Verifying genuine multipartite entanglement of the whole from its separable parts

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Collaboration

Theory:

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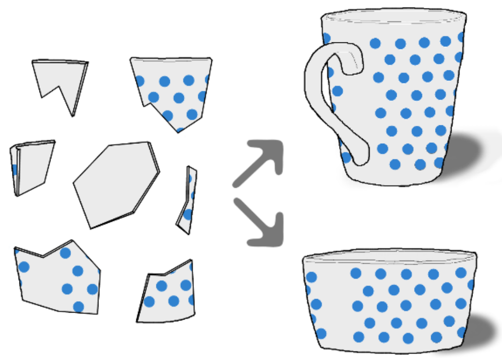
Introduction

Inference of a global property of the whole from fragments.



Inference of a global property of a composite quantum system from reduced density matrices (marginals).

- Experiments with composite systems involving many subsystems.
- Separability and security criteria, calculation of some entanglement measures. (Extensibility of ρ_{AB} : $\exists \quad \rho_{ABE}, \quad \mathrm{Tr}_E(\rho_{ABE}) = \rho_{AB}$.)

Inference of entanglement from marginals

Pure bipartite state:

Mixed ρ_A $\rightarrow$ $|\psi\rangle_{AB} \neq |\chi\rangle_A |\phi\rangle_B$ (entanglement).

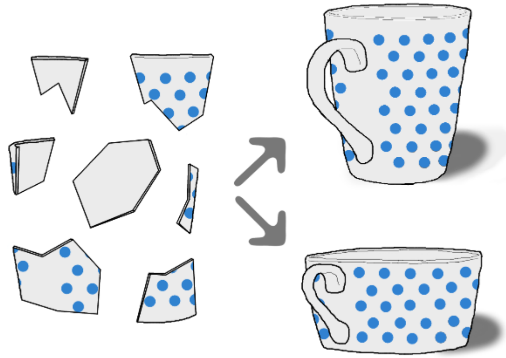
Mixed bipartite state:

ρ_A and ρ_B compatible with $\rho_A \otimes \rho_B$ (inconclusive).

Three systems A, B, C:

Entangled ρ_{AB} $\rightarrow$ ρ_{ABC} entangled across $A|BC$ cut.

Can we infer a property from parts that do not have the property?



``Emergent'' entanglement from separable marginals

G. Tóth, Phys. Rev. A **71**, 010301 (2005);
G. Tóth et. al., Phys. Rev. Lett. **99**, 250405 (2007).

Counterexample: $\frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$ compatible with $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ and $\frac{1}{2}(|000\rangle\langle 000| + |111\rangle\langle 111|)$.

Genuine multipartite entanglement (GME):

$$\rho \neq p_1 \rho^{\text{sep}}_{A|BC} + p_2 \rho^{\text{sep}}_{\text{B}|\text{AC}} + p_3 \rho^{\text{sep}}_{C|AB} \quad \text{(biseparable state)},$$

$$\rho^{\text{sep}}_{X|Y} = \sum_i p_i \rho_X^{(i)} \otimes \rho_Y^{(i)} \quad \text{(separable state)}.$$

Can we infer GME from separable marginals?

∃ separable $\{\rho_{AB}, \rho_{BC}, \rho_{AC}\}$ compatible only with GME ρ_{ABC}.

(L. Chen et al., PRA **90**, 042314 (2014))

Theory

State:

$$\rho = \frac{2}{3}|\xi\rangle\langle\xi| + \frac{1}{3}|\bar{W}\rangle\langle\bar{W}|,$$

$$|\xi\rangle = \frac{1}{3}(e^{i\frac{\pi}{3}}|001\rangle + e^{-i\frac{\pi}{3}}|010\rangle - |100\rangle) + \sqrt{\frac{2}{3}}|111\rangle,$$

$$|\bar{W}\rangle = \frac{1}{\sqrt{3}}(|011\rangle + |101\rangle + |110\rangle).$$

(N. Miklin et al., PRA **93**, 020104 (2016))

Separable marginals:

$$\min[\mathrm{eig}(\rho_{jk}^{T_j})] = 0.0037.$$

GME witness:

$$\begin{aligned} W &= W^\dagger, \\ \mathrm{Tr}(\rho W) &\geq 0 \quad \text{for all biseparable } \rho, \\ \mathrm{Tr}(\rho W) &< 0 \quad \text{for some } \rho. \end{aligned}$$

SDP:

$$\begin{aligned} \underset{W,P_M,Q_M}{\text{minimize}} \quad & \mathrm{Tr}(\rho W) \\ \text{subject to} \quad & \mathrm{Tr}(W) = 1, \\ & W = \sum_{i,j=0}^{3} w_{ij}^{(AB)} \sigma_i^{(A)} \otimes \sigma_j^{(B)} \otimes \mathbb{1}^{(C)} + \text{permutations}, \\ & \text{and for all bipartitions } M|\overline{M}, \\ & W = P_M + Q_M^{T_M}, \quad P_M \geq 0, \quad Q_M \geq 0. \end{aligned}$$

$$\mathrm{Tr}(\rho W) \doteq -1.98 \cdot 10^{-2}$$

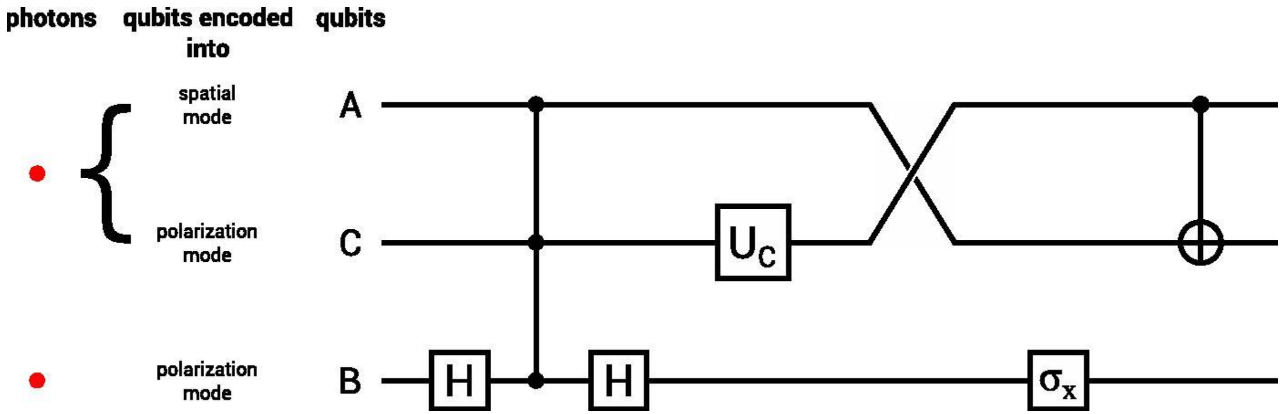
Robustness:

$$p\rho + \frac{(1-p)}{8}\mathbb{1}$$

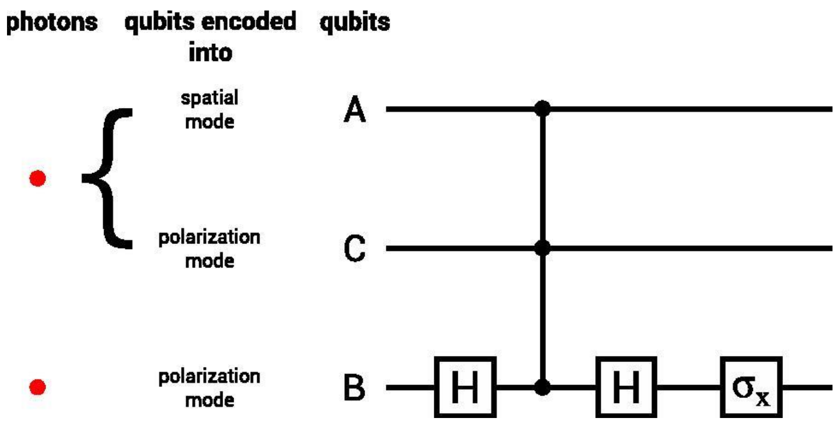
exhibits the effect for up to 13.7% of white noise.

Logical circuit

$|\xi\rangle$



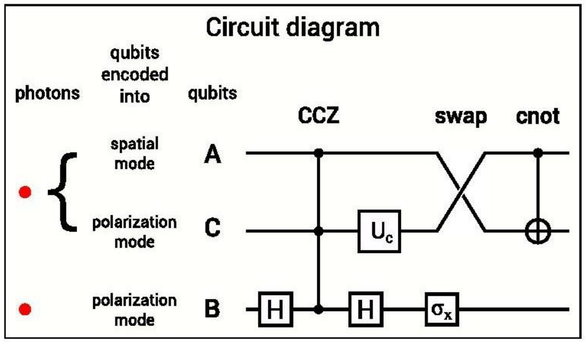
$|\bar{W}\rangle$

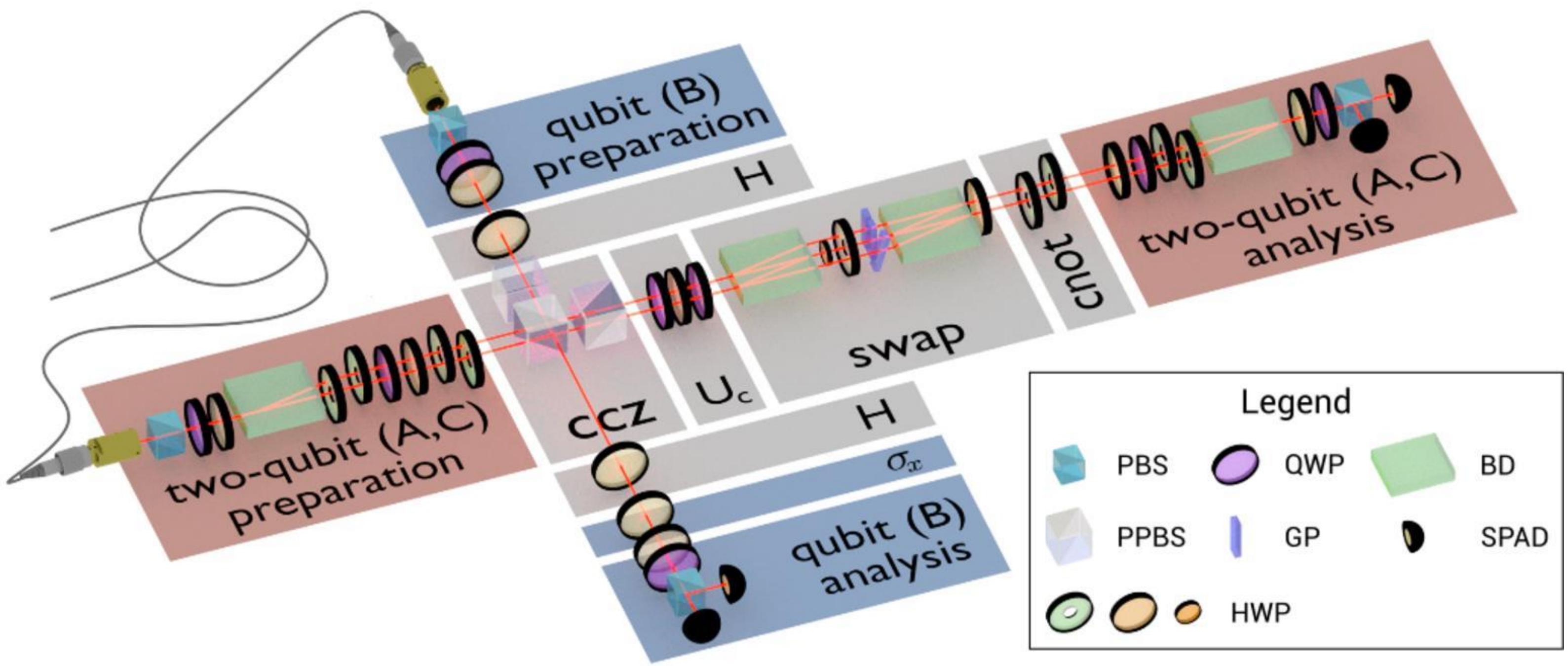


CNOT gate:
Bit flip on the 2nd qubit when the 1st qubit is $|1\rangle$.

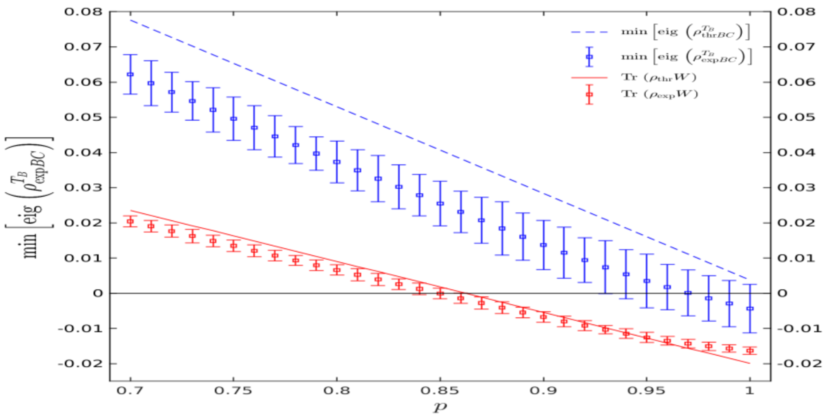
Toffoli gate:
Bit flip on the 3rd qubit when the 1st and 2nd qubit is $|1\rangle$.

Experiment

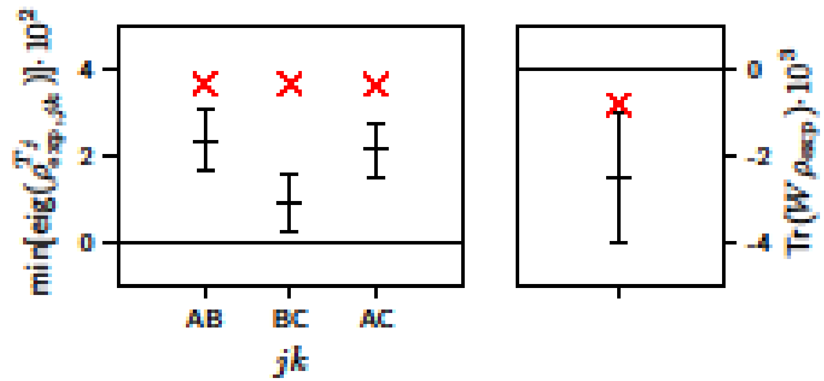

Circuit diagram
photons
qubits encoded into
qubits
CCZ
swap
cnot
spatial mode
polarization mode
polarization mode
A
C
B
H
U$_c$
H
σ$_x$


qubit (B) preparation
H
two-qubit (A,C) preparation
CCZ
U$_c$
swap
cnot
two-qubit (A,C) analysis
H
σ$_x$
qubit (B) analysis
Legend
PBS
QWP
BD
PPBS
GP
SPAD
HWP

Results



p=0.868



Separable marginals

jk	AB	BC	AC
$\min[\mathrm{eig}(\rho_{\mathrm{exp},jk}^{T_j})] \cdot 10^2$	2.4 ± 0.7	0.9 ± 0.6	2.1 ± 0.6

Genuine ME $\mathrm{Tr}(\rho_{\mathrm{exp}} W) = (-3 \pm 2) \cdot 10^{-3}$

Fidelity $\mathcal{F}(\rho_{\mathrm{exp}}, \rho) \equiv \{\mathrm{Tr}[(\rho_{\mathrm{exp}}^{1/2} \rho \rho_{\mathrm{exp}}^{1/2})^{1/2}]\}^2 = 0.939 \pm 0.008.$

Conclusion

- Experimental verification of genuine multipartite entanglement of a global state from its separable marginals.
- Is there a Gaussian version of this phenomenon?
- Is there a classical analogy of this phenomenon?