delta}t \to t$$

Math framework

Dimensionless variables:

$$\frac{g^2(N_{at}/L)}{\Delta} z \to z$$

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$$\left(\frac{\partial}{\partial t} + i\frac{\Omega_0^2}{\Delta} f^2(t-z/c)\right)\hat{b}(t,z) = -i\frac{g\sqrt{N_{at}/L}\,\Omega_0}{\Delta} f(t-z/c)\hat{a}(t,z) + \hat{f}_b(t,z),$$

The writing process:

$$\hat{B}(z) = -i\int_0^{T_W} \hat{a}_{in}(t') f_W(t') exp\left(-i\int_{t'}^{t} f_W^2(t'')dt''\right) J_0\left(2\sqrt{z\int_{t'}^{t} f_W^2(t'')dt''}\right) dt' + vac = \int_0^{T_W} \hat{a}_{in}(t') G_{ab}(t,z) dt + vac,$$

Math framework

Dimensionless variables:

$$\frac{g^2(N_{at}/L)}{\Delta} z \to z$$

$$\frac{\Omega_0^2}{\Delta} t \to t$$

Heisenberg-Langevin equations:

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z} + ic\frac{g^2 N_{at}/L}{\Delta}\right)\hat{a}(t,z) = -ic\frac{g\sqrt{N_{at}/L}\,\Omega_0}{\Delta} f(t-z/c)\hat{b}(t,z) + \hat{f}_a(t,z),$$

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The readout process:

$$\hat{a}^R(t) = -i\int_0^{L} \hat{B}(z') f_R(t) exp\left(-i\int_0^{t} f_R^2(t',z)dt'\right) J_0\left(2\sqrt{(z')\int_0^{t} f_R^2(t',z)dt'}\right) dz' + vac = \int_0^{L} \hat{B}(z) G_{ba}(t,z) dz + vac.$$

Math framework

Dimensionless variables:

$$\frac{g^2(N_{at}/L)}{\Delta}z \to z$$

$$\frac{\Omega_0^2}{\Delta}t \to t$$

Heisenberg-Langevin equations:

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z} + ic\frac{g^2 N_{at}/L}{\Delta}\right)\hat{a}(t,z) = -ic\frac{g\sqrt{N_{at}/L}\,\Omega_0}{\Delta}f(t-z/c)\hat{b}(t,z) + \hat{f}_a(t,z),$$

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The writing process:

$$\hat{B}(z) = -i\int_0^{T_W}\hat{a}_{in}(t')f_W(t')exp\left(-i\int_{t'}^{t}f_W^2(t'')dt''\right)J_0\left(2\sqrt{z\int_{t'}^{t}f_W^2(t'')dt''}\right)dt' + vac = \int_0^{T_W}\hat{a}_{in}(t')G_{ab}(t,z)dt + vac,$$

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Full cycle of quantum memory (writing, storing and readout):

$$\hat{a}^R(t) = -\int_0^{T_W}dt'K(t,t')\hat{a}_{in}(T_W-t').$$

Math framework

Dimensionless variables:

$$\frac{g^2(N_{at}/L)}{\Delta}z \to z$$

$$\frac{\Omega_0^2}{\Delta}t \to t$$

Heisenberg-Langevin equations:

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z} + ic\frac{g^2 N_{at}/L}{\Delta}\right)\hat{a}(t,z) = -ic\frac{g\sqrt{N_{at}/L}\,\Omega_0}{\Delta}f(t-z/c)\hat{b}(t,z) + \hat{f}_a(t,z),$$

$$\left(\frac{\partial}{\partial t} + i\frac{\Omega_0^2}{\Delta}f^2(t-z/c)\right)\hat{b}(t,z) = -i\frac{g\sqrt{N_{at}/L}\,\Omega_0}{\Delta}f(t-z/c)\hat{a}(t,z) + \hat{f}_b(t,z),$$

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The readout process:

$$\hat{a}^R(t) = -i\int_0^{L}\hat{B}(z')f_R(t)exp\left(-i\int_0^{t}f_R^2(t',z)dt'\right)J_0\left(2\sqrt{(z')\int_0^{t}f_R^2(t',z)dt'}\right)dz' + vac = \int_0^{L}\hat{B}(z)G_{ba}(t,z)dz + vac.$$

Full cycle of quantum memory (writing, storing and readout):

$$\hat{a}^R(t) = -\int_0^{T_W}dt'K(t,t')\hat{a}_{in}(T_W-t').$$

the full-cycle kernel:

$$K(t,t') = \int_0^{L}G_{ab}(t,z)G_{ba}(t',z)dz$$

Math framework

Dimensionless variables:

$$\frac{g^2(N_{at}/L)}{\Delta} z \to z$$

$$\frac{\Omega_0^2}{\Delta} t \to t$$

Heisenberg-Langevin equations:

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z} + ic\frac{g^2 N_{at}/L}{\Delta}\right)\hat{a}(t,z) = -ic\frac{g\sqrt{N_{at}/L}\,\Omega_0}{\Delta} f(t-z/c)\hat{b}(t,z) + \hat{f}_a(t,z),$$

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The readout process:

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Full cycle of quantum memory (writing, storing and readout):

$$\hat{a}^R(t) = -\int_0^{T_W} dt' K(t,t')\hat{a}_{in}(T_W - t').$$

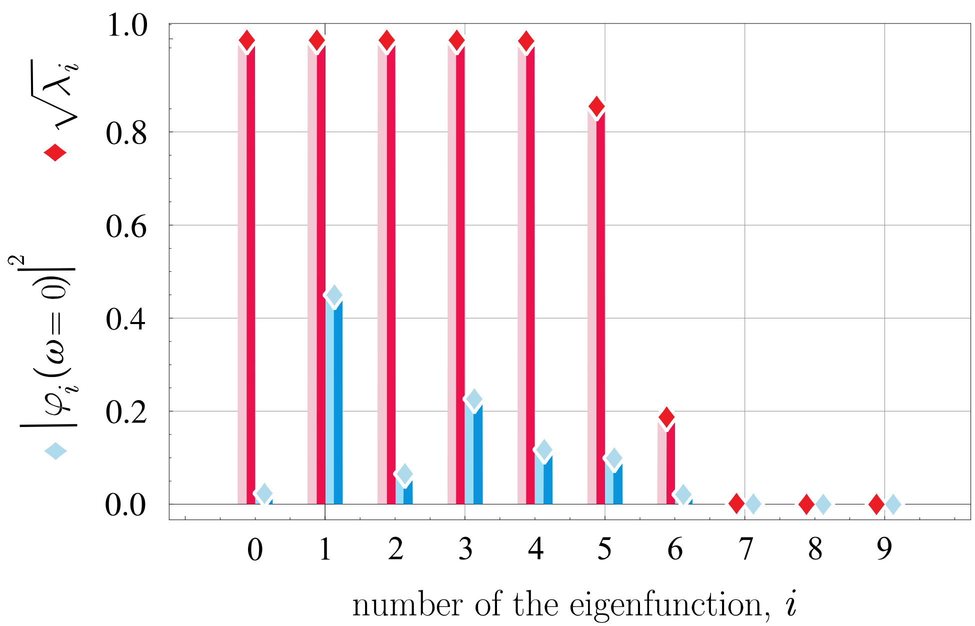
the full-cycle kernel:

$$K(t,t') = \int_0^{L} G_{ab}(t,z) G_{ba}(t',z)dz$$

Complex kernel is "bad" !

(not available to store correlations)

Eigenfunctions (eigenmodes) of the memory



Eigenvalues of the kernel K(t,t') (red bars) and zero spectral components of the eigenfunctions (blue bars) (step-function is considered as a driving profile).
Parameters for the calculation:
N = 90, L = 10, T 0 = 0.1, T = 10000.

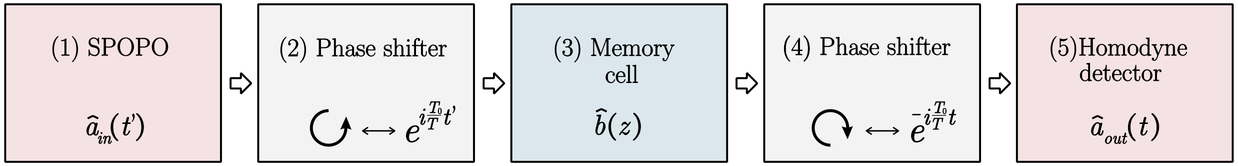
Schmidt mode decomposition:

$$K(t,t') = \sum_i \sqrt{\lambda_i}\,\varphi_i(t)\varphi_i(t').$$

the offered memory scheme:
- allows one to preserve the SPOPO light correlations at the quantum level, including the genuine multipartite entanglement embedded in this light.
- demonstrates the high number of independent quantum degrees of freedom

- eigenvalues
- zero spectral components of the eigenfunctions

Preservation of quantum correlations



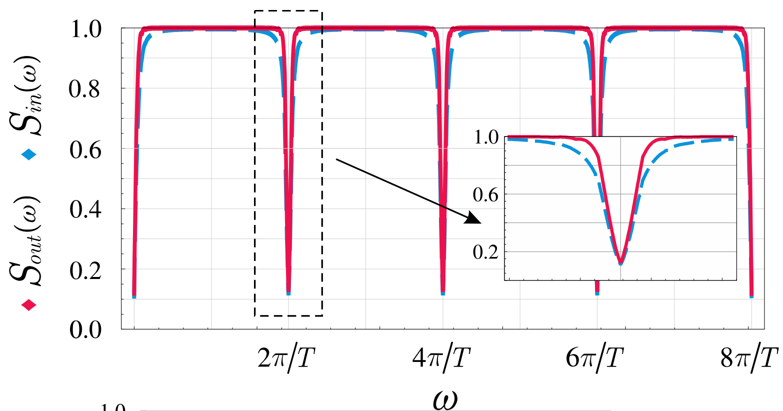
$$K(t,t') \to G(t,t')$$

real

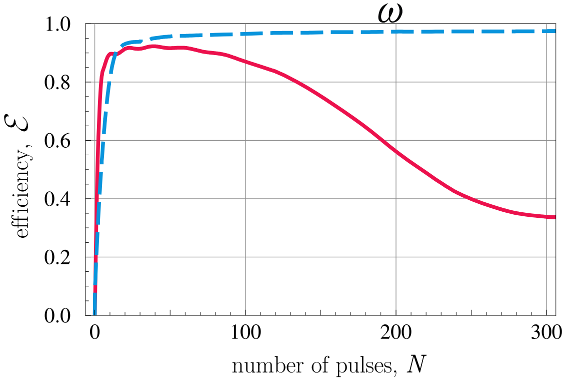
A block diagram of the quantum memory with the phase shifters.

Real kernel is "good" !

(available to store correlations)



Reduction of the short noise in spectrum of the photocurrent fluctuations under homodyne detection of the Y-quadrature of N consecutive pulses of the input (blue dashed line) and output (red solid line) signal fields. The time profile of the homodyne β(t) is a step function in both cases. Parameters of the calculation:
N = 90, T0 = 0.1, T = 10000, κsT = 0.1, L = 10.

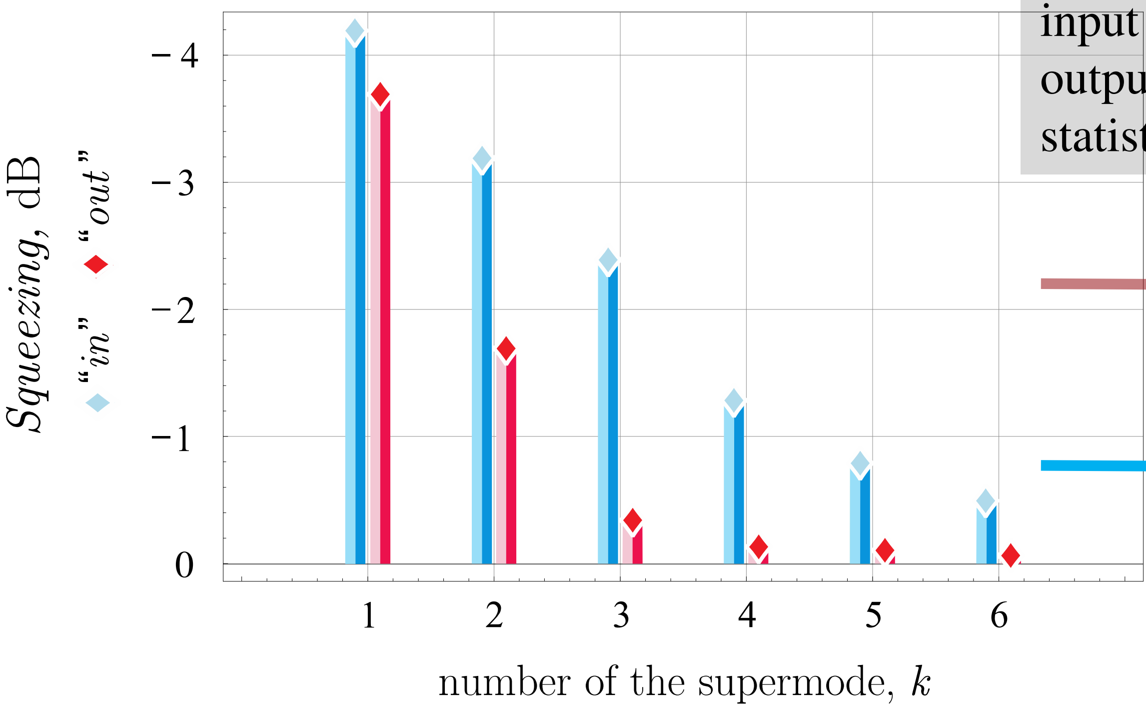


Dependence of the efficiency of the recording stage on the number of pulses in a comb under the following parameters:
L = 10, 30, 50; T0 = 0.1; T = 10 000.

A.D. Manukhova, K.S. Tikhonov, T.Yu. Golubeva, and Yu.M. Golubev.
Preservation of quantum correlations in a femtosecond light pulse train within an atomic ensemble
// Phys. Rev. A, 2017, 95, 013801.

The squeezing of the supermodes in the restored light

The squeezed supermodes of the input signal can be restored in the output light and keep its quantum statistics.



- after the full cycle of the memory.

- before the full cycle of the memory.

Preservation of the squeezing in the first six supermodes. Values of the maximum squeezing in decibels for SPOPO light before (blue bars; the values are taken from the experiment) and after [red bars] the full cycle of the memory.

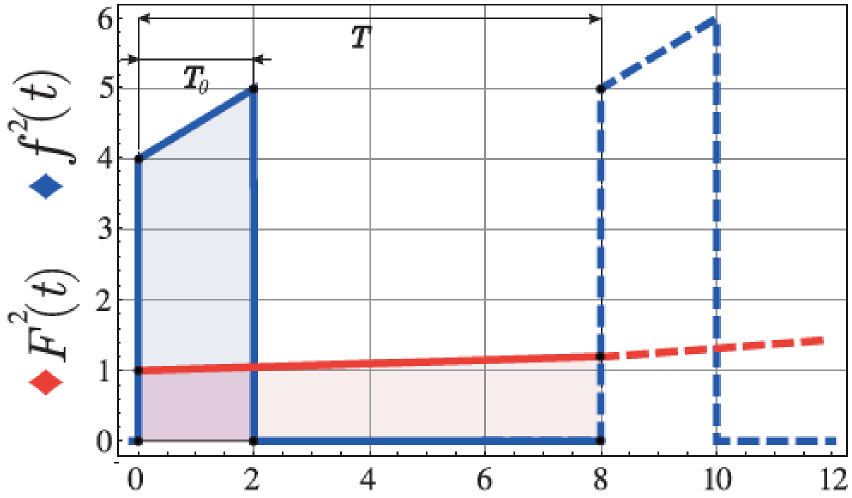
Parameters of the calculation:
L = 10, T0 = 0.1, T = 10 000.

J. Roslund, R. Medeiros de Araujo, S. Jiang, C. Fabre and N. Treps. Wavelength-multiplexed quantum networks with ultrafast frequency combs// Nat. Photon. Lett. – 2014 – Vol. 8. – P. 109–112.

The passage from the pulsed structure of the fields to their envelopes

N>>1 - the number of pulses is large

Graphic presentation of the passage from the pulsed structure of the fields to their envelopes:



$f(t)$ - pulsed structure

$F(t)$ - the envelope

ensure :
- commutation relations
- normalization

$\sqrt{T_0/T}$ - normalization factor

$$Q(t) = \int_0^t F^2(t')dt'$$

Amplitudes of the signal field and collective coherence are given by the annihilation bosonic operators through the envelopes:

Kernels (for writing and readout processes):

$$\hat{B}(z) = \int_0^{T_W} dt\ \hat{A}_{in}(t)\, G_{ab}(t,z)\ + vac, \qquad G_{ab}(t,z) = F_W(t) J_0\left(2\sqrt{Q_W(t)z}\right),$$

$$\hat{A}_{out}(t) = \int_0^L dz \hat{B}(z)\, G_{ba}(t,z)\ + vac, \qquad G_{ba}(t,z) = F_R(t) J_0\left(2\sqrt{Q_R(t)z}\right).$$

Shaping of the driving for the efficient writing of single supermode

We want to be able to write only one (**any** of our choice) mode from all the SPOPO radiation's modes

Full memory cycle:

$$\hat{A}_{out}(t) = \int_0^{T_W} dt' \hat{A}_{in}(t') G(t,t') + vac,$$

the writing and the reading are performed with the same driving field profiles:

$$F_W(t)=F_R(t)$$

Schmidt mode decomposition of the kernel:

$$G(t,t') = \sum_{k=1}^{\infty} \sqrt{\lambda_k} \varphi_k(t) \varphi_k(t')$$

$$\Rightarrow \quad \boxed{G(t,t') = \varphi_1(t)\varphi_1(t') = L_i(t)L_i(t')}$$

To find a proper driving profile:

$$L_i^2(t) = F_i^2(t) \int_0^L dz \left[J_0 \left(2\sqrt{z \int_0^t dt_1 \, F_i^2(t_1)} \right) \right]^2 .$$

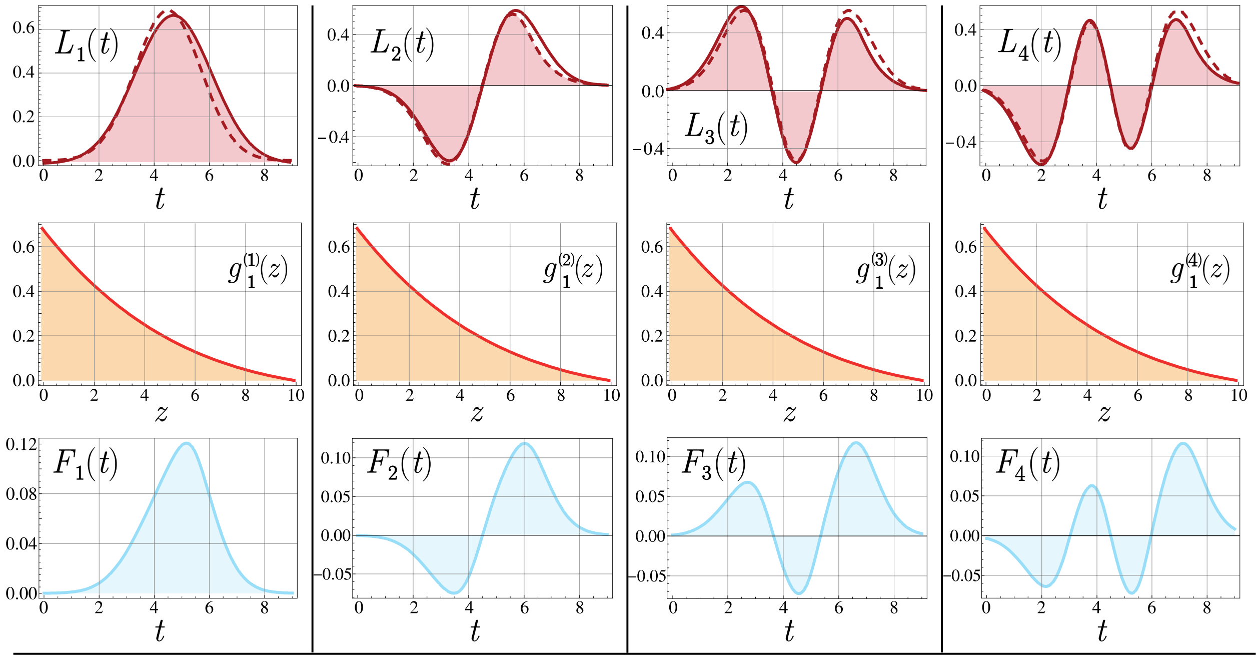
semi-analytic and **numeric** solutions

Iterative procedure:

$$F_i^{(j)}(t) = L_i(t) \left(\int_0^L dz \left[J_0 \left(2\sqrt{z \int_0^t dt_1 \, (F_i^{(j-1)}(t_1))^2} \right) \right]^2 \right)^{-1/2}$$

$$F_i^{(0)}(t) = L_i(t)$$

the response of the medium while the efficient writing of a single supermode



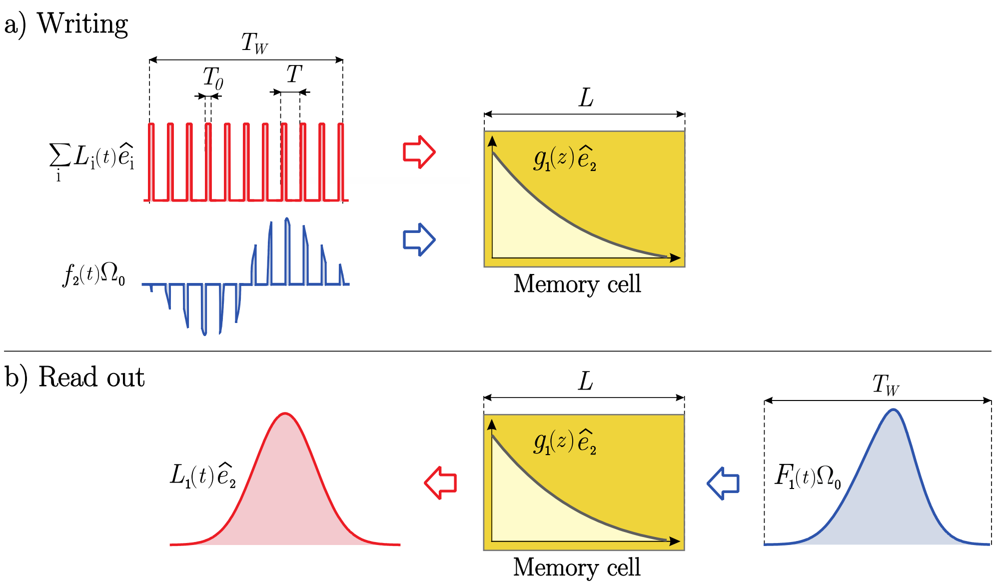
the response of the medium under such excitation does not depend on the mode number and provided the readout of the signal with the same driving field

Profiles of the first four SPOPO supermodes (top row, dashed curves) and the amplitudes $A_{out}(t)$ (top row, solid curves) restored with an appropriate choice of the driving field shapes (bottom row) found according to the iterative procedure. The middle frames demonstrate the identity of all response functions mapped by the fields in the medium.

A method for searching the driving field profile has been developed, which makes it possible to restore and transform the signal amplitude with an accuracy (estimated as the overlap integral of the initial and reconstructed modes) of about **95%**

III. Noiseless converting of a quantum field shape

Writing and readout of the orthogonal modes



We write **the second** supermode with quantum statistics of the **second** supermode

We read the field with the profile **the first** supermode, but with quantum statistics of **the second** one

Schematic representation of the shape converter for the signal field.

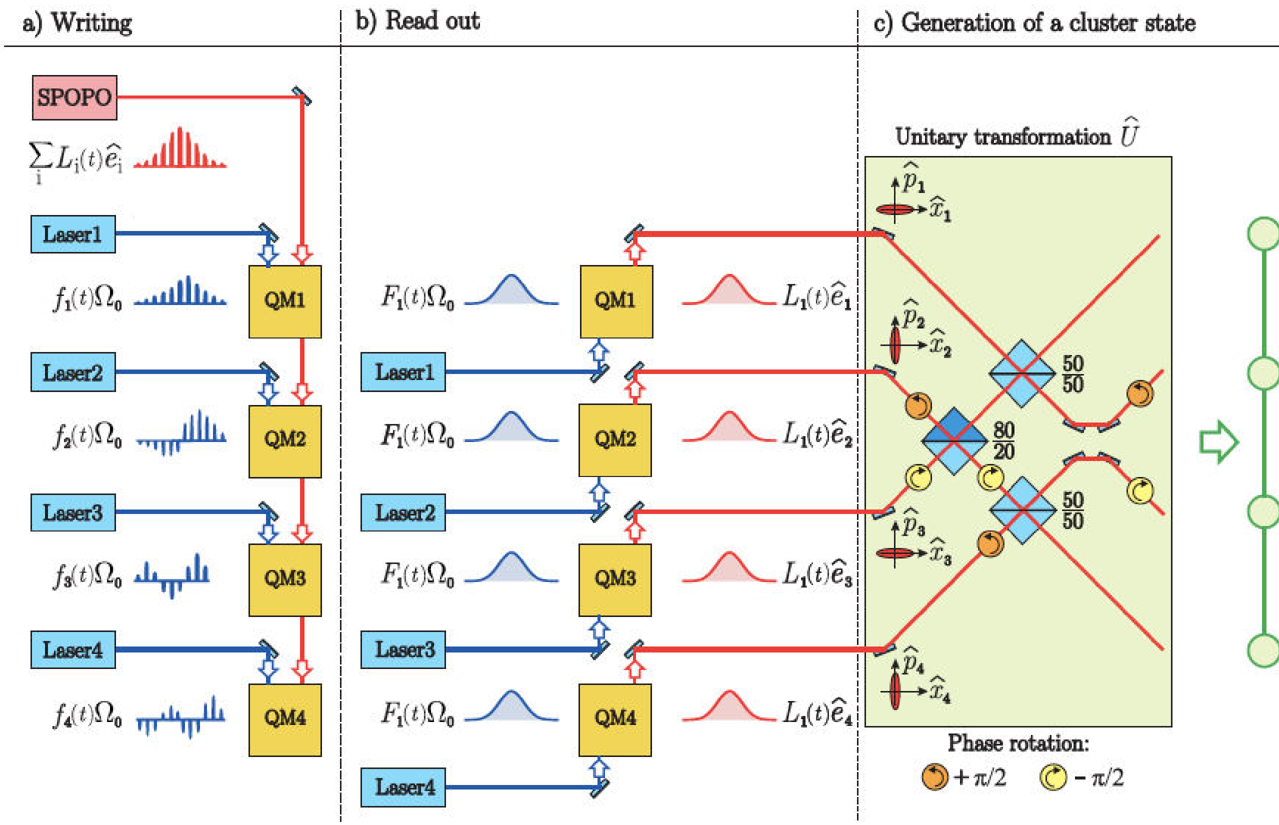
$$\hat{A}_{out}(t) = \int_0^{T_W}\int_0^{L} dzdt' \; \hat{A}_{in}(t')\, G^{(2)}_{ab}(t', z) G^{(1)}_{ba}(t, z)$$

$$= \int_0^{T_W}\int_0^{L} dzdt' \; \sum_k L_k(t')\hat{e}_k \; g_1(z) L_2(t') \; g_1(z) L_1(t) = L_1(t)\hat{e}_2.$$

A protocol for converting the waveform is developed with the preservation of quantum statistics based on the memory cell.

IV. Cluster state generation

Scheme of the cluster state generation



A linear quantum multimode cluster state of light has been generated on the basis of the SPOPO's supermodes using the developed quantum memory cells.

Scheme of the thought experiment on generation of a light linear cluster consisting of four nodes.

A.D. Manukhova, K.S. Tikhonov, T.Yu. Golubeva, and Yu.M. Golubev.
Noiseless signal shaping and cluster-state generation with a quantum memory cell.
// *Phys. Rev. A*, 2017, 96, 023851.

Conclusions:

The Raman protocol of quantum memory provides effective writing and readout of a femtosecond pulse train and demonstrates a significant number of quantum degrees of freedom.

Quantum correlations of the essentially multimode radiation of a synchronously pumped optical parametric oscillator can be effectively preserved provided that a phase correction scheme is used.

The squeezed supermodes of the input signal can be restored in the output light and keep its quantum statistics.

A method for searching the driving field profile enabling to restore and transform the signal amplitude with a high accuracy has been developed

A protocol for converting the waveform of the quantum signal with the preservation of its quantum statistics has been developed basing on the memory cell

A linear quantum multimode cluster state of light has been generated on the basis of the SPOPO's supermodes using the developed quantum memory cells.

Storage and manipulation of a frequency light comb for quantum cluster state generation

A. D. Manukhova, K. S. Tikhonov, T. Yu. Golubeva, and Yu. M. Golubev

THANK YOU
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