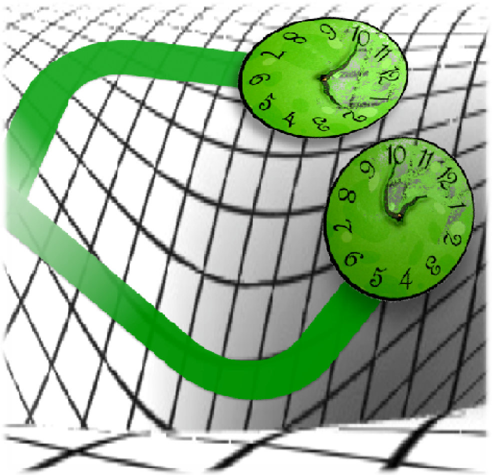
Faculty of Physics, University of Vienna &
Institute for Quantum Optics and Quantum Information, Vienna

Quantum clocks, mirrors and Alice and Bob in Gravity

Časlav Brukner



Olomouc, May 10th, 2012

Motivation

Quantum Mechanics

- entanglement
- single particle interference
- Bohr's complementarity principle
- Born's rule

Newtonian gravity sufficient
(if any gravity effects seen at all!)

General Relativity

- Einstein's equations
- gravity as space-time geometry
- gravitational time dilation
- black holes

consistent with classical mechanics

Motivation

Quantum Mechanics

General Relativity

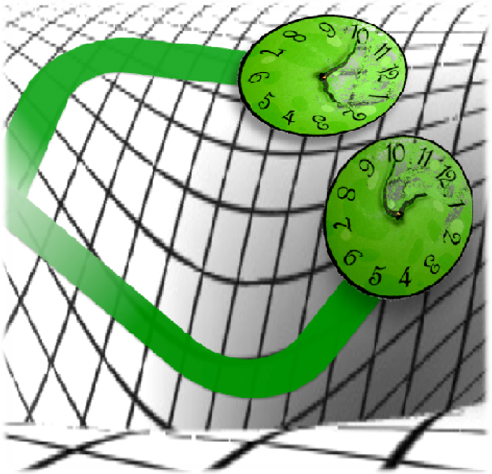
1. Effects that require both theories to be explained?

2. Effects that require an unified framework („quantum gravity“)?

Outline

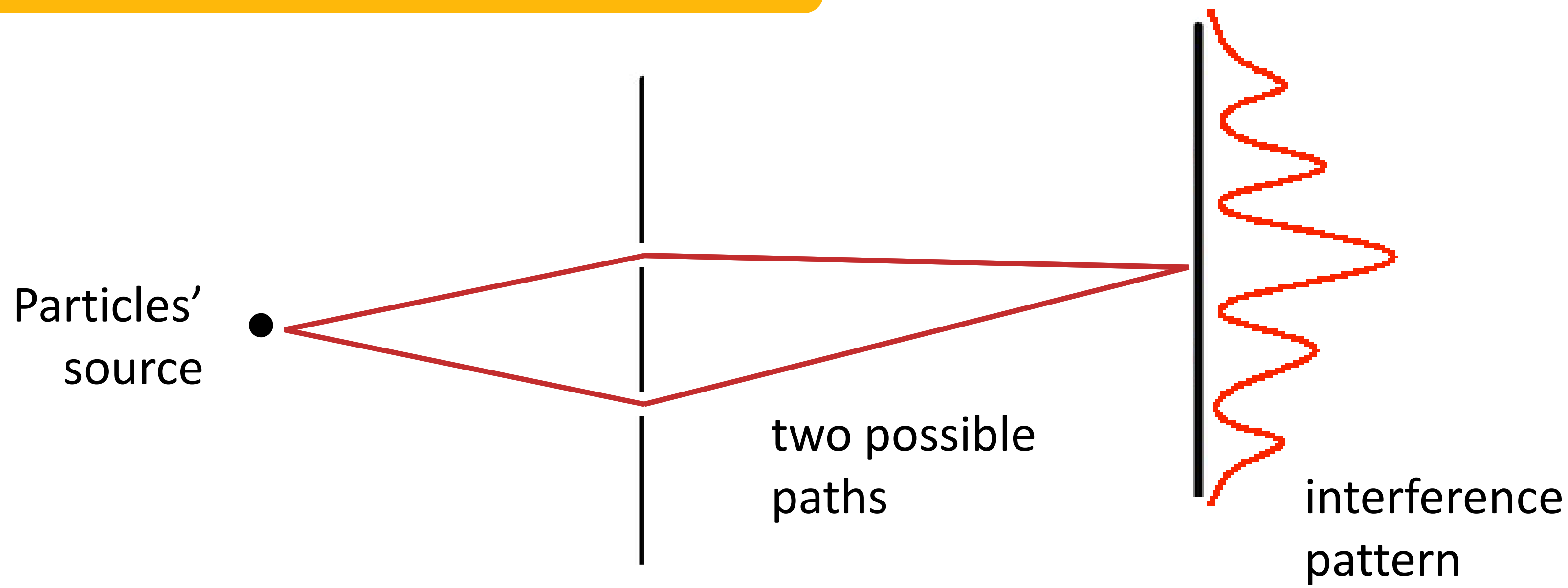
- Introduction & motivation
- [1] Gravitational readshift and quantum complementarity
- [2] Quantum correlations with no-causal order
- [3] Probing Planck-scale physics with quantum optics
- Conclusion

[1] Gravitational redshift and quantum complementarity

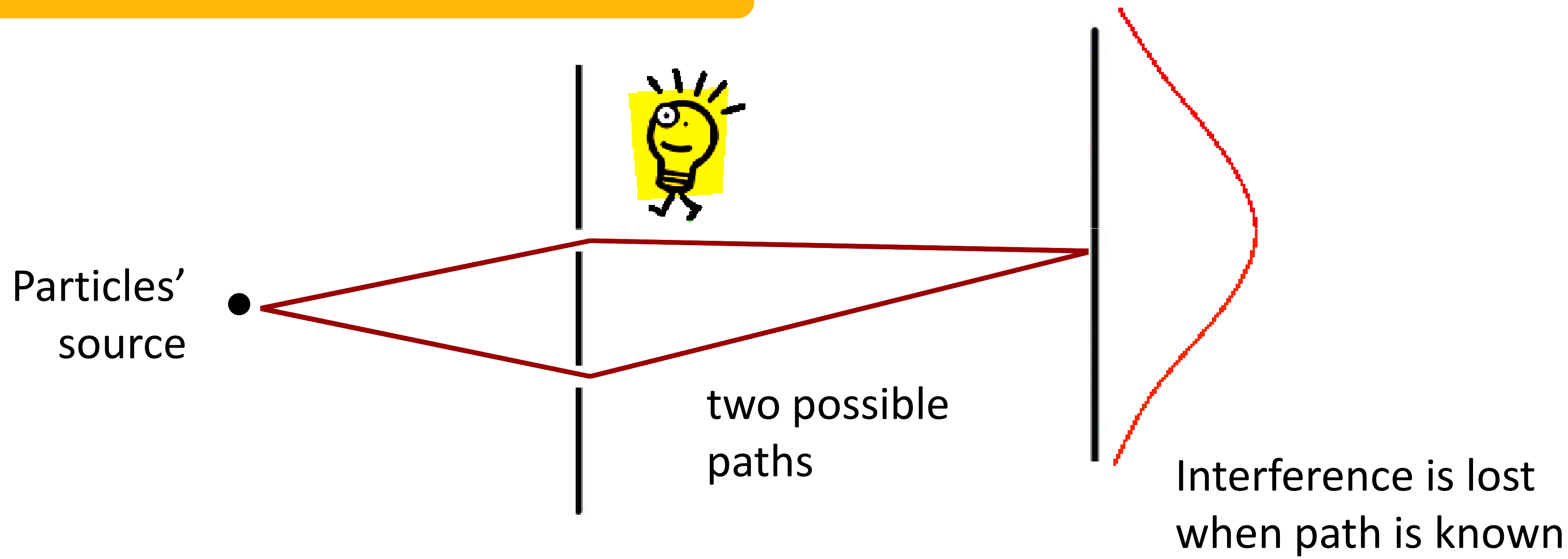


M. Zych, F. Costa, I. Pikovski, Č. Brukner:
Nature Communication 2:505
doi: 10.1038/ncomms1498 (2011)

Quantum Complementarity Principle

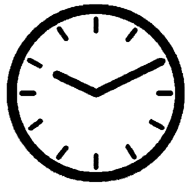


Quantum Complementarity Principle



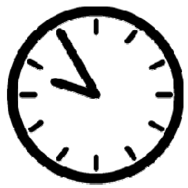
It is **not possible** to simultaneously know the path of the particle and observe its interference.

Gravitational time dilation



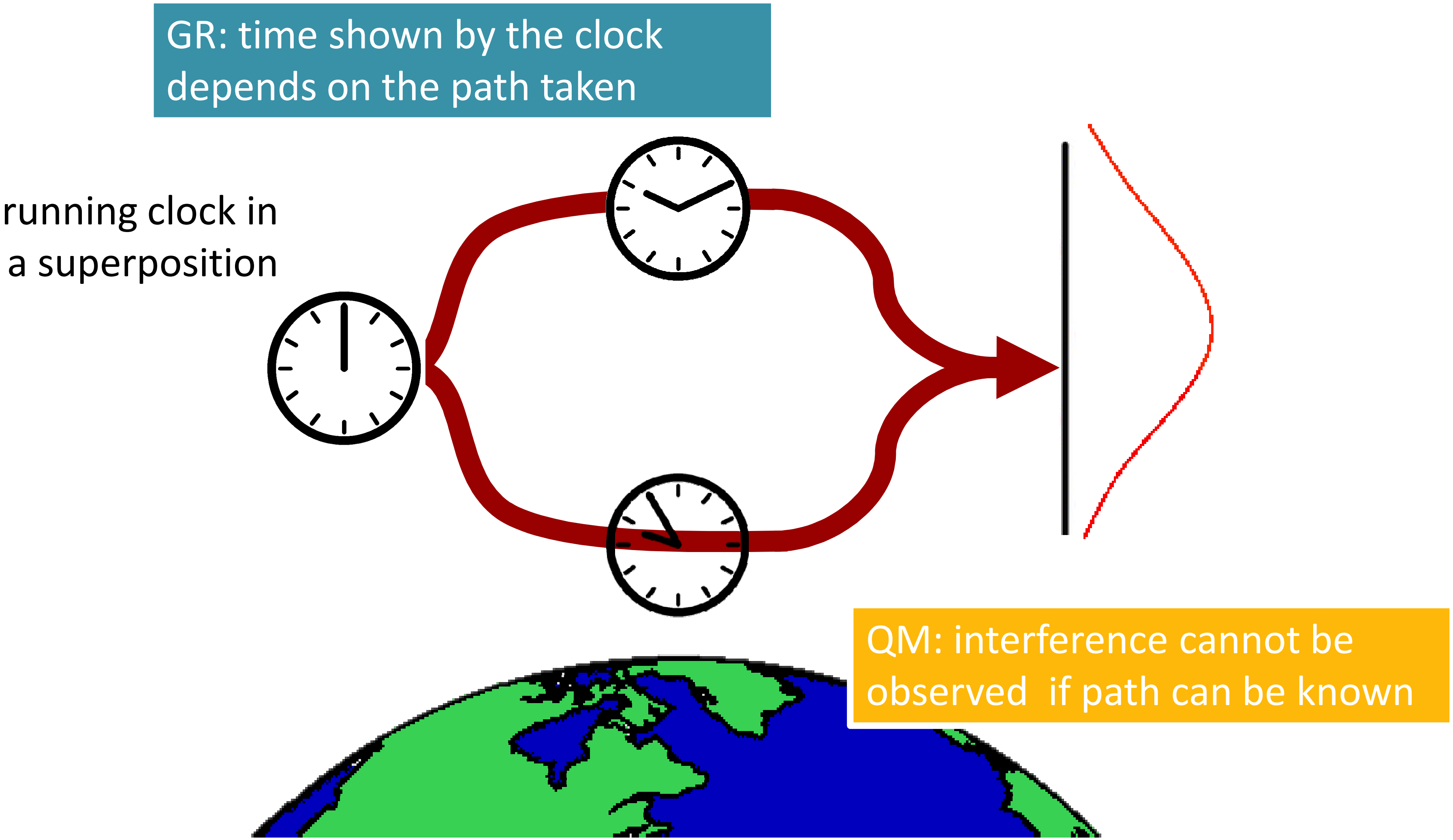
Two initially synchronized clocks placed at different gravitational potentials.

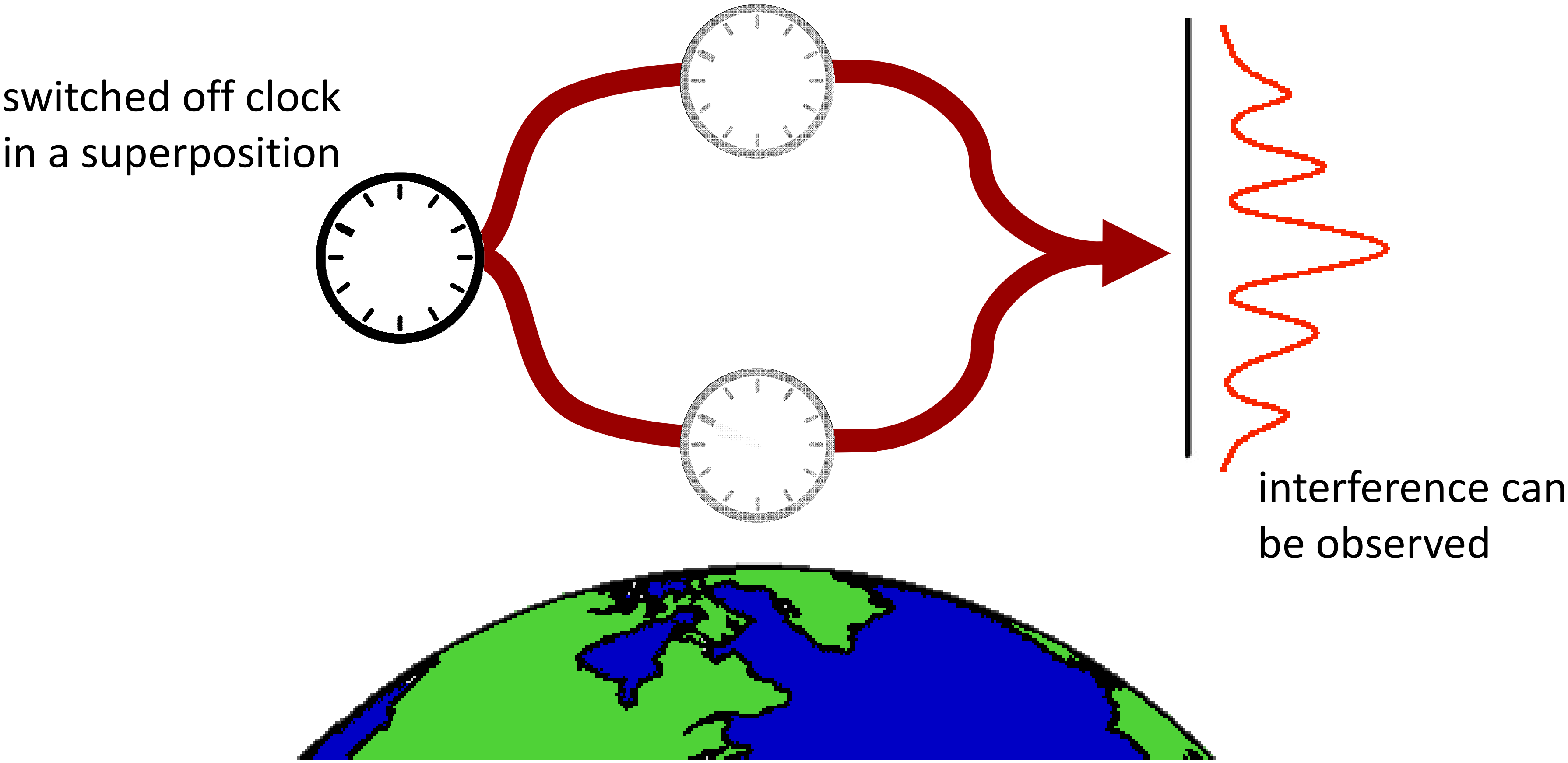
Clock closer to a massive body ticks slower than the clock further away from the mass.



Initially synchronized clocks will eventually show **different times** when placed at different gravitational potentials.

Interference of clocks

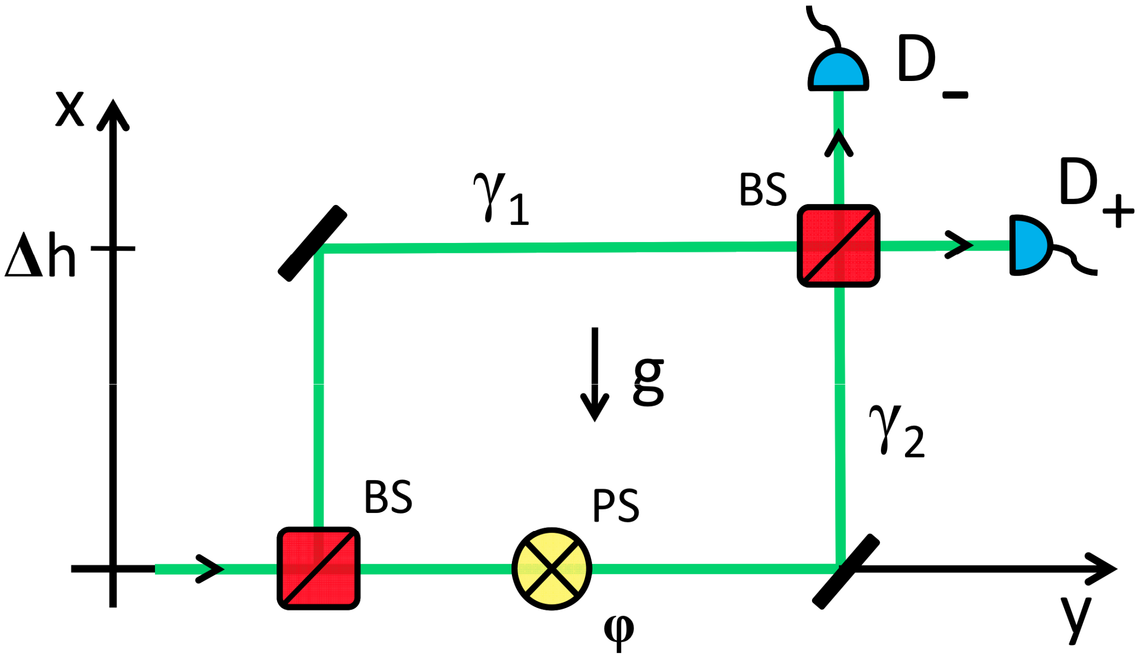




quantum complementarity + time dilation =

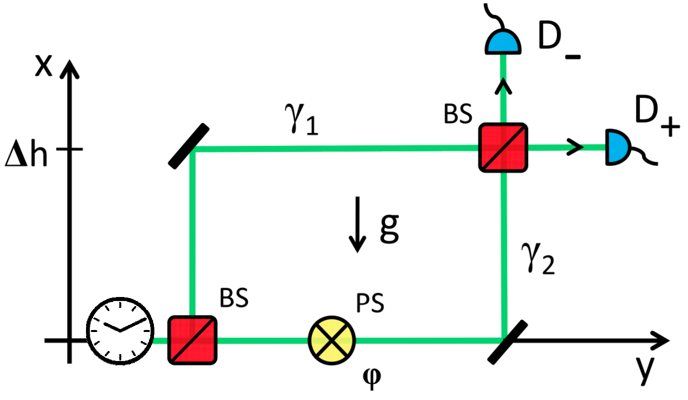
= drop in the interferometric visibility

Mach-Zehnder interferometer in a gravitational field



$\gamma_{1,2}$: two possible paths through the setup,
g : homogeneous gravitational field,
Δh : separation between the paths

Quantum Complementarity



"**clock**"- a system with an evolving in time degree of freedom

modes associated with the path γ_1

state of the "clock", which followed path γ_1

$$|\Psi_{MZ}\rangle = \frac{1}{\sqrt{2}}\left(i|r_1\rangle|\tau_1\rangle e^{-i\phi_1} + |r_2\rangle|\tau_2\rangle e^{-i\phi_2+i\varphi}\right)$$

Probabilities of detection

$$\langle\tau_1|\tau_2\rangle = |\langle\tau_1|\tau_2\rangle|e^{i\alpha}$$

$$P_\pm = \frac{1}{2} \pm \frac{1}{2}|\langle\tau_1|\tau_2\rangle|\cos\left(\Delta\phi + \alpha + \varphi\right)$$

Visibility of the interference pattern: $\mathcal{V} = |\langle\tau_1|\tau_2\rangle|$

Distunguishability of the paths: $\mathcal{D} = \sqrt{1 - |\langle\tau_1|\tau_2\rangle|^2}$

Interferometric visibility drops to the extent to which path information becomes available from the "clock"

Results

$$H_{\odot} = E_0 |0\rangle\langle 0| + E_1 |1\rangle\langle 1|$$

$$|\tau^{in}\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

- $\Delta E := E_1 - E_0$
- $\Delta V := g\Delta h$, gravitational potential
- Δh: distance between the paths
- ΔT: time for which the particle travels in superposition at constant heights

$$P_+(\varphi, m, \Delta E, \Delta V, \Delta T) =$$

$$= \frac{1}{2} \pm \frac{1}{2}\cos\left(\frac{\Delta E \Delta V \Delta T}{2\hbar c^2}\right)\cos\left(\left(mc^2 - \langle H_{\odot}\rangle + \bar{E}^{GR}_{corr}\right)\frac{\Delta V \Delta T}{\hbar c^2} + \varphi\right)$$

relative phase from the Newtonian potential

GR corrections to the relative phase from the path d.o.f.

new effects appearing with the "clock":

change in the interferometric visibility

$$\mathcal{V} = \left|\cos\left(\frac{\Delta E \Delta V \Delta T}{2\hbar c^2}\right)\right|$$

phase shift proportional to the average internal energy

Results

$$H_{\odot} = E_0 |0\rangle\langle 0| + E_1 |1\rangle\langle 1|$$

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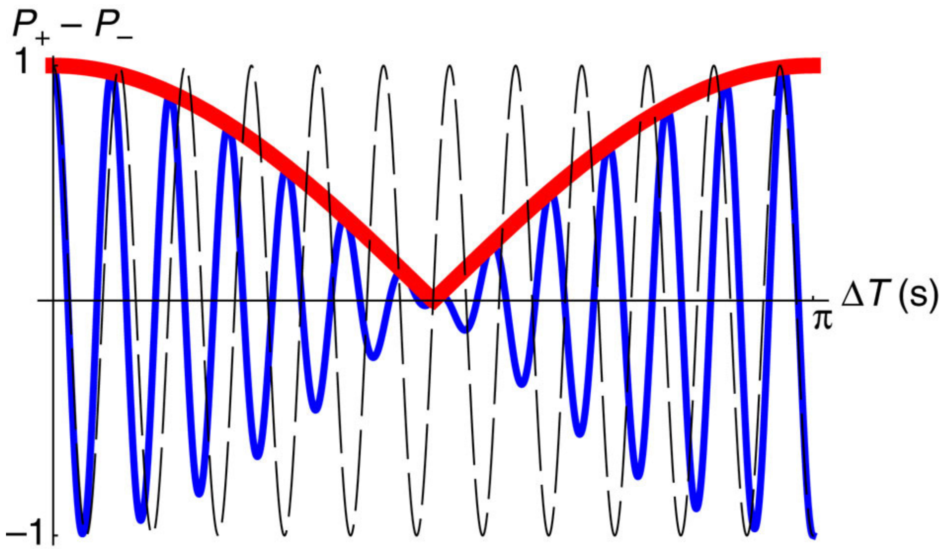


- dashed, black line - interference with the "clock" switched off
- blue line - phase with the "clock" switched on
- thick, red line - modulation in the visibility

Phase shift vs Drop of visibility

$$|\Psi_{MZ}\rangle = \frac{1}{\sqrt{2}}\left(i|r_1\rangle|\tau_1\rangle e^{-i\phi_1} + |r_2\rangle|\tau_2\rangle e^{-i\phi_2+i\varphi}\right)$$

Phase Shift

Drop in Visibility

Explanable by:
- a potential force in absolute time (possible non-Newtonian)
- analogue to a charged particle in EM field
- Flat space-time: no redshift
- independent of whether a particle is a „clock“ or a rock

Not explanaible without:
- gravity as metric theory,
- proper time τ flows at different rates – redshift
- curved space-time geometry
- iff a particle is an operationally well defined „clock“

Colella, R., Overhauser, A. W. & Werner, S. A. *Phys. Rev. Lett*. 34, 1472–1474 (1975).

Müller, H., Peters, A. & Chu, S. *Nature* 463, 926–929 (2010).

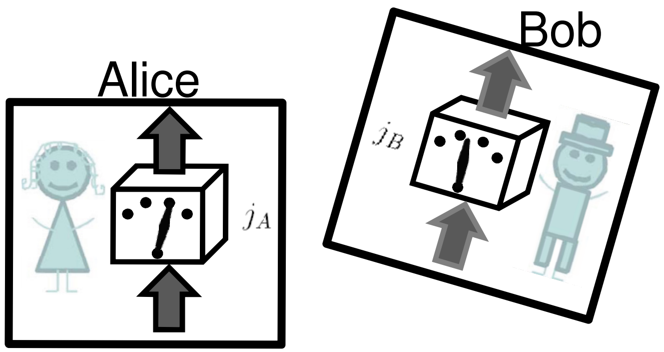
Experiment challenging (2-3 orders of magnitude)

MZ3

MZ3 such an interpretation was recently propsed in: H. Müller, A. Peters, & S. Chu, A precision measurement of the gravitational redshift by the interference of matter waves. Nature 463, 926–929 (2010).

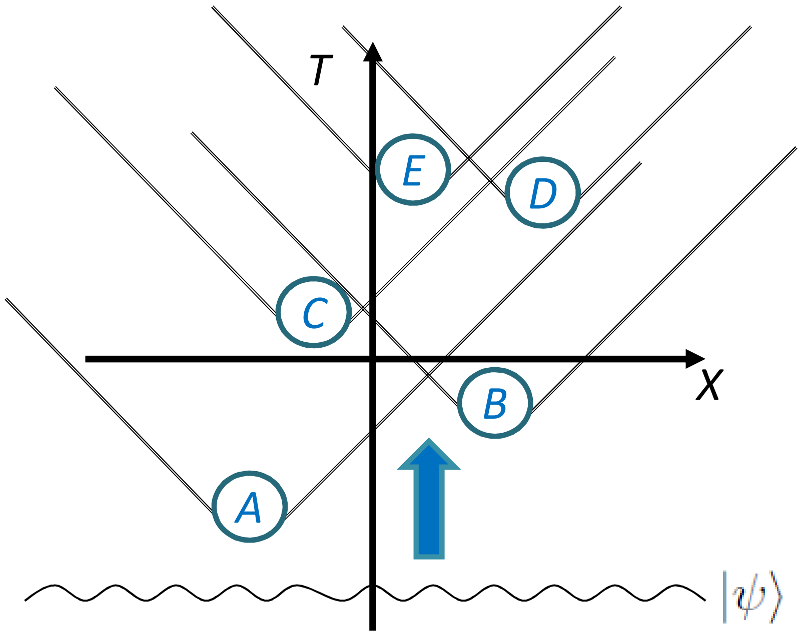
Magdalena Zych; 23.1.2012

[2] Quantum correlations with no causal order



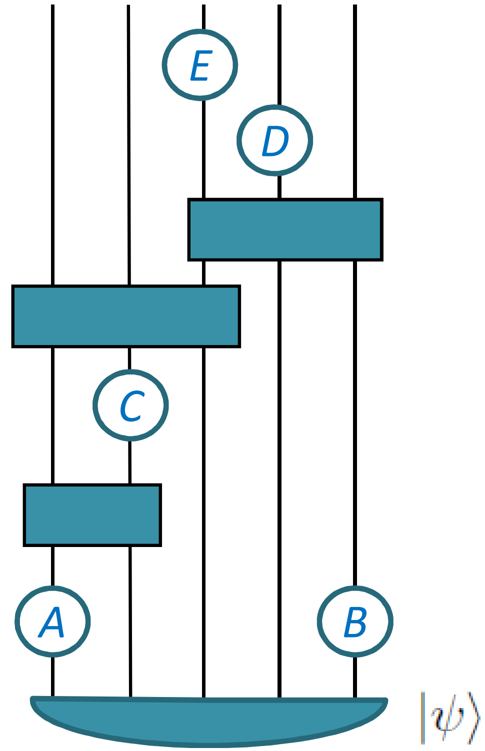
O. Oreshkov, F. Costa, Č. Brukner:
arXiv:1105.4464

Measurements in space-time



- Fix positions
- Define initial state
- Follow Eqs of motion
- Include causal influences
- Find joint probabilities
  P(*A, B, C, D, E*)

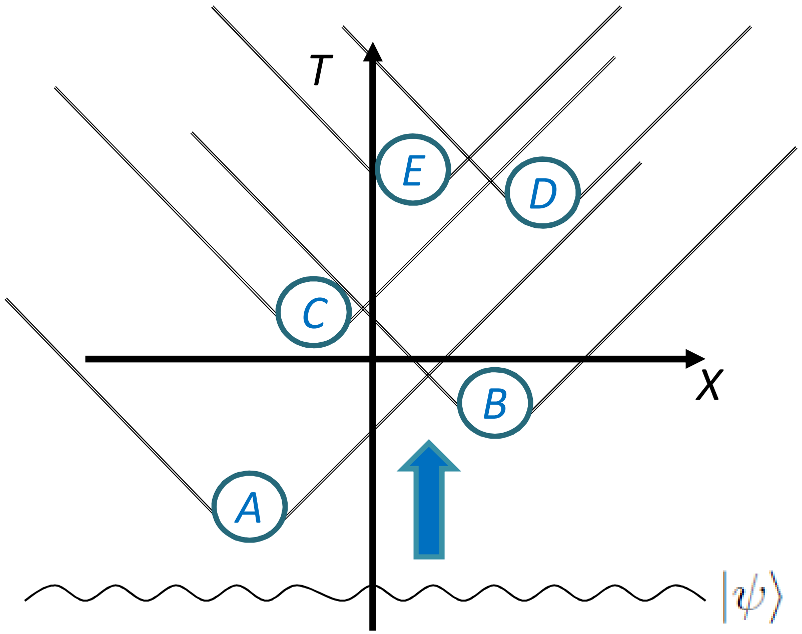
Circuit model



Space-time & definite causal structure are pre-existing entities.

What happens if one removes global time and causal structure from quantum mechanics? What new phenomenology is implied?

Measurements in space-time



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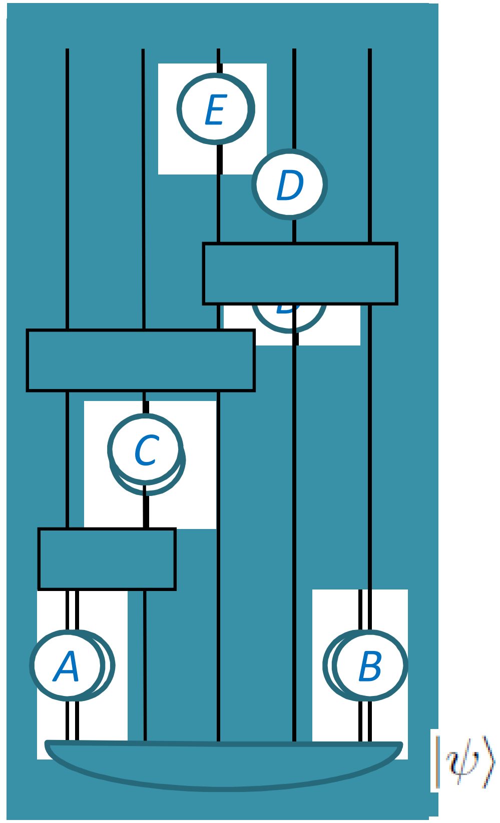
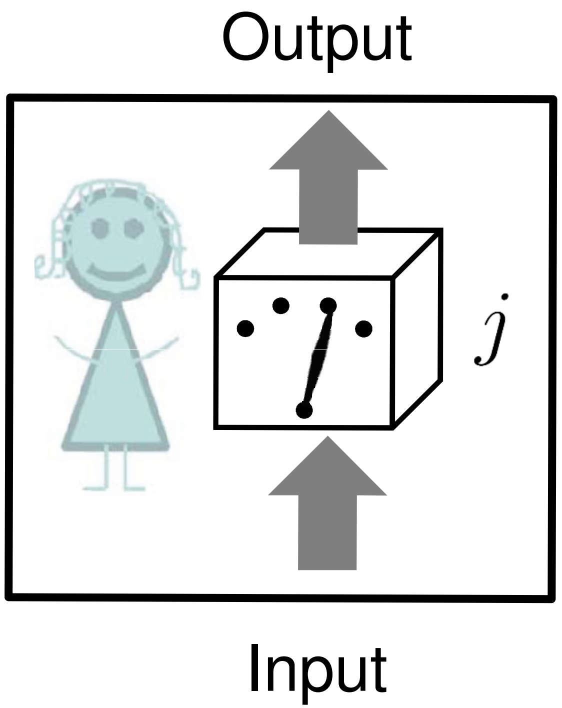
Circuit model



New computational model?

New phenomenology?

Operational approach



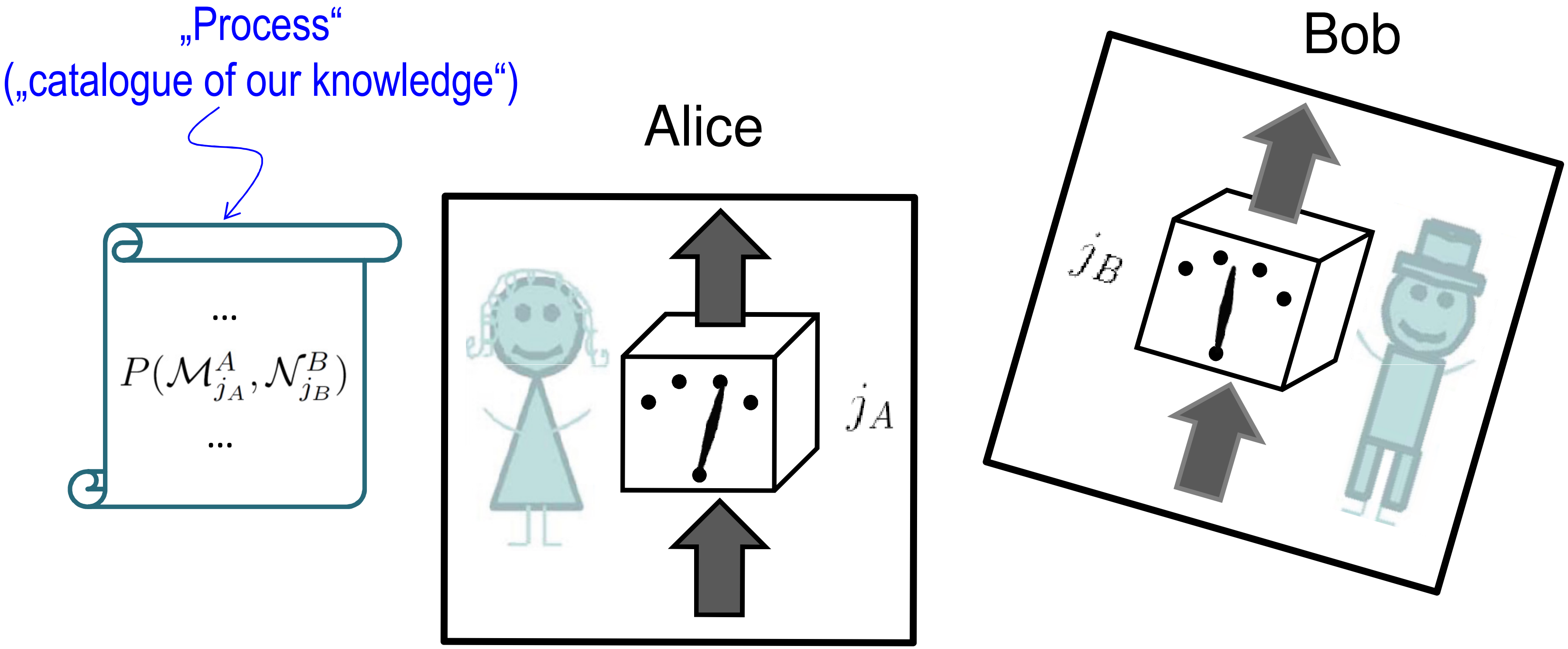
The system exits the lab.

One out of a set of possible transformations (CP-maps) is performed.

A system enters the lab.

This is the **only** way how the labs interact with the “outside world”.

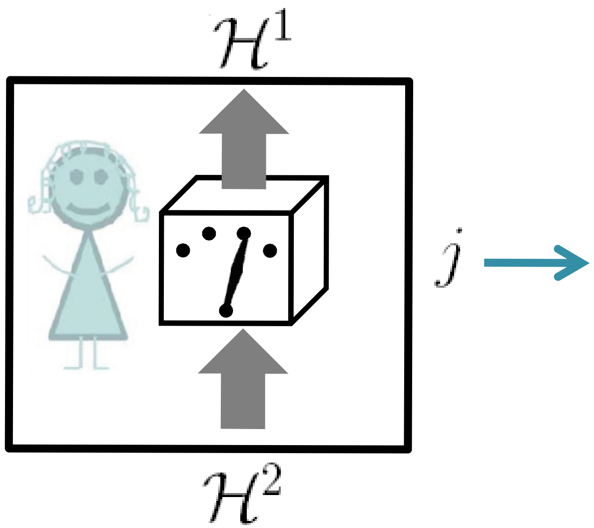
Operational approach



No prior assumption of pre-existing causal structure, in particular of the pre-existing background time.

Main premise:

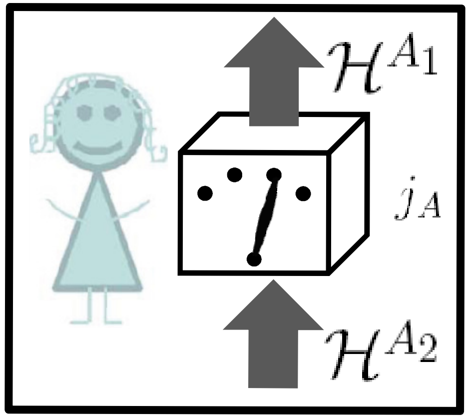
Local quantum mechanics: The local operations of each party are described by quantum mechanics.

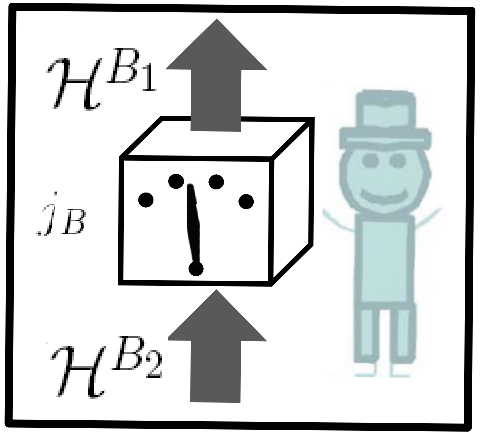


Transformations = completely positive (CP) trace non increasing maps

$$\mathcal{M}_j : \mathcal{L}(\mathcal{H}^2) \to \mathcal{L}(\mathcal{H}^1)$$

Two parties





$$\mathcal{M}^A_{j_A} : \mathcal{L}(\mathcal{H}^{A_2}) \to \mathcal{L}(\mathcal{H}^{A_1}) \qquad \mathcal{M}^B_{j_B} : \mathcal{L}(\mathcal{H}^{B_2}) \to \mathcal{L}(\mathcal{H}^{B_1})$$

Probabilities are bilinear functions of the CP maps

Choi-Jamilkowski representation of the CP maps:

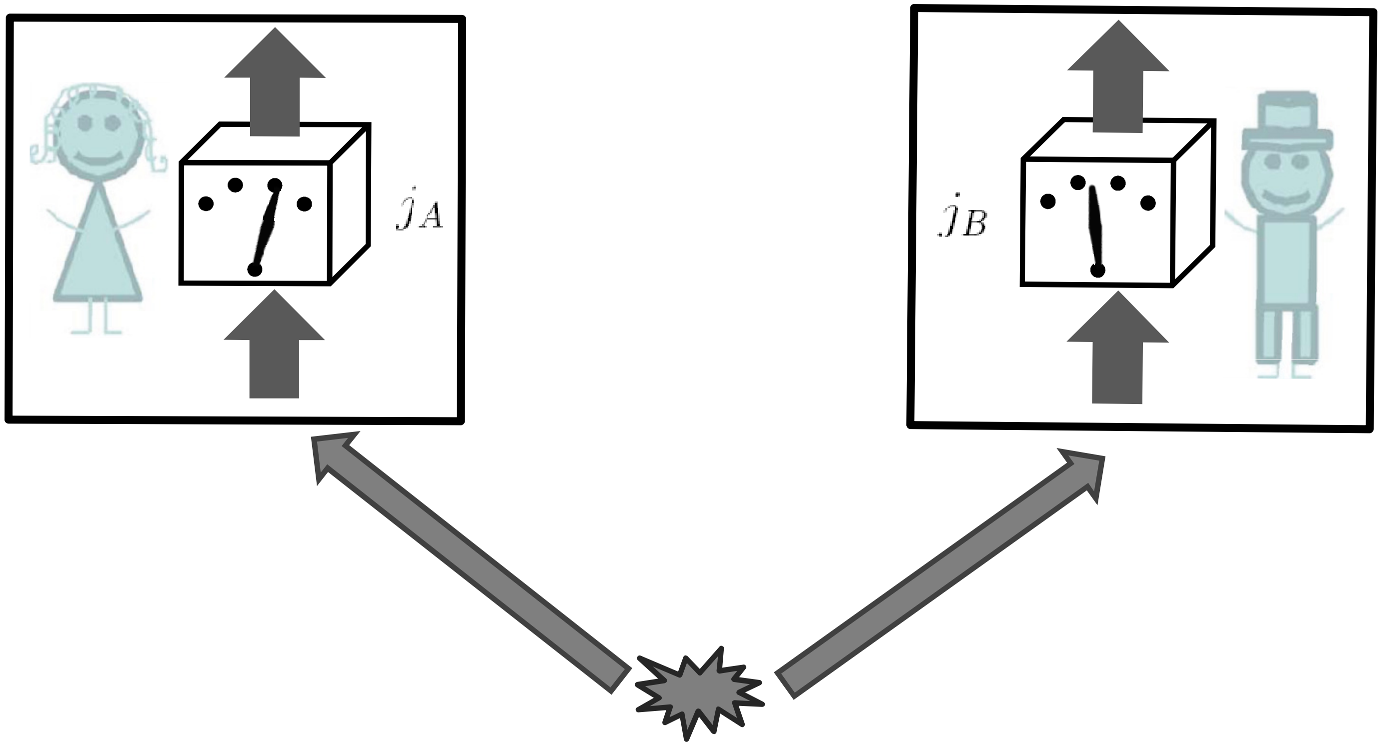
$$P(\mathcal{M}^A, \mathcal{M}^B) = \mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2}\left(\rho^{A_1 A_2}_{\mathcal{M}^A} \otimes \rho^{B_1 B_2}_{\mathcal{M}^B}\right)\right]$$

„Process Matrix"

CP maps

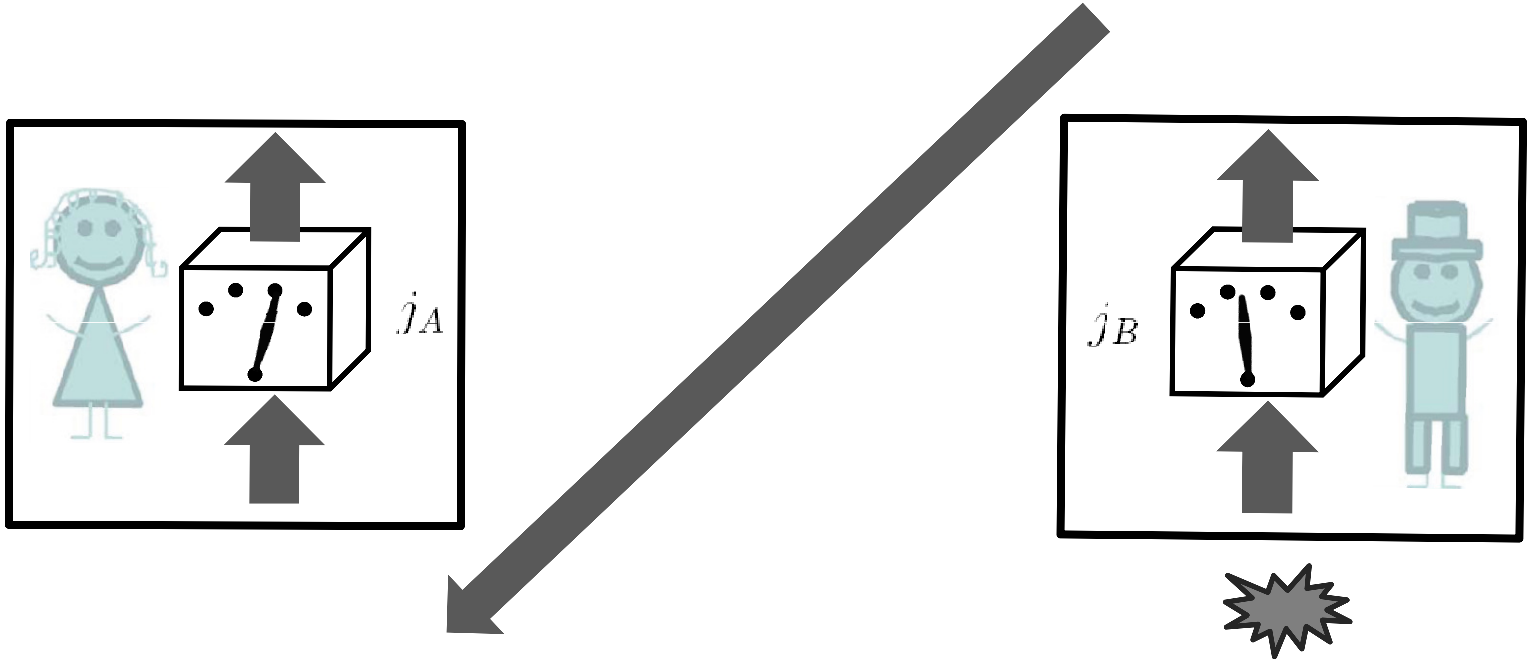
Chiribella et al, Phys. Rev. A. 81, 062348 (2010)

Bipartite state



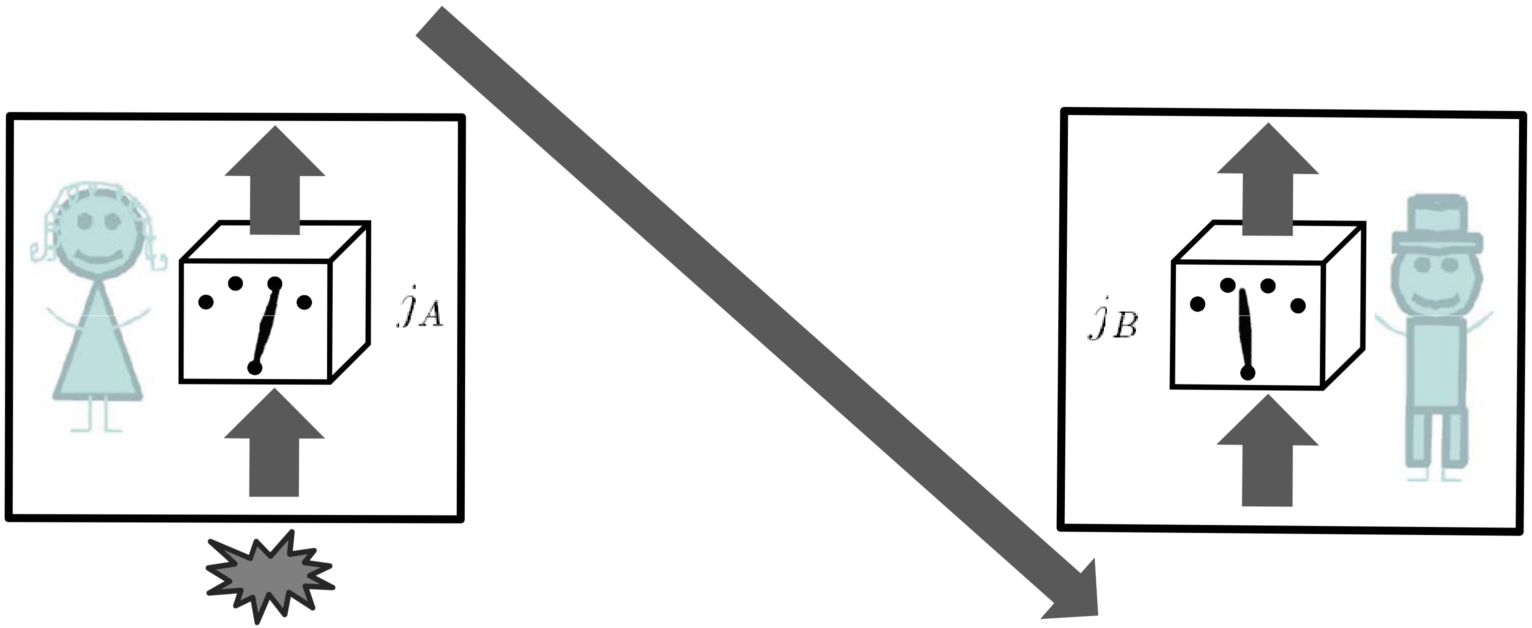
Sharing a joint state; No signalling

Channel B→A



Sending a state from B to A; Possibility of signalling

Channel A→B



Sending a state from A to B; Possibility of signalling

Mixtures of different orders also possible

Most general causally separable situation: probabilistic mixture of ordered ones:

$$W^{A_1A_2B_1B_2} = qW^{B\not\leq A} + (1-q)W^{A\not\leq B}$$

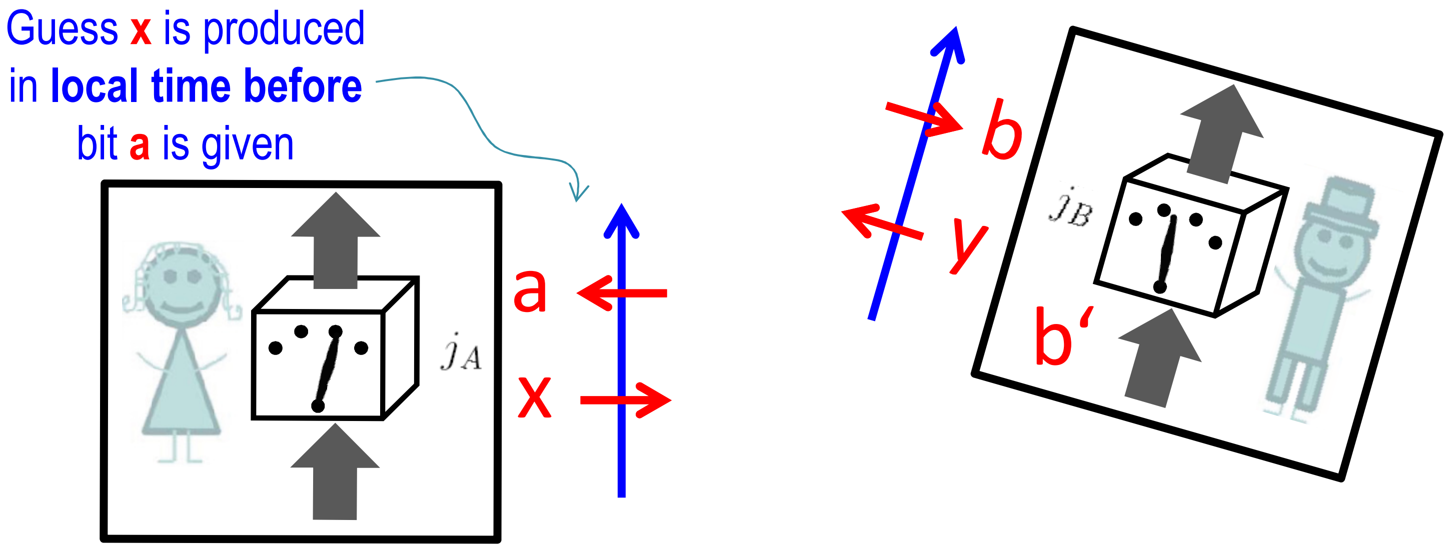
Signalling only from A to B or causally independent

Signalling only from B to A or causally independent

Do all possible processes W respect definite causal order?

NO!

Causal Game

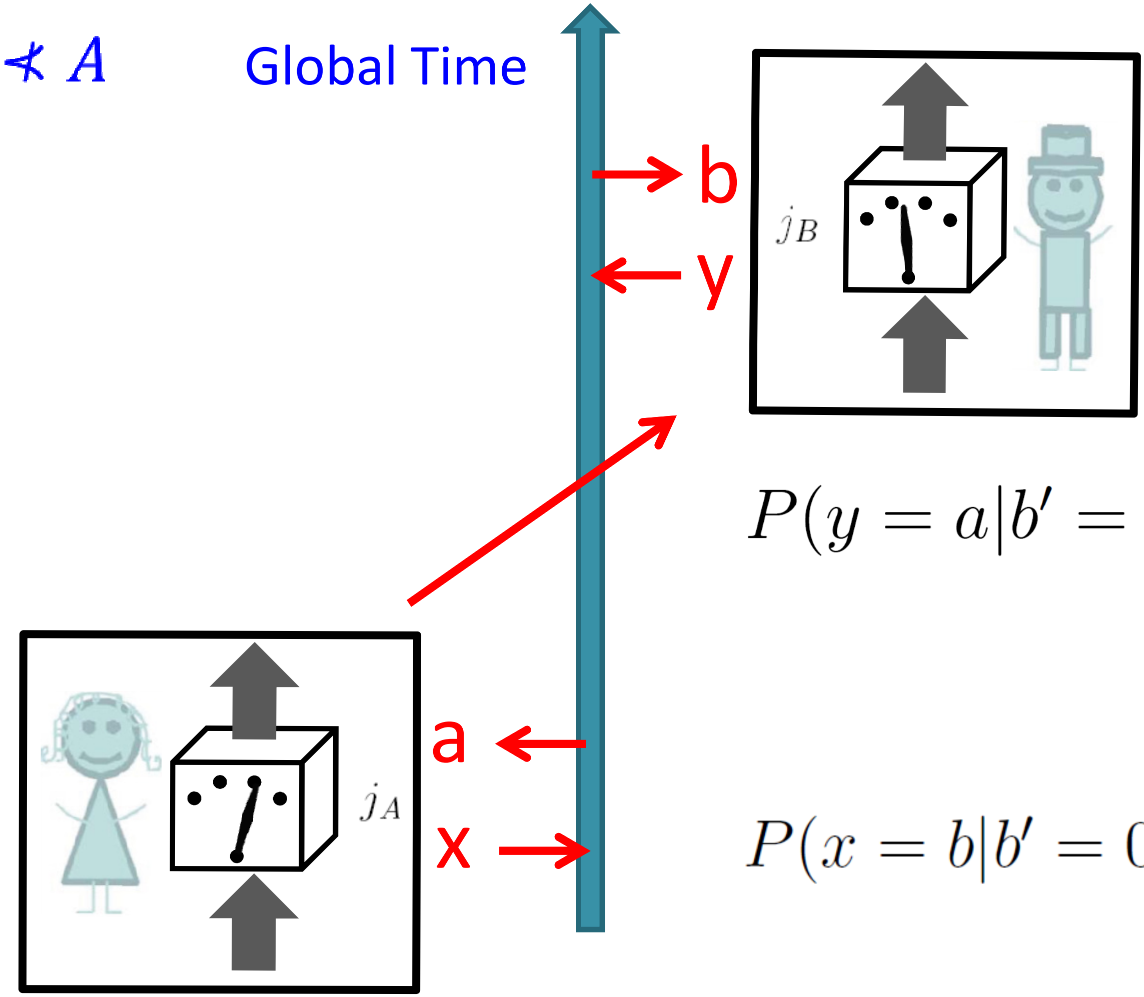


- Alice is given bit **a** and Bob bit **b**.
- Alice produces **x** and Bob **y**, which are their best guesses for the value of the bit given to the other.
- Bob is given an additional bit **b'** that tells him whether he should guess her bit (**b'=1**) or she should guess his bit (**b'=0**).
- The goal is to maximize the probability for correct guess:

$$p_{succ} := \frac{1}{2}\left[P(x = b|b' = 0) + P(y = a|b' = 1)\right]$$

Causally ordered situation

Case: $B \not\prec A$



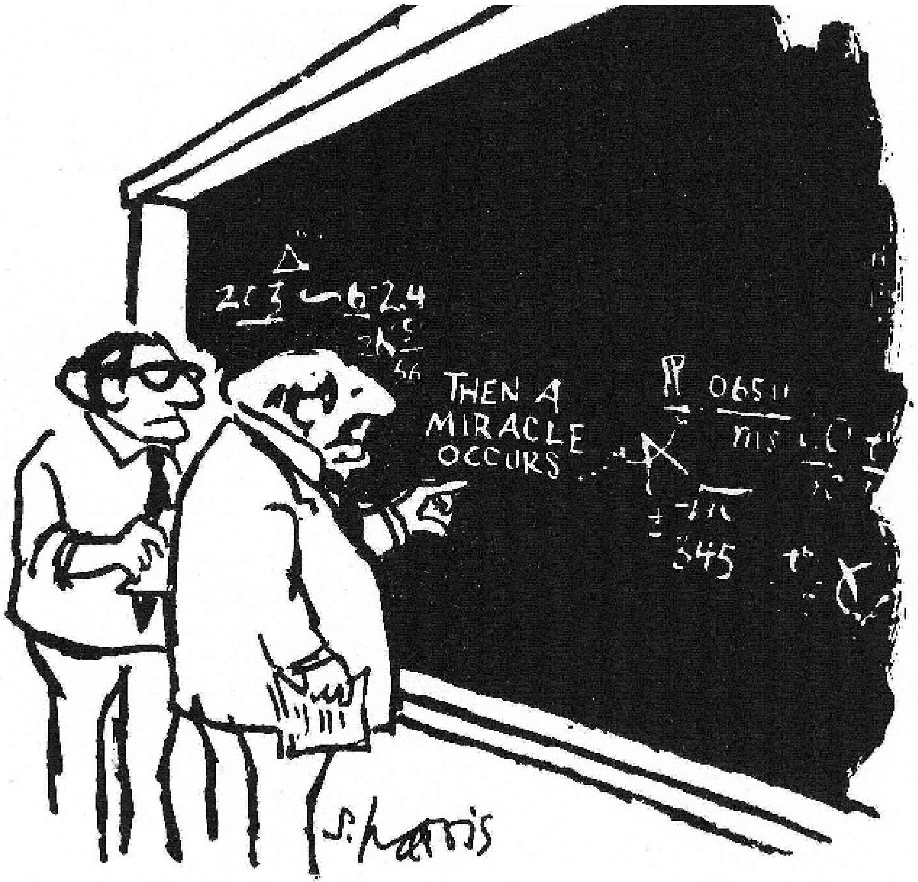
$P(y = a|b' = 1) = 1$

$P(x = b|b' = 0) = 1/2$

$$p_{succ} = P(x = b|b' = 0) + P(y = a|b' = 1) \leq \frac{3}{4}$$

Causally non-separable situation

$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_1}\sigma_z^{B_2} + \sigma_z^{A_2}\sigma_z^{B_1}\sigma_x^{B_2}\right)\right]$$



"I think you should be more explicit here in step two."

The probability of success is

$$p_{succ} = \frac{2+\sqrt{2}}{4} > \frac{3}{4}$$

"Tsirlason bound for non-causal correlations" ??

This process cannot be realized as a probabilistic mixture of causally ordered situations!

[2] Probing Planck physics with quantum optics

I. Pikovski, M. R. Vanner, M. Aspelmeyer, M. S. Kim and Č. Brukner:
Nature Physics (2012) doi:10.1038/nphys2262

Experimental quantum gravity?

Effects largely believed to be relevant at the **Planck-scale:**

$$E_{Planck} = \sqrt{\frac{\hbar c^5}{G}} = 1.956 \times 10^9 \text{ J}$$

$$\frac{E_{exp}}{E_{Planck}} \approx 10^{-15}$$

High-energy scattering experiments

$$L_{Planck} = \sqrt{\frac{\hbar G}{c^3}} = 1.6161 \times 10^{-35} \text{ m}$$

$$\frac{x_{exp}}{L_{Planck}} \approx 10^{17}$$

High-precision quantum metrology

$$m_{Planck} = \sqrt{\frac{\hbar c}{G}} = 22\ \mu\text{g}$$

Optomechanics:

Mirrors can have mass of pg – kg!

Modified uncertainty relation

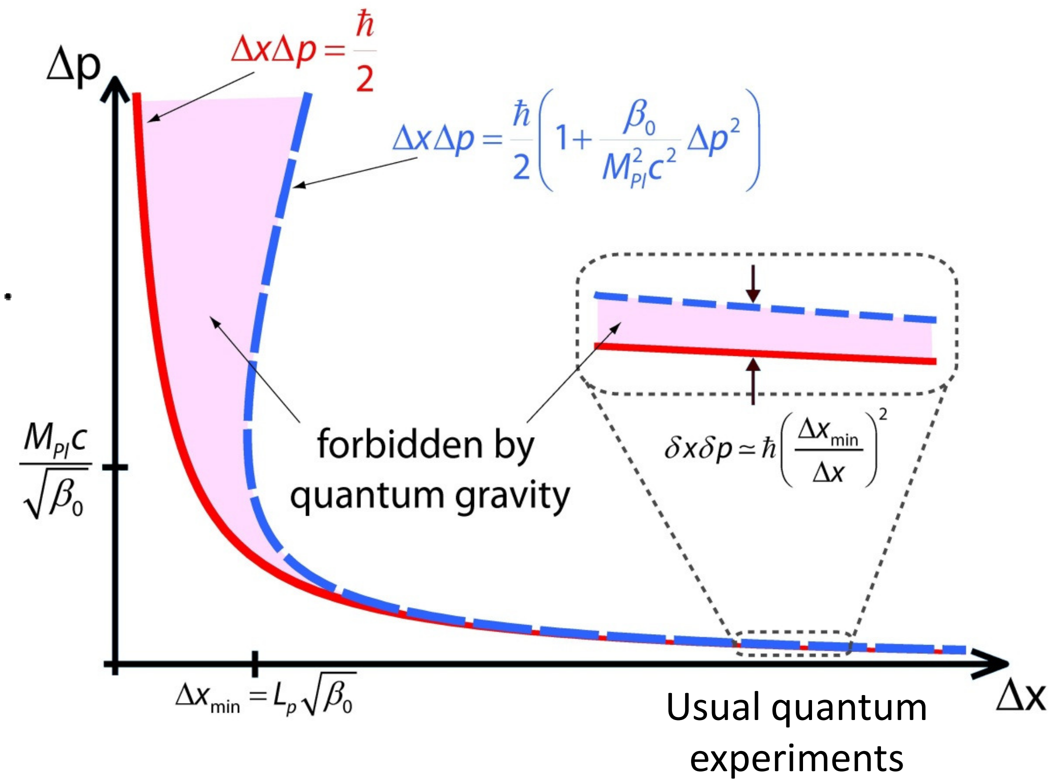
- A minimal measurable length scale Δx_{min}~Planck-length $L_P \sim 10^{-35}\ m.$
- Thus $\Delta x\ \Delta p = \frac{\hbar}{2}$ cannot hold for $\Delta p \rightarrow \infty$
- Typical modification in QGR:

$$\Delta x\ \Delta p\ \geq \frac{\hbar}{2}(1+\beta_0 \frac{\Delta p^2}{M_{Pl}^2 c^2})$$

standard QM

Modification: $\mathrm{M_{Pl}} \approx 22\mu g$ Planck-mass, β_0 dimensionless parameter

(L. Garay, Int. J. Mod. Phys. A10, 145 (1995))



Current experimental bound: $\beta_0 < 10^{33}$

(S. Das & E. C. Vagenas, PRL 101, 221301 (2008))

Possible commutator modifications

$\Delta x\, \Delta p \geq \frac{\hbar}{2}(1+\beta\Delta p^2)$ implies a modified commutator. E.g.:

- $\left[\hat{X},\hat{P}\right]_\beta = i\left(1+\beta_0\frac{\hat{P}^2}{M_{Pl}^2c^2}\right)$ *(A. Kempf, G. Mangano and R. Mann, PRD, 52, 2 (1995))*
- $\left[\hat{X},\hat{P}\right]_\mu = i\sqrt{1+2\mu_0\frac{(\hat{P}/c)^2+m^2}{M_{Pl}^2}}$ *(M. Maggiore, Phys. Lett. B, 319 (1993))*
- $\left[\hat{X},\hat{P}\right]_\gamma = i\left(1-\gamma_0\frac{\hat{P}}{M_{Pl}c}+{\gamma_0}^2\frac{\hat{P}^2}{M_{Pl}^2c^2}\right)$ *(A. F. Ali, S. Das and E. C. Vagenas, Phys. Lett. B, 678 (2009))*

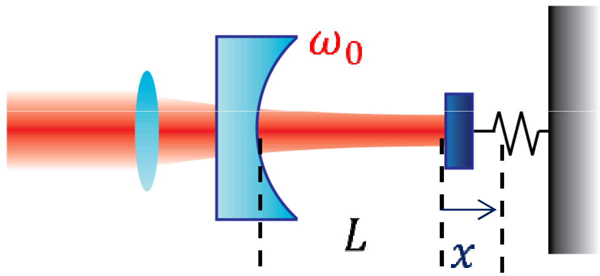
Note: ground-state $p_0 = \sqrt{m\omega\hbar}$, mass-dependent

Opto-mechanics

Control of a massive systems with light

Opto-mechanical interaction:

$$\hat{H} = \hbar\omega_m \hat{n}_m + \hbar\omega_0 \hat{n}_L - \hbar g_0 \hat{n}_L \hat{X}_m,$$

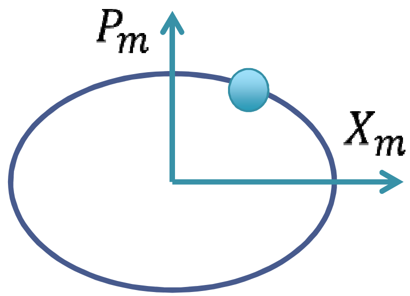


$g_0 = \frac{\omega_0}{L}\sqrt{\frac{\hbar}{m\omega_m}}$ Opto-mechanical coupling rate

- Pulsed interactions (duration $\tau \ll \omega_m$): *(Vanner, et al., PNAS 108, 16182 (2011))*

$$\hat{H} \approx -\hbar g_0 \hat{n}_L \hat{X}_m$$

- Harmonic evolution: $\hat{X}_m(t) = \hat{X}_m \cos(\omega_m t) - \hat{P}_m \sin(\omega_m t)$



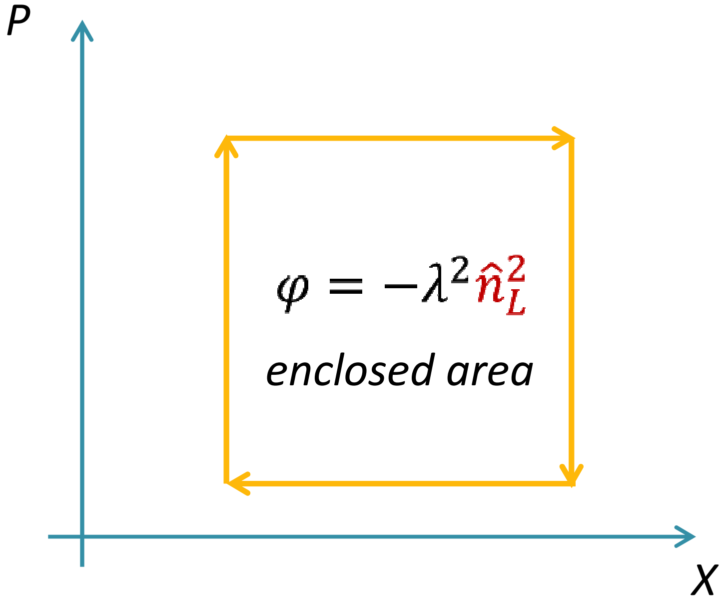
Loop in a phase space

Displacements of a quantum system around a loop in phase space via an ancillary (light) system:

$$\hat{\xi} = e^{i\lambda\hat{n}_L\hat{P}}\, e^{-i\lambda\hat{n}_L\hat{X}}\, e^{-i\lambda\hat{n}_L\hat{P}}\, e^{i\lambda\hat{n}_L\hat{X}}$$
$$= e^{-i\,\lambda^2\hat{n}_L^2}$$

Four pulses separated by $\omega_m t = \pi/2$:

- Resulting phase changes the ancilla, but is state-independent
- Mechanics remains unaffected, and is fully disentangled from the ancilla



Phase due to QG

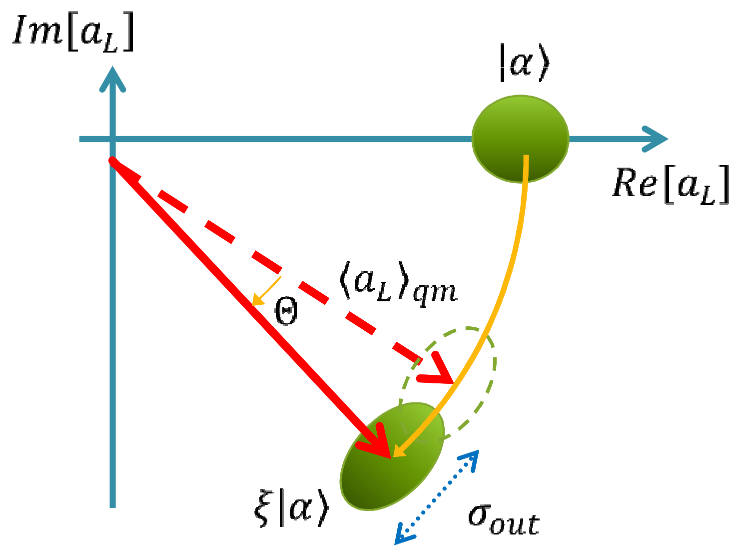
$$\hat{\xi} = e^{i\lambda\hat{n}_L\hat{P}}\, e^{-i\lambda\hat{n}_L\hat{X}}\, e^{-i\lambda\hat{n}_L\hat{P}}\, e^{i\lambda\hat{n}_L\hat{X}} = e^{\sum_{k=1}^{\infty}\frac{(-i\lambda\hat{n}_L)^{k+1}}{k!}[\hat{X},\hat{P}]_k}$$

where $[\hat{X},\hat{P}]_k \equiv \underbrace{[\hat{X},[\hat{X},\ldots,\hat{P}]}_{k}$

- QM: $[\hat{X},\hat{P}] = i \Rightarrow \hat{\xi}_{QM} = e^{-i\lambda^2\hat{n}_L^2}$
- QG: $[\hat{X},\hat{P}] = i\,F(\hat{X},\hat{P})$

Any arbitrary deformed algebra will show in $\hat{\xi}$!

By measuring the ancilla (initially in $|\alpha\rangle$) one can obtain a measure of the commutator.



$$\langle\hat{a}_L\rangle = \langle\alpha|\hat{\xi}^+\hat{a}_L\hat{\xi}|\alpha\rangle \cong \langle\hat{a}_L\rangle_{QM}\, e^{-i\,\Theta([\hat{X},\hat{P}]_{mod})}$$

Example: $\Theta(\beta) \simeq \frac{4}{3}\beta N_p^3\lambda^4\, e^{-i6\lambda^2}$

Table 2 | Experimental parameters to measure quantum gravitational deformations of the canonical commutator.

	$[X_m, P_m]$ $\lvert\Theta\rvert$	Equation (2) $\mu_0 \frac{32\hbar\mathcal{F}^2 m N_p}{M_P^2 \lambda_L^2 \omega_m}$	Equation (3) $\gamma_0 \frac{96\hbar^2\mathcal{F}^3 N_p^2}{M_P c \lambda_L^3 m\omega_m}$	Equation (1) $\beta_0 \frac{1024\hbar^3\mathcal{F}^4 N_p^3}{3M_P^2 c^2 \lambda_L^4 m\omega_m}$
Finess	$\mathcal{F}$	10^5	2×10^5	4×10^5
Mass	m	10^{-11} kg	10^{-9} kg	10^{-7} kg
Mech. Frequency	$\omega_m/2\pi$	10^5 Hz	10^5 Hz	10^5 Hz
Optical wavelength	λ_L	1,064 nm	1,064 nm	532 nm
Photon number	N_p	10^8	5×10^{10}	10^{14}
Measurement runs	N_r	1	10^5	10^6
Measur. precision	$\delta\langle\Phi\rangle$	10^{-4}	10^{-8}	10^{-10}

The parameters are chosen such that a precision of $\delta\mu_0 \sim 1$, $\delta\gamma_0 \sim 1$ and $\delta\beta_0 \sim 1$ can be achieved, which amounts to measuring Planck-scale deformations.

Quantum Information Meets Gravity

Summary

1. New paradigm for tests of genuine general relativistic effects in quantum mechanics:
   - **Drop in the visibility of quantum interference due to gravitational time dilation**
2. Quantum formalism for indefinite causal structures
   - **Quantum correlations with no-causal order**
3. Possibility to probe phenomenological predictions of quantum gravity in massive quantum systems:
   - **Measurement of the deformation of commutation relation of the center-of-mass modes**

Borivoje Dakic

Igor Pikovski

Fabio Costa

Magdalena Zych

Ognyan Oreshkov (U. Brüssels)

C.B.

CoQuS Complex Quantum Systems

FQXi Foundational Questions Institute

John Templeton Foundation

Thank you for your attention

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