Towards realistic super-resolution of incoherent point sources

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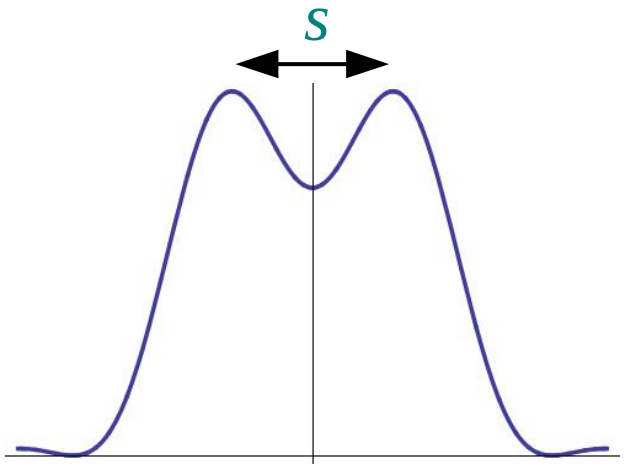
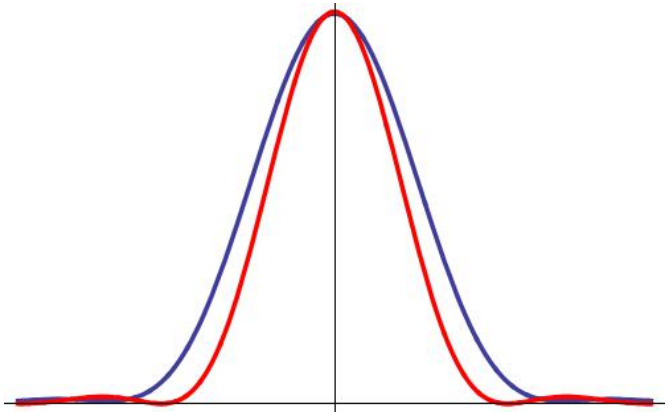
two-point resolution

- linear invariant system
- two mutually incoherent point sources



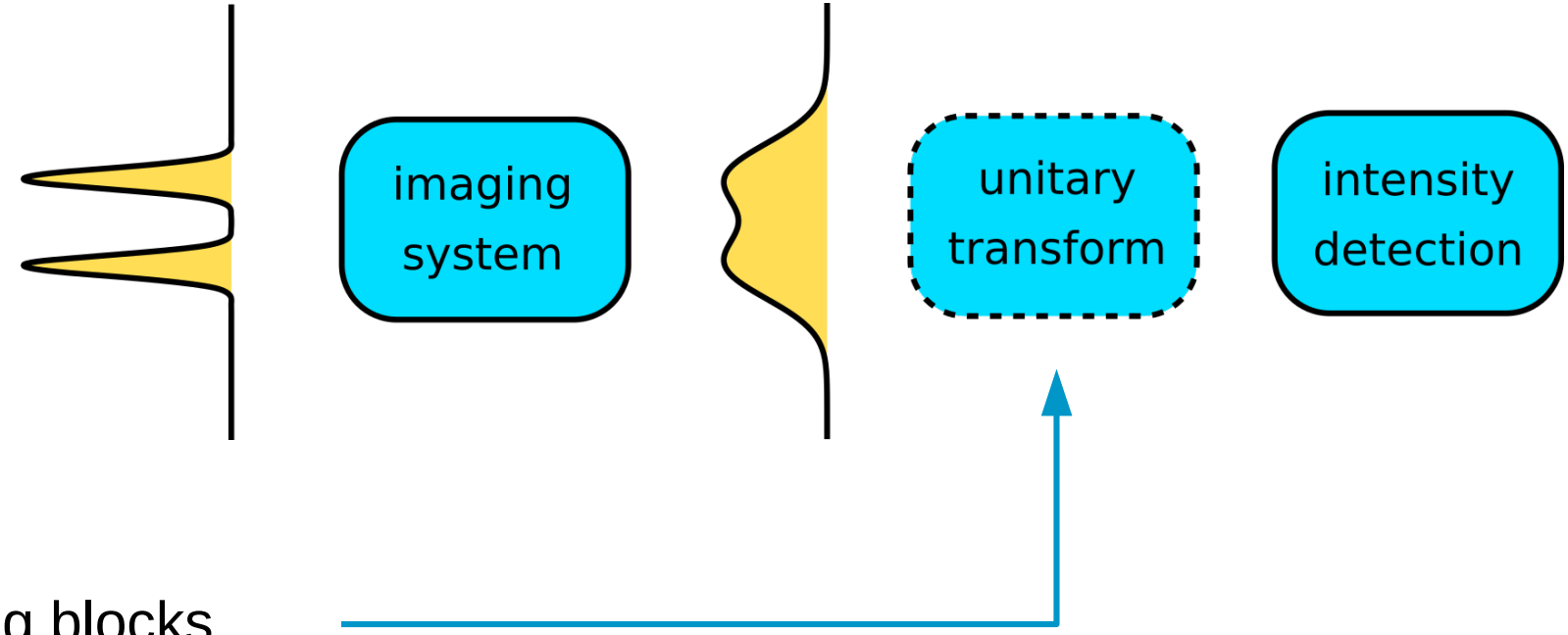
standard resolution limit



super-resolution

- detection noise sets the ultimate limits
- is direct imaging optimal?

standard vs optimal detection

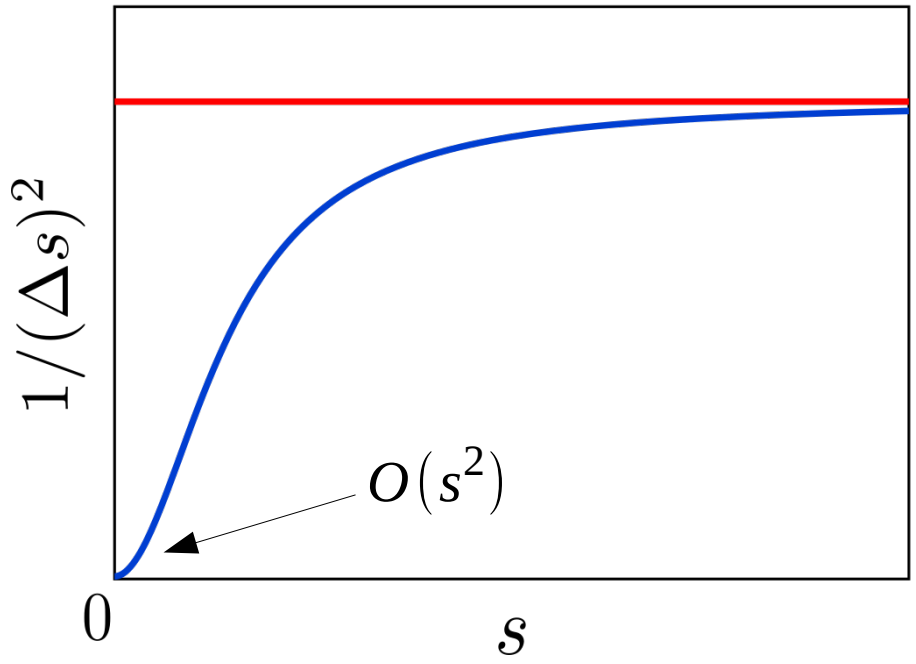


building blocks

- phase modulation
- propagation (FT)

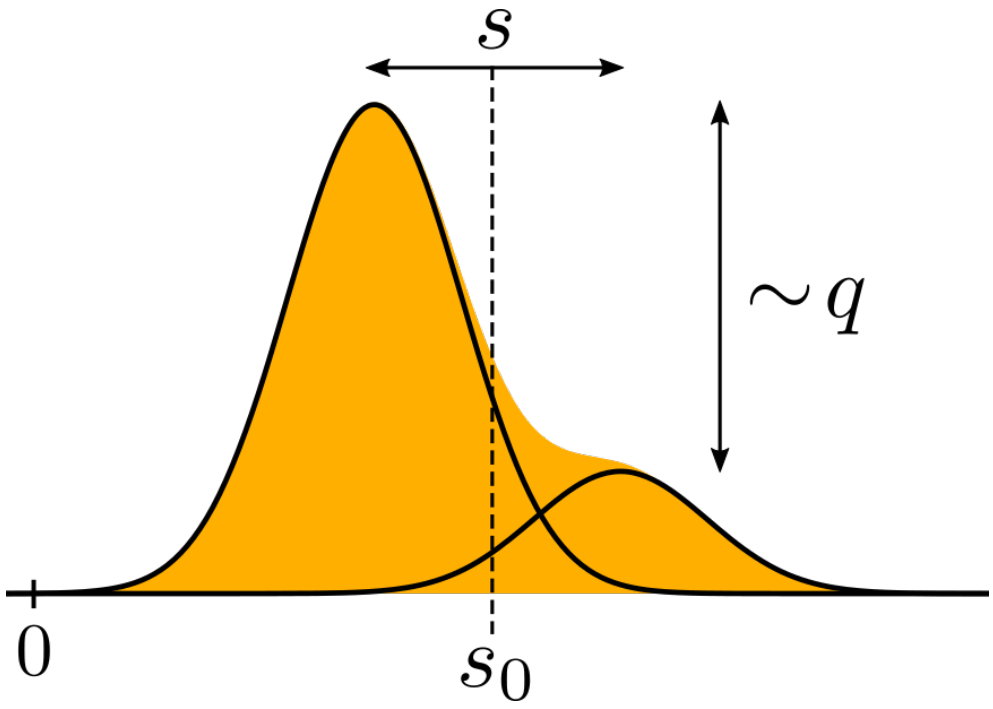
single–parameter case

- performance of ***a given*** measurement is bounded by FI/CRLB
- performance of ***the best*** measurement is bounded by qFI/qCRLB



- super-resolution with direct detection is hard → Rayleigh's curse
- but becomes easy for optimal measurement (M. Tsang and others)

multi–parameter case



real impulse response $\psi(x) = \langle x|\psi\rangle = \psi(x)^*$

signal $\rho(s_0, s, q) = q|\psi_+\rangle\langle\psi_+| + (1-q)|\psi_-\rangle\langle\psi_-|$

signal components $|\psi_\pm\rangle = e^{-i(s_0 \pm s/2)P}|\psi\rangle$

qFI

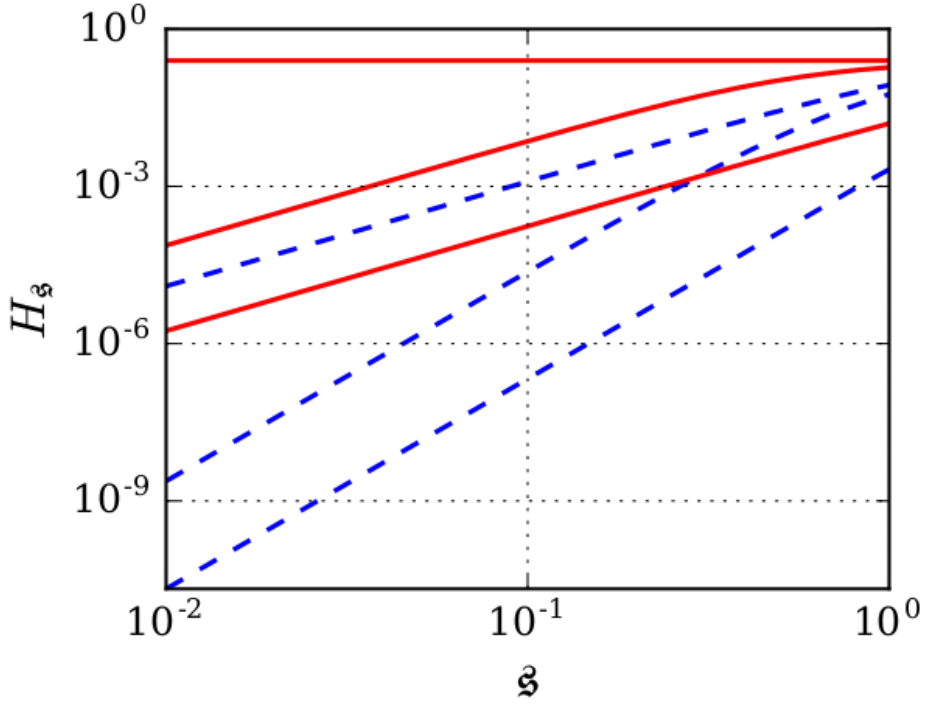
- separation is coupled to other parameters
- all parameters must be estimated simultaneously
- quantum bounds for unbalanced sources

$$1/(\Delta s_0)^2 \propto s^2$$
$$1/(\Delta s)^2 \propto s^2 \quad , \quad s \ll 1$$
$$1/(\Delta q)^2 \propto s^4$$

- Rayleigh's curse reappears in the multi–dimensional case

example: Gauss PSF

Gauss PSF $\sigma = 1$



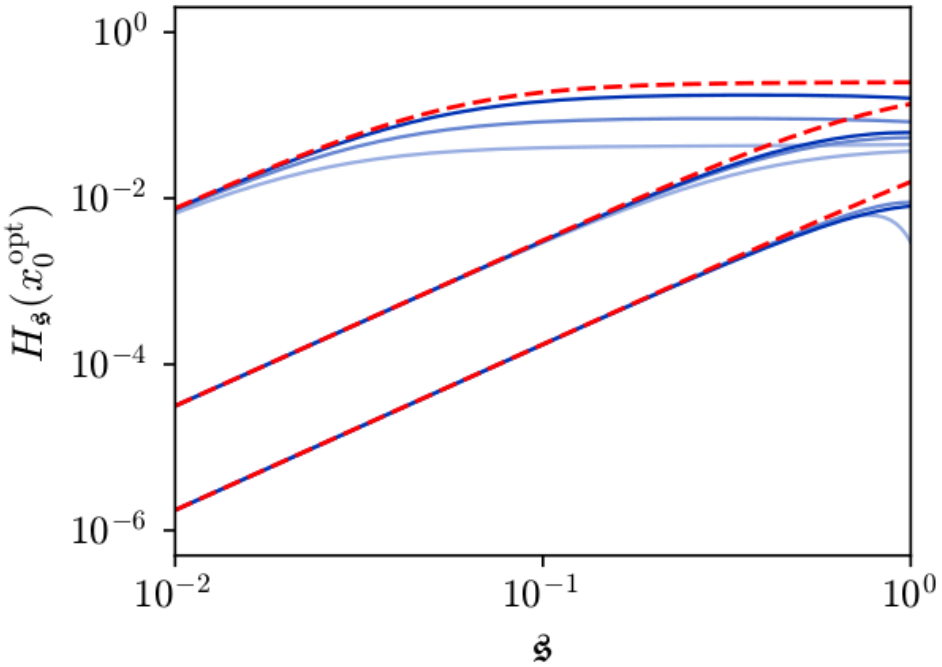
- quantum improvement is always $\propto 1/s^2$
- improvement gets larger for more unbalanced signals

optimal measurements

super–resolution regime $s \ll 1$

3 parameters ⟶ 4-channel minimal POVM

one particular family of optimal measurements applied to a Gauss PSF



conclusions

- QFI can be derived for multi–parameter two-point resolution with arbitrary real PSFs
- Rayleigh’s curse reappears in multiparameter scenarios
- optimal detection provides $1/s^2$ improvement over the direct CCD imaging
- 4-channel measurements can be constructed achieving the quantum limit asymptotically in the super-resolution regime